THERMODYNAMICS

OF THE

STEAM-ENGINE

AND

OTHER HEAT-ENGINES

BY

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SIXTH EDITION, REVISED AND ENLARGED
TOTAL ISSUE, THIRTEEN THOUSAND

NEW YORK
JOHN WILEY & SONS
LONDON: CHAPMAN & HALL, LIMITED
1911

Stanhope Press
F. H. GILSON COMPANY
BOSTON, U.S.A.

PREFACE TO SIXTH EDITION.

WHEN this work was first in preparation the author had before him the problem of teaching thermodynamics so that students in engineering could use the results immediately in connection with experiments in the Engineering Laboratories of the Massachusetts Institute of Technology. The acceptance of the book by teachers of engineering appears to justify the general plan, which will be adhered to now that the development of engineering and presentation of new physical investigations call for a complete revision.

The author is still of the opinion that the general mathematical presentation due to Clausius and Kelvin is the most satisfactory, and carries with it the ability to read current thermodynamic investigation by physicists and engineers. At the same time it is recognized that recent determinations of the properties of both saturated and superheated steam so far narrow the applications of the general method that there is justification for those who prefer special methods for those applications. To provide for both views of this subject, the general mathematical discussion is presented in a separate chapter, which may be omitted at the first reading (or altogether), provided that the special methods which are also given in the proper places are taken to be sufficient.

Following the method of the first edition the original experimental data, on which the working equations whether logical or empirical must be based, are given with particularity, to afford an idea of the degree of precision to be attributed to calculations made by their aid. The properties of perfect gases and of both saturated and superheated steam are now determined with a satisfactory degree of certainty and precision, so that the physical

constants and the tables required by engineers can be used with confidence, and may be expected to have permanence. The author's "Tables of the Properties of Steam, etc.," were originally computed to accompany the first edition of the work; this present edition is accompanied by a new edition of the "Steam and Entropy Tables" which have been entirely recomputed from new and precise experimental data. The temperature-entropy table will be found to give ready and exact solutions of all adiabatic problems for both saturated and superheated steam, and of many other problems. It allows of the use of certain rapid and refined computations for steam turbines which cannot be so readily determined without it.

There is presented in this book, firstly, a presentation of the fundamental conceptions and processes of thermodynamics; secondly, a statement of the properties and characteristics of gases and of vapors, and their treatment by thermodynamic methods so as to provide solutions of problems that arise in engineering; thirdly, a discussion of steam-engines, internal combustion engines, air-compressors, refrigerating machines, injectors and steam-turbines. As far as possible the work is so arranged that any individual discussion may be read by itself in any order. Discretion can, therefore, be used in teaching, and in reading or rereading the several subjects discussed.

A feature which has been given prominence in the first and in all subsequent editions is the presentation of the results of tests on steam engines and other machines, with the idea of qualifying the student to comprehend the significance of such results, and to judge for himself the performance to be expected from engines and machines under test and in practice. Adequate laboratory experience, such as is now given in all well-equipped engineering schools, is essential for the proper conception of this side of the subject; and conversely, an adequate theoretical training, such as this book aims to give, is essential for efficient laboratory instruction.

Whenever direct quotations have been made, references have been given in footnotes. It does not appear necessary to add

other acknowledgments of assistance from well-known authors, further than to say that their writings have been diligently searched in the preparation of this book, since any text-book must be largely an adaptation of their work to the needs of instruction.

C. H. P.

JUNE, 1909.

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THERMODYNAMICS OF THE STEAM-ENGINE.

CHAPTER I.

THERMAL CAPACITIES.

THE object of thermodynamics, or the mechanical theory of heat, is the solution of problems involving the action of heat, and, for the engineer, more especially those problems presented by the steam-engine and other thermal motors. The substances in which the engineer has the most interest are gases and vapors, more especially air and steam. Fortunately an adequate treatment can be given of these substances for engineering purposes.

First General Principle. — In the development of the theory of thermodynamics it is assumed that if any two characteristics or properties of a substance are known these two, treated as independent variables, will enable us to calculate any third property.

As an example, we have from the combination of the laws of Boyle and Gay-Lussac the general equation for gases,

$$pv = RT,$$

in which p is the pressure, v is the volume, T is the absolute temperature by the air-thermometer, and R is a constant which for air has the value 53.35 when English units are used. It is probable that this equation led to the general assumption just quoted. That assumption is purely arbitrary, and is to be justified by its results. It may properly be considered to be the first general principle of the theory of thermodynamics; the other two general principles are the so-called first and second laws of thermodynamics, which will be stated and discussed later.

Characteristic Equation. — An equation which gives the relations of the properties of any substance is called the characteristic equation for that substance. The properties appearing in a characteristic equation are commonly pressure, volume, and temperature, but other properties may be used if convenient. The form of the equation must be determined from experiments, either directly or indirectly.

The characteristic equation for a gas is, as already quoted,

$$pv = RT.$$

The characteristic equation for an imperfect gas, like superheated steam, is likely to be more complex; for example, the equation given by Knoblauch, Linde, and Klebe is

$$pv = BT - p(1 + ap)\left[C\left(\frac{373}{T}\right)^3 - D\right].$$

On the other hand, the properties of saturated steam, especially if mixed with water, cannot be represented by a single equation.

Specific Pressure. — The pressure is assumed to be a hydrostatic pressure, such as a fluid exerts on the sides of the containing vessel or on an immersed body. The pressure is consequently the pressure exerted *by* the substance under consideration rather than the pressure *on* that substance. For example, in the cylinder of a steam-engine the pressure of the steam is exerted on the piston during the forward stroke and does work on the piston; during the return stroke, when the steam is expelled from the cylinder, it still exerts pressure on the piston and abstracts work from it.

For the purposes of the general theory pressures are expressed in terms of pounds on the square foot for the English system of units. In the metric system the pressure is expressed in terms of kilograms on the square metre. A pressure thus expressed is called the *specific pressure*. In engineering practice other terms are used, such as pounds on the square inch, inches of mercury, millimetres of mercury, atmospheres, or kilograms on the square centimetre.

Specific Volume. — It is convenient to deal with one unit of weight of the substance under discussion, and to consider the volume occupied by one pound or one kilogram of the substance; this is called the *specific volume*, and is expressed in cubic feet or in cubic metres. The specific volume of air at freezing-point and under the normal atmospheric pressure is 12.39 cubic feet; the specific volume of saturated steam at 212° F. is 26.6 cubic feet; and the specific volume of water is about $\frac{1}{62.4}$, or nearly 0.016 of a cubic foot.

Temperature is commonly measured by aid of a mercurial thermometer which has for its reference-points the freezing-point and boiling-point of water. A centigrade thermometer has the volume of the stem between the reference-points divided into one hundred equal parts called degrees. The Fahrenheit thermometer differs from the centigrade in having one hundred and eighty degrees between the freezing-point and the boiling-point, and in having its zero thirty-two degrees below freezing.

The scale of a mercurial thermometer is entirely arbitrary, and its indications depend on the relative expansion of glass and mercury. Indications of such thermometers, however carefully made, differ appreciably, mainly on account of the varying nature of the glass. For refined investigations thermometric readings are reduced to the air-thermometer, which has the advantage that the expansion of air is so large compared with the expansion of glass that the latter has little or no effect.

It is convenient in making calculations of the properties of air to refer temperatures to the absolute zero of the scale of the air-thermometer. To get a conception of what is meant by this expression we may imagine the air-thermometer to be made of a uniform glass tube with a proper index to show the volume of the air. The position of the index may be marked at boiling-point and at freezing-point as on the mercurial thermometer, and the space between may be divided into one hundred parts or degrees. If the graduations are continued to the closed end of the tube there will be found to be 273 of them. It will be

shown later that there is reason to suppose that the absolutc zero of temperature is 273° centigrade below the freezing-point of water. Speculations as to the meaning of absolute zero and discussions concerning the nature of substances at that temperature are not now profitable. It is sufficient to know that equations are simplified and calculations are facilitated by this device. For example, if temperature is reckoned from the arbitrary zero of the centigrade thermometer, then the characteristic equation for a perfect gas becomes

$$pv = \left(\frac{1}{\alpha} + t\right) R,$$

in which α is the coefficient of dilatation and $\frac{1}{\alpha} = 273$ nearly.

In order to distinguish the absolute temperature from the temperature by the thermometcr we shall designate the former by T and the latter by t, bearing in mind that

$$T = t + 273^\circ \text{ centigrade,}$$
$$T = t + 459.5 \text{ Fahrenheit.}$$

Physicists give great weight to the discussion of a scale of temperature that can be connected with the fundamental units of length and weight like the foot and the pound. Such a scale, since it does not depend on the properties of any substance (glass, mercury, or air), is considered to be the *absolute scale* of temperature. The differences between such a scale and the scale of the air-thermometer are very small, and are difficult to determine, and for the engineer are of little moment. At the proper place the conception of the absolute scale can be easily stated.

Graphical Representation of the Characteristic Equation. — Any equation with three variables may be represented by a geometrical surface referred to co-ordinate axes, of which surface the variables are the co-ordinates. In the case of a perfect gas which conforms to the equation

$$pv = RT,$$

the surface is such that each section perpendicular to the axis of T is a rectangular hyperbola (Fig. 1).

Returning now to the general case, it is apparent that the characteristic equation of any substance may be represented by a geometrical surface referred to co-ordinate axes, since the equation is assumed to contain only three variables; but the surface will in general be less simple in form than that representing the combined laws of Boyle and Gay-Lussac.

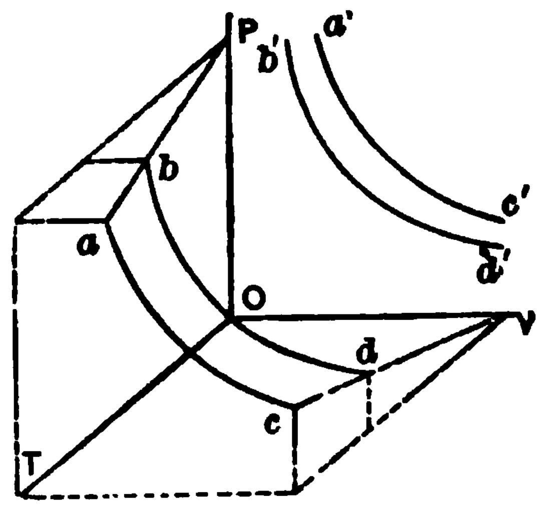

FIG. 1.

If one of the variables, as T, is given a special constant value, it is equivalent to taking a section perpendicular to the axis of T; and a plane curve will be cut from the surface, which may be conveniently projected on the (p, v) plane. The reason for choosing the (p, v) plane is that the curves correspond with those drawn by the steam-engine indicator.

Considerable use is made of such thermal curves in explaining thermodynamic conceptions. As a rule, a graphical process or representation is merely another way of presenting an idea that has been, or may be, presented analytically; there is, however, an advantage in representing a condition or a change to the eye by a diagram, especially in a discussion which appears to be abstract. A number of thermal curves are explained on page 16.

Standard Temperature. — For many purposes it is convenient to take the freezing-point of water for the standard temperature, since it is one of the reference-points on the thermometric scale; this is especially true for air. But the properties of water change rapidly at and near freezing-point and are very imperfectly known. It has consequently become customary to take 62° F. for the standard temperature for the English system of units; there is a convenience in this, inasmuch as the pound and yard are standards at that temperature. For the metric system 15° C. is used, though the kilogram and metre are standards at freezing-point.

Thermal Unit. — Heat is measured in calories or in British thermal units (B. T. U.). A British thermal unit is the heat required to raise one pound of water from 62° F. to 63° F.; in like manner a calorie is the heat required to raise one kilogram of water from 15° C. to 16° C.

Specific Heat is the number of thermal units required to raise a unit of weight of a given substance one degree of temperature. The specific heat of water at the standard temperature is, of course, unity.

If the specific heat of a given substance is constant, then the heat required to raise one pound through a given range of temperature is the product of the specific heat by the increase of temperature. Thus if c is the specific heat and $t - t_1$ is the range of temperature the heat required is

$$Q = c\,(t - t_1), \text{ and } c = \frac{Q}{t - t_1}.$$

If the specific heat varies the amount of heat must be obtained by integration — that is,

$$Q = \int c\,dt,$$

and conversely

$$c = \frac{dQ}{dt}.$$

It is customary to distinguish two specific heats for perfect gases; specific heat at constant pressure and specific heat at constant volume, which may be represented by

$$c_p = \left(\frac{\delta Q}{\delta t}\right)_p \text{ and } c_v = \left(\frac{\delta Q}{\delta t}\right)_v;$$

the subscript attached to the parenthesis indicates the property which is constant during the change. It is evident that the specific heats just expressed are partial differential coefficients.

Latent Heat of Expansion is the amount of heat required to increase the volume of a unit of weight of the substance by one

cubic foot, or one cubic metre, at constant temperature. It may be represented by

$$l = \left(\frac{\delta Q}{\delta v}\right)_t.$$

Thermal Capacities. — The two specific heats and the latent heat of expansion are known as thermal capacities. It is customary to use three other properties suggested by those just named which are represented as follows:

$$m = \left(\frac{\delta Q}{\delta p}\right)_t; \quad n = \left(\frac{\delta Q}{\delta p}\right)_v \text{ and } o = \left(\frac{\delta Q}{\delta v}\right)_p.$$

The first represents the amount of heat that must be applied to one pound of a substance (such as air) to increase the pressure by the amount of one pound per square foot at constant temperature; this property is usually negative and represents the heat that must be abstracted to prevent the temperature from rising. The other two can be defined in like manner if desired, but it is not very important to state the definitions nor to try to gain a conception as to what they mean, as it is easy to express them in terms of the first three, for which the conceptions are not difficult. They have no names assigned to them, which is, on the whole, fortunate, as, of the first three, two have names that have no real significance, and the third is a misnomer.

General Equations of the Effects Produced by Heat. — In order to be able to compute the amount of heat required to produce a change in a substance by aid of the characteristic equation, it is necessary to admit that there is a functional relation between the heat applied and some two of the properties that enter into the characteristic equation. It will appear later in connection with the discussion of the first law of thermodynamics that an integral equation cannot in general be written directly, but we may write a differential equation in one of the three following forms:

$$dQ = \left(\frac{\delta Q}{\delta t}\right)_v dt + \left(\frac{\delta Q}{\delta v}\right)_t dv,$$

$$dQ = \left(\frac{\delta Q}{\delta t}\right)_p dt + \left(\frac{\delta Q}{\delta p}\right)_t dp,$$

$$dQ = \left(\frac{\delta Q}{\delta p}\right)_v dp + \left(\frac{\delta Q}{\delta v}\right)_p dv,$$

or substituting for the partial differential coefficients the letters which have been selected to represent them,

$$dQ = c_v dt + l dv \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$dQ = c_p dt + m dp \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$dQ = n dp + o dv \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

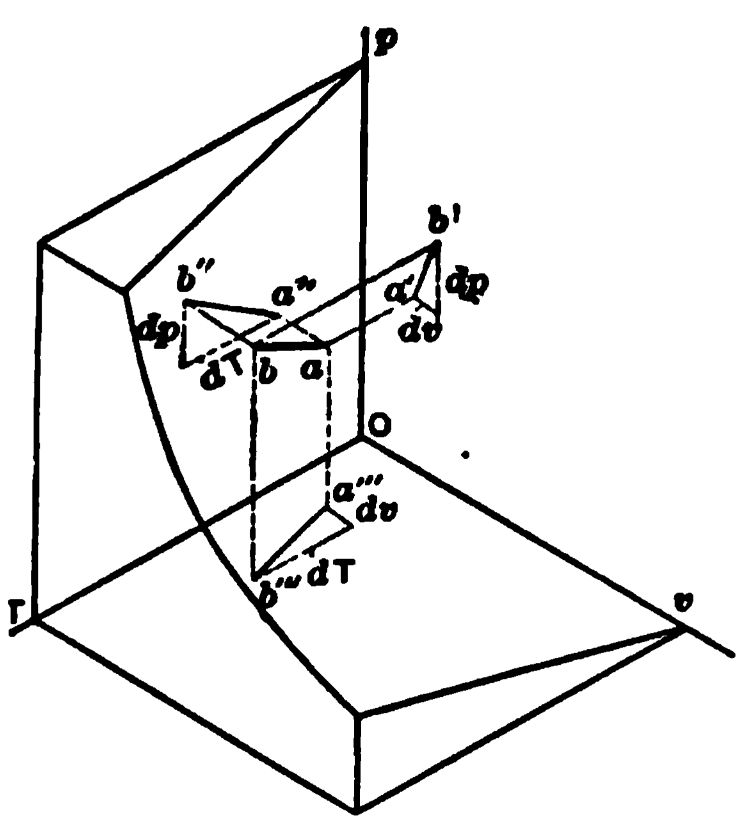

Fig. 2.

This matter may perhaps be clearer if it is presented graphically as in Fig. 2, where ab is intended to represent the path of a point on the characteristic surface in consequence of the addition of the heat dQ. There will in general be a change of temperature volume and pressure as indicated on the figure.

Now the path ab, which for a small change may be considered to be a straight line, will be projected on the three planes at $a'b'$, $a''b''$ and $a'''b'''$. The projection on the (v,T) plane may be resolved into the components δv and δT; the first represents a change of volume at constant temperature requiring the heat ldv, and the second represents a change of temperature at constant volume requiring the heat $c_p dt$. Consequently the heat required for the change in terms of the volume and temperature is

$$dQ = c_v dt + l dv.$$

Relations of the Thermal Capacities. — The three equations (1), (2), and (3), show the changes produced by the addition of an amount of heat dQ to a unit of weight of a substance, the difference coming from the methods of analyzing the changes. We may conveniently find the relations of the several thermal capacities by the method of undetermined coefficients. Thus equating the right-hand members of equations (1) and (2),

$$c_v dt + l dv = c_p dt + m dp \quad . \quad . \quad . \quad . \quad . \quad (4)$$

From the characteristic equation we shall have in general

$$v = F\,(p, T),$$

as, for example, for air we have

$$v = \frac{RT}{p},$$

and consequently we may write

$$dv = \frac{\delta v}{\delta t} dt + \frac{\delta v}{\delta p} dp,$$

which substituted in equation (4) gives,

$$c_p dt + m dp = c_v dt + l\left(\frac{\delta v}{\delta t} dt + \frac{\delta v}{\delta p} dp\right).$$

$$\therefore\ c_p dt + m dp = \left(c_v + l\,\frac{\delta v}{\delta t}\right) dt + l\,\frac{\delta v}{\delta p} dp \quad . \quad . \quad (5)$$

It will be noted that, as T differs from t only by the addition of a constant, the differential dt may be used in all cases, whether we are dealing with absolute temperatures, or temperatures on the ordinary thermometer.

In equation (5) p and T are independent variables, and each may have all possible values; consequently we may equate like coefficients.

$$\therefore\ c_p = c_v + l\,\frac{\delta v}{\delta t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Also, equating the remaining coefficients,

$$l \frac{\delta v}{\delta p} = m \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

If the characteristic equation is solved for the pressure we shall have

$$p = F_1 (T, v),$$

so that

$$dp = \frac{\delta p}{\delta t} dt + \frac{\delta p}{\delta v} dv \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

which substituted in equation (4) gives

$$c_p dt + m \left(\frac{\delta p}{\delta t} dt + \frac{\delta p}{\delta v} dv \right) = c_v dt + l dv.$$

$$\therefore \left(c_p + m \frac{\delta p}{\delta t} \right) dt + m \frac{\delta p}{\delta v} dv = c_v dt + l dv.$$

Equating like coefficients,

$$c_p + m \frac{\delta p}{\delta t} = c_v \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

$$- m \frac{\delta p}{\delta t} = c_p - c_v \quad . \quad . \quad . \quad . \quad . \quad (10)$$

From equations (2) and (3)

$$c_p dt + m dp = n dp + o dv \quad . \quad . \quad . \quad . \quad (11)$$

and from an equation

$$T = F_2 (v, p)$$

$$dt = \frac{\delta t}{\delta v} dv + \frac{\delta t}{\delta p} dp;$$

which latter substituted in equation (11) gives

$$c_p \frac{\delta t}{\delta v} dv + c_p \frac{\delta t}{\delta p} dp + m dp = n dp + o dv.$$

Equating coefficients of dv,

$$o = c_p \frac{\delta t}{\delta v} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

Finally, from equations (1) and (3),

$$c_v dt + l dv = n dp + o dv \quad . \quad . \quad . \quad . \quad . \quad (13)$$

Substituting for dt as above,

$$c_v \frac{\delta t}{\delta v} dv + c_v \frac{\delta t}{\delta p} dp + l dv = n dp + o dv.$$

Equating coefficients of dp,

$$n = c_v \frac{\delta t}{\delta p} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

For convenience the several relations of the thermal capacities may be assembled as follows:

$$l = (c_p - c_v) \frac{\delta t}{\delta v}; \qquad m = -(c_p - c_v) \frac{\delta t}{\delta p}$$

$$n = c_v \frac{\delta t}{\delta p}; \quad o = c_p \frac{\delta t}{\delta v}$$

$$m = l \frac{\delta v}{\delta p}.$$

They are the necessary algebraic relations of the literal functions growing out of the first general principle, and are independent of the scale of temperature, or of any other theoretical or experimental principle of thermodynamics other than the one already stated — namely, that any two properties of a given substance, treated as independent variables, are sufficient to allow us to calculate any third property.

Of the six thermal capacities the specific heat at constant pressure is the only one that is commonly known by direct experiment. For perfect gases this thermal capacity is a constant, and, further, the ratio of the specific heats

$$\frac{c_p}{c_v} = \kappa$$

is a constant, so that c_v is readily calculated. The relations of the thermal capacities allow us to calculate values for the

other thermal capacities, l, m, n, and o, provided that we can first determine the several partial differential coefficients which appear in the proper equations. But for a perfect gas the characteristic equation is

$$pv = RT,$$

from which we have

$$\frac{\delta v}{\delta t} = \frac{R}{p}; \quad \frac{\delta p}{\delta t} = \frac{R}{v};$$

$$\frac{\delta t}{\delta p} = \frac{v}{R}; \quad \frac{\delta t}{\delta v} = \frac{p}{R}.$$

Substituting these values in the equations for the thermal capacities, we have

$$l = \frac{p}{R}(c_p - c_v); \quad -m = \frac{v}{R}(c_p - c_v);$$

$$n = \frac{v}{R}c_v; \quad o = \frac{p}{R}c_p;$$

by aid of which the several thermal capacities may be calculated numerically, or, what is the usual procedure, may be represented in terms of the specific heats.

CHAPTER II.

FIRST LAW OF THERMODYNAMICS.

THE formal statement of the first law of thermodynamics is: *Heat and mechanical energy are mutually convertible, and heat requires for its production and produces by its disappearance a definite number of units of work for each thermal unit.*

This law, which may be considered to be the second general principle of thermodynamics, is the statement of a well-determined physical fact. It is a special statement of the general law of the conservation of energy, i.e., that energy may be transformed from one form to another, but can neither be created nor destroyed. It should be stated, however, that the general law of conservation of energy, though universally accepted, has not been proved by direct experiment in all cases; there may be cases that are not susceptible of so direct a proof as we have for the transformation of heat into work.

The best determinations of the mechanical equivalent of heat were made by Rowland, whose work will be considered in detail in connection with the properties of steam and water. From his work it appears that 778 foot-pounds of work are required to raise one pound of water from 62° to 63° Fahrenheit; this value of the mechanical equivalent of heat is now commonly accepted by engineers, and is verified by the latest determinations by Joule and other experimenters.

The values of the mechanical equivalent of heat for the English system and for the metric system are:

1 B. T. U. = 778 foot-pounds.
1 calorie = 426.9 metre-kilograms.

This physical constant is commonly represented by the letter J; the reciprocal is represented by A.

In older works on thermodynamics the values of J are commonly quoted as 772 for the English system and 424 for the metric system. The error of these values is about one per cent.

Effects of the Transfer of Heat. — Let a quantity of any substance of which the weight is one unit — i.e., one pound or one kilogram — receive a quantity of heat dQ. It will, in general, experience three changes, each requiring an expenditure of energy. They are: (1) The temperature will be raised, and, according to the theory that sensible heat is due to the vibrations of the particles of the body, the kinetic energy will be increased. Let dS represent this change of sensible heat or vibration work expressed in units of work. (2) The mean positions of the particles will be changed; in general the body will expand. Let dI represent the units of work required for this change of internal potential energy, or work of disgregation. (3) The expansion indicated in (2) is generally against an external pressure, and to overcome the same — that is, for the change in external potential energy — there will be required the work dW.

If during the transmission no heat is lost, and if no heat is transformed into other forms of energy, such as sound, electricity, etc., then the first law of thermodynamics gives

$$dQ = A(dS + dI + dW) \quad . \quad . \quad . \quad . \quad . \quad (15)$$

It is to be understood that any or all of the terms of the equation may become zero or may be negative. If all the terms become negative heat is withdrawn instead of added, and dQ is negative. It is not easy to distinguish between the vibration work and the disgregation work, and for many purposes it is unnecessary; consequently they are treated together under the name of intrinsic energy, and we have

$$dQ = A(dS + dI + dW) = A(dE + dW) \quad . \quad . \quad (16)$$

The inner work, or intrinsic energy, depends on the state of the body, and not at all on the manner by which it arrived at

that state; just as the total energy of a falling body, with reference to a given plane consisting of kinetic energy and potential energy, depends on the velocity of the body and the height above the plane, and not on the previous history of the body.

The external work is assumed to be done by a fluid-pressure; consequently

$$dW = pdv \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

$$W = \int_{v}^{v_2} pdv \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

where v_2 and v_1 are the final and initial volumes.

In order to find the value of the integral v in equation (18) it is necessary to know the manner in which the pressure varies with the volume. Since the pressure may vary in different ways, the external work cannot be determined from the initial and final states of the body; consequently the heat required to effect a change from one state to another depends on the manner in which the change is effected.

Assuming the law of the variation of the pressure and volume to be known, we may integrate thus:

$$Q = A\left(E_2 - E_1 + \int_{v_1}^{v_2} pdv\right) \quad . \quad . \quad . \quad . \quad (19)$$

In order to determine E for any state of a body it would be necessary to deprive it entirely of vibration and disgregation energy, which would of course involve reducing it to a state of absolute cold; consequently the direct determination is impossible. However, in all our work the substances operated on are changed from one state to another, and in each state the intrinsic energy depends on the state only; consequently the change of intrinsic energy may be determined from the initial and final states only, without knowing the manner of change from one to the other.

In general, equations will be arranged to involve differences

of energy only, and the hypothesis involved in a separation into vibration and disgregation work avoided.

Thermal Lines. — The external work can be determined only when the relations of p and v are known, or, in general, when the characteristic equation is known. It has already been shown that in such case the equation may be represented by a geometrical surface, on which so-called thermal lines can be drawn representing the properties of the substance under consideration. These lines are commonly projected on the (p, v) plane. It is convenient in many cases to find the relation of p and v under a given condition and represent it by a curve drawn directly on the (p, v) plane.

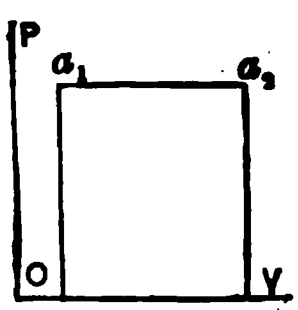

FIG. 3.

Lines of Equal Pressure. — The change of condition takes place at constant pressure, and consists of a change of volume, as represented in Fig. 3. The tracing-point moves from a_1 to a_2, and the volume changes from v_1 to v_2. The work done is represented by the rectangular area under a_1a_2, or by

$$W = p\int_{v_1}^{v_2} dv = p(v_2 - v_1) \quad . \quad . \quad . \quad . \quad (20)$$

During the change the temperature may or may not change; the diagram shows nothing concerning it.

Lines of Equal Volume. — The pressure increases at constant volume, and the tracing-point moves from a_1 to a_2. The temperature usually increases meanwhile. Since dv is zero,

$$W = \int_{v_1}^{v_2} pdv = 0 \quad . \quad . \quad . \quad . \quad . \quad (21)$$

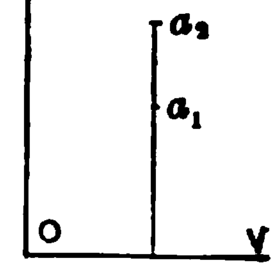

FIG. 4.

Isothermal Lines, or Lines of Equal Temperature. — The temperature remains constant, and a line is drawn, usually convex, toward the axis OV. The pressure of a mixture of a

liquid and its vapor is constant for a given temperature; consequently the isothermal for such a mixture is a line of equal pressure, represented by Fig. 3. The isothermal of a perfect gas, on the other hand, is an equilateral hyperbola, as appears from the law of Boyle, which may be written

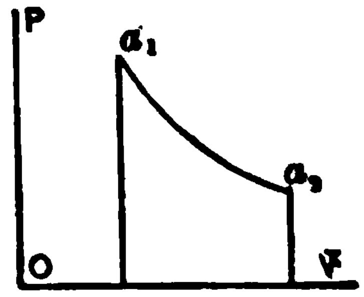

FIG. 5.

$$pv = C.$$

Isodynamic or Isoenergic Lines are lines representing changes during which the intrinsic energy remains constant. Consequently all the heat received is transformed into external work. It will be seen later that the isodynamic and isothermal lines for a gas are the same.

Adiabatic Lines. — A very important problem in thermodynamics is to determine the behavior of a substance when a change of condition takes place in a non-conducting vessel. During the change — for example, an increase of volume or expansion — some of the heat in the substance may be changed into work; but no heat is transferred to or from the substance through the walls of the containing vessel. Such changes are called *adiabatic* changes.

Very rapid changes of dry air in the cylinder of an air-compressor or a compressed-air engine are very nearly adiabatic. Adiabatic changes never occur in the cylinder of a steam-engine on account of the rapidity with which steam is condensed on or vaporized from the cast-iron walls of the cylinder.

Since there is no transmission of heat to (or from) the working substance, equation (19) becomes

$$Q = A(E_2 - E_1 + \int_{v_1}^{v_2} pdv) \quad . \quad . \quad . \quad . \quad (22)$$

$$E_1 - E_2 = \int_{v_1}^{v_2} pdv \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

that is, the external work is done wholly at the expense of the intrinsic energy of the working substance, as must be the case in conformity with the assumption of an adiabatic change.

Relation of Adiabatic and Isothermal Lines. — An important property of adiabatic lines can be shown to advantage at this place, namely, that such a line is steeper than an isothermal line on the (p, v) plane where they cross, as represented in Fig. 6. The essential feature of adiabatic expansion is that no heat is supplied and that consequently the external work of expansion is done at the expense of the intrinsic energy which consequently decreases. The intrinsic energy is the sum of the vibration energy and the disgregation energy, both of which in general decrease during an adiabatic expansion; in particular the decrease of vibration energy means a loss of temperature. Conversely an adiabatic compression is accompanied by an increase of temperature. If an isothermal compression is represented by cb, then an adiabatic compression will be represented by a steeper line like ca, crossing the constant pressure line ba to the right of b, and thus indicating that at that pressure there is a greater volume, as must be the case for a body which expands during a rise of temperature at constant pressure.

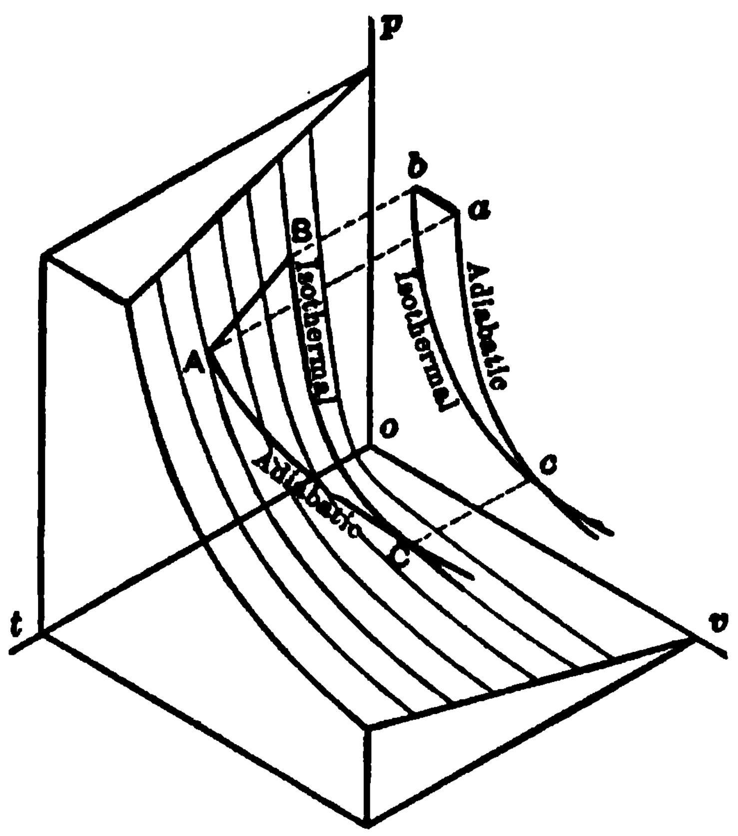

FIG. 6.

It is very instructive to note the relation of these lines on the surface which represents the characteristic equation for a perfect gas. In Fig. 6, which is an isometric projection, the general form of the surface can be recognized from the following conditions: — a horizontal section representing constant pressure cuts the surface in a straight line which indicates that the volume increases proportionally to the absolute temperature, and this line is projected as a horizontal line on the (p, v) plane; a vertical section parallel to the (p, t) plane shows that the pressure in this case increases as the absolute temperature, and the line of intersection with the surface is projected as a vertical line on the

(p, v) plane; finally vertical sections parallel to the (p, v) plane are rectangular hyperbolæ which are projected in their true form on the (p, v) plane. If AC is an adiabatic curve on the characteristic surface, its loss of temperature is properly represented by the fact that it crosses a series of isothermals in passing from A to C; AB is a line of constant pressure showing a decrease of temperature between the isothermals through A and through C; finally the projection of ABC on to the (p, v) plane shows that the adiabatic line ac is steeper than the isothermal line bc. Attention should be called to the fact that the first statement of this relation is the more general as it holds for all substances that expand with rise of temperature at constant pressure whatever may be the form of the characteristic equation.

Thermal Lines and their Projections. — The treatment given of thermal lines is believed to be the simplest and to present the features that are most useful in practice. There is, however, both interest and instruction in considering their relation in space and their projections on the three thermal planes. It is well to look attentively at Fig. 6, which is a correct isometric projection of the characteristic surface of a gas following the law of Boyle and Gay-Lussac, noting that every section by a plane parallel to the (p, v) plane is a rectangular hyperbola which has the same form in space and when projected on the (p, v) plane. The sections by a plane parallel to the (p, t) plane are straight lines and are of course projected as straight lines on that plane and on the (p, v) plane; in like manner the sections by planes parallel to the (t, v) plane are straight lines. The adiabatic line in space and as projected on the (p, v) plane is probably drawn a little too steep, but the divergence from truth is not evident to the eye.

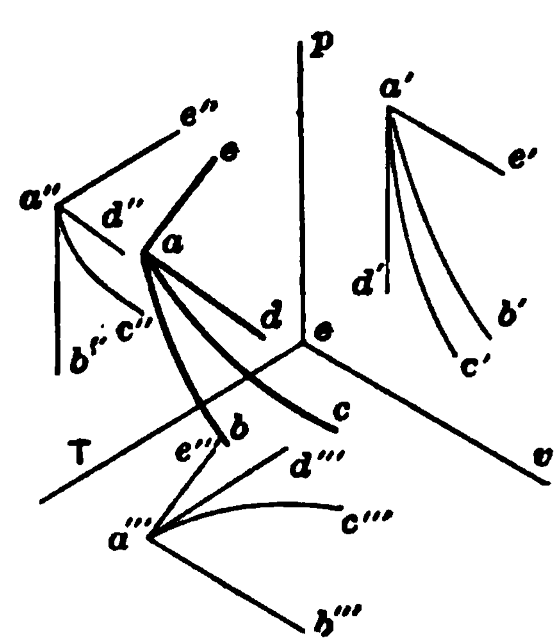

FIG. 7.

In Fig. 7 the same method of projection is used, but other lines are added together with their projections on the several

planes. Beginning at the point a in space the line ab is an isothermal which is projected as a rectangular hyperbola $a'b'$ on the (p, v) plane, and as straight lines $a''b''$ and $a'''b'''$ on the (p, t) and (t, v) plane. The adiabatic line ac is steeper than the isothermal, both in space and on the (p, v) plane, as already explained; it is projected as a curve ($a''c''$ or $a'''c'''$) on the other planes. The section showing constant pressure is represented in space by the straight line ae which projected on the (p, t) plane is parallel to the axis ot, and on the (t, v) plane is parallel to the line itself in space; on the (p, v) plane it is horizontal, as shown in Fig. 3. In much the same way ad is the section by a plane parallel to the (t, p) plane, and $a'd'$, $a''d''$ and $a'''d'''$ are its projections.

Graphical Representations of Change of Intrinsic Energy. — Professor Rankine first used a graphical method of representing a change of intrinsic energy, employing adiabatic lines only, as follows:

Suppose that a substance is originally in the state A (Fig. 8), and that it expands adiabatically; then the external work is done at the expense of the intrinsic energy; hence if the expansion has proceeded to A_1 the area AA_1a_1a, which represents the external work, also represents the change of intrinsic energy. Suppose that the expansion were to continue indefinitely; then the adiabatic will approach the axis OV indefinitely, and the area representing the work will be included between the curve $A\alpha$ produced indefinitely, the ordinate Aa, and the axis OV; this area will represent all the work that can be obtained by the expansion of the substance; and if it be admitted that during the expansion all the intrinsic energy is transformed into work, so that at the end the intrinsic energy is zero, it represents also the intrinsic energy. In cases for which the equation of the adiabatic can be found it is easy to show that

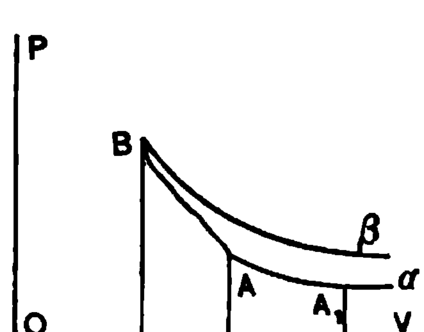

FIG. 8.

$$E_1 = \int_{v_1}^{\infty} pdv \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

is a finite quantity; and in any case, if we admit an absolute zero of temperature, it is evident that the intrinsic energy cannot be infinite. On the other hand, if an isothermal curve were treated in the same way the area would be infinite, since heat would be continually added during the expansion.

Now suppose the body to pass from the condition represented by A to that represented by B, by any path whatever — that is, by any succession of changes whatever — for example, that represented by the irregular curve AB. The intrinsic energy in the state B is represented by the area $VbB\beta$. The change of intrinsic energy is represented by the area $\beta BbaA\alpha$, and this area does not depend on the form of the curve AB. This graphical process is only another way of saying that the intrinsic energy depends on the state of the substance only, and that change of intrinsic energy depends on the final and initial states only.

Another way of representing change of intrinsic energy by aid of isodynamic lines avoids an infinite diagram. Suppose the change of state to be represented by the curve AB (Fig. 9). Draw an isodynamic line AC through the point A, and an adiabatic line BC through B, intersecting at C; in general the isoenergic line is distinct from the isothermal line; for example, the isothermal line for a saturated vapor is a straight line parallel to the OV axis, and the isoenergic line is represented approximately by the equation

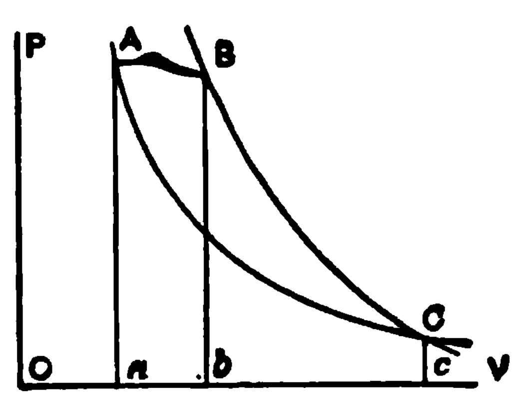

FIG. 9.

$$pv^{1.0456} = \text{const.}$$

Then the area $ABba$ represents the external work, and the area $bBCc$ represents the change of intrinsic energy; for if the body be allowed to expand adiabatically till the intrinsic energy is reduced to its original amount at the condition represented by A the external work $bBCc$ will be done at the expense of the intrinsic energy.

CHAPTER III.

SECOND LAW OF THERMODYNAMICS.

Heat-engines are engines by which heat is transformed into work. All actual engines used as motors go through continuous cycles of operations, which periodically return things to the original conditions. All heat-engines are similar in that they receive heat from some *source*, transform part of it into work, and deliver the remainder (minus certain losses) to a *refrigerator*.

The source and refrigerator of a condensing steam-engine are the furnace and the condenser. The boiler is properly considered as a part of the engine, and receives heat from the source.

Carnot's Engine. — It is convenient to discuss a simple ideal engine, first described by Carnot.

Let P of Fig. 10 represent a cylinder with non-conducting walls, in which is fitted a piston, also of non-conducting material, and moving without friction; on the other hand, the bottom of the cylinder is supposed to be of a material that is a perfect conductor. There is a non-conducting stand C on which the cylinder can be placed while adiabatic changes take place. The source of heat A at a temperature t is supposed to be so maintained that in operations during which the cylinder is placed on it, and draws heat from it, the temperature is unchanged. The refrigerator B at the temperature t_1 in like manner can withdraw heat from the cylinder, when it is placed on it, at a constant temperature.

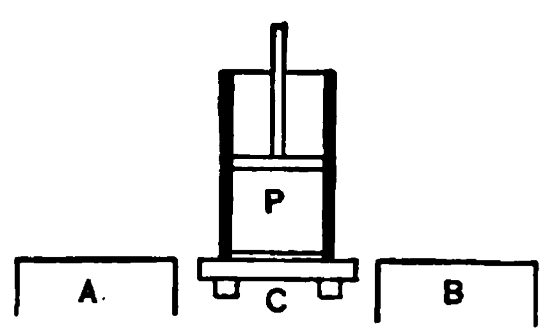

FIG. 10.

Let there be a unit of weight (for example, one pound) of a certain substance in the cylinder at the temperature t of the source of heat. Place the cylinder on the source of heat A

(Fig. 10), and let the substance expand at the constant temperature t, receiving heat from the source A.

If the first condition of the substance be represented by A (Fig. 11), then the second will be represented by B, and AB will be an isothermal. If E_a and E_b are the intrinsic energies at A and B, and if W_{ab}, represented by the area $aABb$, be the external work, the heat received from A will be

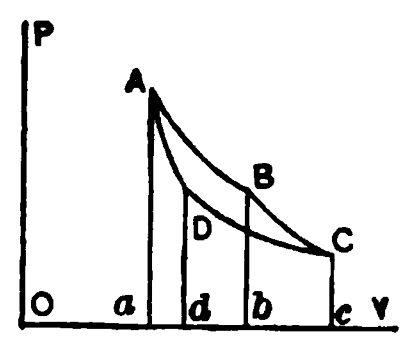

FIG. 11.

$$Q = A\ (E_b - E_a + W_{ab}) \quad . \quad . \quad . \quad . \quad . \quad (25)$$

Now place the cylinder on the stand C (Fig. 10), and let the substance expand adiabatically until the temperature is reduced to t_1, that of the refrigerator, the change being represented by the adiabatic BC (Fig. 11). If E_c is the intrinsic energy at C, then, since no heat passes into or out of the cylinder,

$$0 = A\ (E_c - E_b + W_{bc}) \quad . \quad . \quad . \quad . \quad . \quad (26)$$

where W_{bc} is the external work represented by the area $bBCc$. Place the cylinder on the refrigerator B, and compress the substance till it passes through the change represented by CD, yielding heat to the refrigerator so that the temperature remains constant. If E_d is the intrinsic energy at D, then

$$- Q_1 = A\ (E_d - E_c - W_{cd}) \quad . \quad . \quad . \quad . \quad (27)$$

is the heat yielded to the refrigerator, and W_{cd}, represented by the area $cCDd$, is the external work, which has a minus sign, since it is done on the substance.

The point D is determined by drawing an adiabatic from A to intersect an isothermal through C. The process is completed by compressing the substance while the cylinder is on the stand C (Fig. 10) till the temperature rises to t, the change being represented by the adiabatic DA. Since there is no transfer of heat,

$$0 = A\ (E_a - E_d - W_{da}) \quad . \quad . \quad . \quad . \quad . \quad (28)$$

Adding together the several equations, member to member,

$$Q - Q_1 = A\ (W_{ab} + W_{bc} - W_{cd} - W_{da}) \quad . \quad . \ (29)$$

or, if W be the resulting work represented by the area $ABCD$, then

$$Q - Q_1 = AW \quad . \quad . \quad . \quad . \quad . \quad . \quad . \ (30)$$

that is, the difference between the heat received and the heat delivered to the refrigerator is the heat transformed into work.

A Reversible Engine is one that may run either in the usual manner, transforming heat into work, or reversed, describing the same cycle in the opposite direction, and transforming work into heat.

A Reversible Cycle is the cycle of a reversible engine.

Carnot's engine is reversible, the reversed cycle being $ADCBA$ (Fig. 11), during which work is done by the engine on the working substance. The engine then draws from the refrigerator a certain quantity of heat, it transforms a certain quantity of work into heat, and delivers the sum of both to the source of heat.

No actual heat-engine is reversible in the sense just stated, for when the order of operations can be reversed, changing the engine from a motor into a pump or compressor, the reversed cycle differs from the direct cycle. For example, the valve-gear of a locomotive may be reversed while the train is running, and then the cylinders will draw gases from the smoke-box, compress them, and force them into the boiler. The locomotive as ordinarily built is seldom reversed in this way, as the hot gases from the smoke-box injure the surfaces of the valves and cylinders. Some locomotives have been arranged so that the exhaust-nozzles can be shut off and steam and water supplied to the exhaust-pipe, thus avoiding the damage from hot gases when the engine is reversed in this way. Such an engine may then have a reversed cycle, drawing steam into the cylinders, compressing and forcing it into the boiler; but in any case the

reversed cycle differs from the direct cycle, and the engine is not properly a reversible engine.

A Closed Cycle is any cycle in which the final state is the same as the initial state. Fig. 12 represents such a cycle made up of four curves of any nature whatever. If the four curves are of two species only, as in the diagram representing the cycle of Carnot's engine, the cycle is said to be simple.

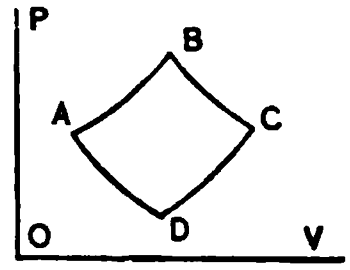

FIG. 12.

In general we shall have for a cycle like that of Fig. 12,

$$Q_{ab} + Q_{bc} - Q_{cd} - Q_{da} = \Sigma Q = A\Sigma W$$
$$= A\ (W^{a}{}_{b} + W_{bc} - W_{cd} - W_{da}).$$

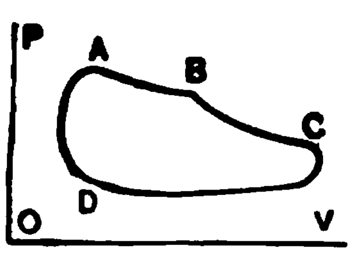

FIG. 13.

A closed curve of any form may be considered to be the general form of a closed cycle, as that in Fig. 13. For such a cycle we have $\int dQ = A\int dW$, which is one more way of stating the first law of thermodynamics.

It may make this last clearer to consider the cycle of Fig. 14 composed of the isothermals AB, CD, and EG, and the adiabatics BC, DE, and GA. The cycle may be divided by drawing the curve through from C to F. It is indifferent whether the path followed be $ABCDEGA$ or $ABCFCDEGA$, or, again, $ABCFGA + CDEFC$.

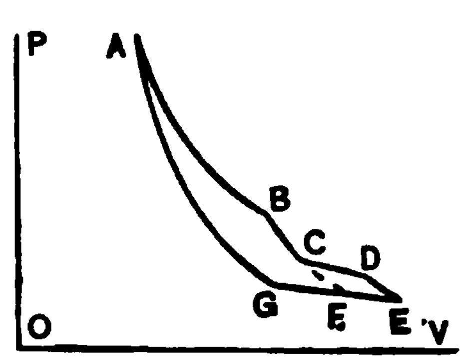

FIG. 14.

Again, an irregular figure may be imagined to be cut into elementary areas by isothermals and adiabatic lines, as in Fig. 15. The summation of the areas will give the entire area, and the summation of the works represented by these will give the entire work represented by the entire area.

The Efficiency of an engine is the ratio of the heat changed into work to the entire heat applied; so that if it be represented by e,

$$e = \frac{AW}{Q} = \frac{Q - Q'}{Q} \quad \ldots \ldots \ldots \quad (31)$$

for the heat Q' rejected to the refrigerator is what is left after AW thermal units have been changed into work.

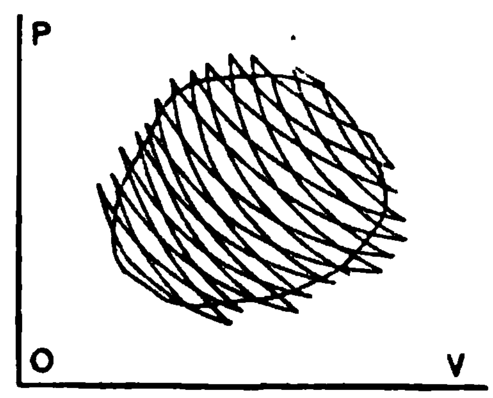

FIG. 15.

Carnot's Principle. — It was first pointed out by Carnot that the efficiency of a reversible engine does not depend on the nature of the working substance, but that it depends on the temperatures of the source of heat and the refrigerator.

Let us see what would be the consequence if this principle were not true. Suppose there are two reversible engines R and A, each taking Q thermal units per second from the source of heat, of which A is the more efficient, so that

$$\frac{AW_a}{Q} = \frac{Q - Q_a'}{Q} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

is larger than

$$\frac{AW_r}{Q} = \frac{Q - Q_r'}{Q} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

this can happen only because Q_a' is less than Q_r', for Q is assumed to be the same for each engine. Let the engine R be reversed and coupled to A, which can run it and still have left the useful work $W_a - W_r$. This useful work cannot come from the source of heat, for the engine R when reversed gives to the source Q thermal units per second, and A takes the same amount in the same time. It must be assumed to come from the refrigerator, which receives Q_a' thermal units per second, and gives up Q_r' thermal units per second, so that it loses

$$Q_r' - Q_a' = A\ (W_a - W_r)$$

thermal units per second. This equation may be derived from equations (32) and (33) by subtraction.

Now it cannot be proved by direct experiment that such an action as that just described is impossible. Again, the first law of thermodynamics is scrupulously regarded, and there is no

contradiction or formal absurdity of statement. And yet when the consequences of the negation of Carnot's principles are clearly set forth they are naturally rejected as improbable, if not impossible. The justification of the principle is found in the fact that theoretical deductions from it are confirmed by experiments.

Second Law of Thermodynamics. — The formal statement of Carnot's principle is known as the second law of thermodynamics. Various forms are given by different investigators, none of which are entirely satisfactory, for the conception is not simple, as is that of the first law.

The following are some of the statements of the second law:

(1) *All reversible engines working between the same source of heat and refrigerator have the same efficiency.*

(2) *The efficiency of a reversible engine is independent of the working substance.*

(3) *A self-acting machine cannot convey heat from one body to another at a higher temperature.*

The second law is the third general principle of thermodynamics; it differs from each of the others and is independent of them. Summing up briefly, the first general principle is a pure assumption that thermodynamic equations may contain only two independent variables; the second is the statement of an experimental fact; the third is a choice of one of two propositions of a dilemma. The first and third are justified by the results of the applications of the theory of thermodynamics.

So far as efficiency is concerned, the second law of thermodynamics shows that it would be a matter of indifference what working substance should be chosen; we might use air or steam in the same engine and get the same efficiency from either; there would, however, be a great difference in the power that would be obtained. In order to obtain a diagram of convenient size and distinctness, the adiabatics are made much steeper than the isothermals in Fig. 11; as a matter of fact the diagram drawn correctly is so long and attenuated that it would be practically

worthless even if it could be obtained with reasonable approximation in practice, as the work of the cycle would hardly overcome the friction of the engine. The isothermals for a mixture of water and steam are horizontal, and the diagram takes the form shown by Fig. 16. In practice a diagram closely resembling Carnot's cycle is chosen as the ideal, differing mainly in that steam is assumed to be supplied and exhausted. In a particular case an engine working between the temperatures 362°.2 F. and 158° F. had an actual thermal efficiency of 0.18; the ideal cycle had an efficiency of 0.23, and Carnot's cycle had an efficiency of 0.25. The ratio of 0.18 to 0.23 is about 0.81, which compares favorably with the efficiency of turbine waterwheels.

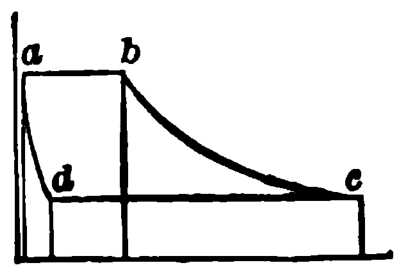

FIG. 16.

Carnot's Function. — Carnot's principle asserts that the efficiency of a reversible engine is independent of the nature of the working substance; consequently the expression for the efficiency will not include such properties of the working substance as specific volume and specific pressure. But the principle asserts also that the efficiency depends on the temperatures of the source of heat and the refrigerator, which indeed are the only properties of the source and refrigerator that can affect the working of the engine.

We may then represent the efficiency as a function of the temperatures of the source of heat and the refrigerator, or, what amounts to the same thing, as a function of the superior temperature and the difference of the temperatures, and may write

$$e = \frac{AW}{Q} = \frac{Q - Q'}{Q} = F\,(t, t - t')$$

where Q is the heat received, Q' the heat rejected, and t and t' are the temperatures of the source of heat and of the refrigerator on any scale whatsoever, absolute or relative.

If the temperature of the refrigerator approaches near that of

the source of heat $Q - Q'$ and $t - t'$ become ΔQ and Δt, and at the limit dQ and dt, so that

$$\frac{dQ}{Q} = F\,(t, dt) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (34)$$

It is convenient to assume that the equation can be expressed in the form

$$\frac{dQ}{Q} = f\,(t)\,dt.$$

The function $f\,(t)$ is known as Carnot's function, and physicists consider that the isolation of this function and the relation of the function to temperature are of great theoretical importance.

Absolute Scale of Temperature. — It is convenient and customary to assign to Carnot's function the form $\frac{1}{T}$, where T is the temperature by the absolute scale referred to on page 3, measured from the absolute zero of temperature. This assumption is justified by the facts that the theory of thermodynamics is much simplified thereby, and that the difference between such a scale of temperature and the scale of the air-thermometer is very small.

Kelvin's Graphical Method. — This treatment of Carnot's function was first proposed by Lord Kelvin, who illustrated the general conception by the following graphical construction:

In Fig. 17 let ak and bi be two adiabatic lines, and let the substance have its condition represented by the point a. Through a and d draw isothermal lines; then the diagram $abcd$ represents the cycle of a simple reversible engine. Draw the isothermal line fe, so that the area $dcef$ shall be equal to $abcd$; then the diagram $dcef$

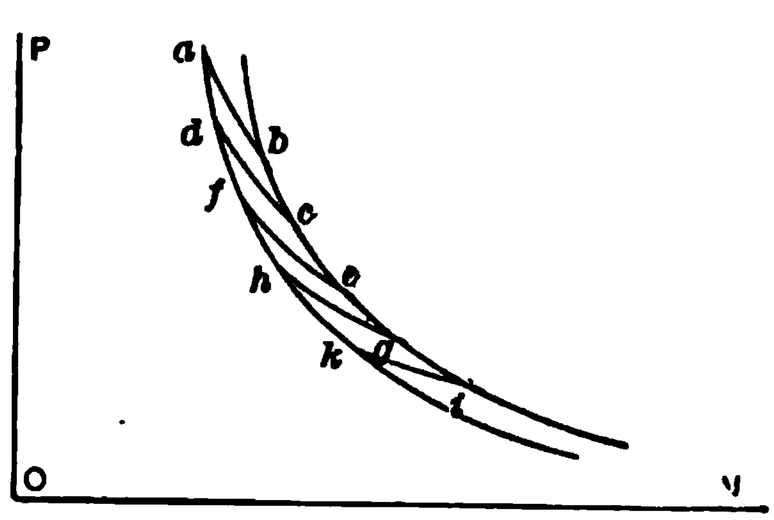

FIG. 17.

represents the cycle of a reversible engine, doing the same amount of work per stroke as that engine whose cycle is repre-

sented by *abcd*; and the difference between the heat drawn from the source and delivered to the refrigerator — i.e., the heat transformed into work — is the same. The refrigerator of the first engine might serve for the source of heat for the second.

Suppose that a series of equal areas are cut off by isothermal lines, as *fegh*, *hgik*, etc., and suppose there are a series of reversible engines corresponding; then there will be a series of sources of heat of determinate temperatures, which may be chosen to establish a thermometric scale. In order to have the scale correspond with those of ordinary thermometers, one of the sources of heat must be at the temperature of boiling water, and one at that of melting ice; and for the centigrade scale there will be one hundred, and for the Fahrenheit scale one hundred and eighty, such cycles, with the appropriate sources of heat, between boiling-point and freezing-point. To establish the absolute zero of the scale the series must be imagined to be continued till the area included between an isothermal and the two adiabatics, continued indefinitely, shall not be greater than one of the equal areas.

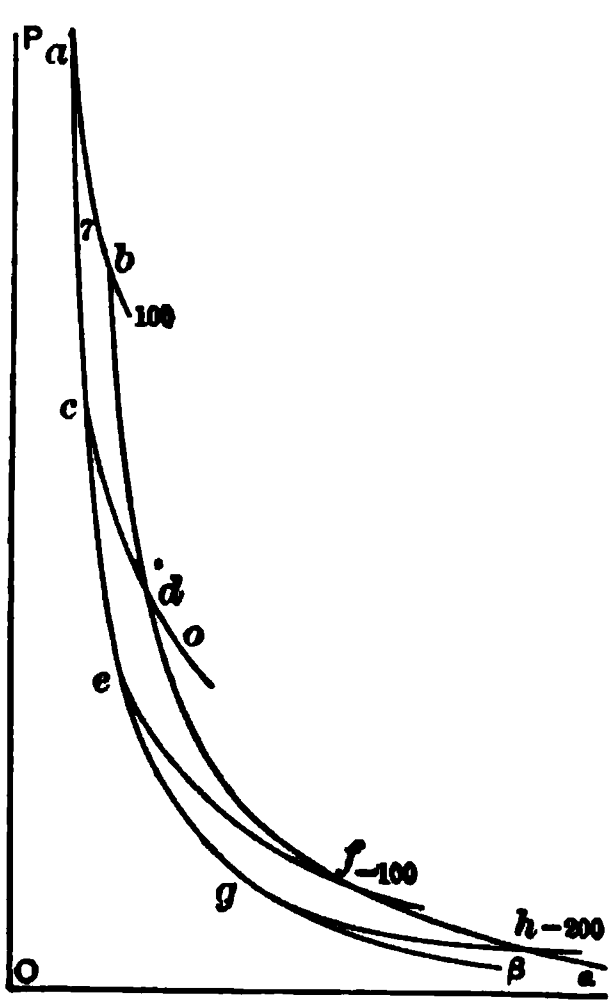

Fig. 18.

This conception of the absolute zero may be made clearer by taking wide intervals of temperature, as on Fig. 18, where the cycle *abcd* is assumed to extend between the isothermals of 0° and 100° C.; that is, from freezing-point to boiling-point. The next cycle, *cdef*, extends to — 100° C., and the third cycle, *efgh*, extends to — 200° C. The remaining area, which is of infinite length and extremely attenuated, is bounded by the isothermal *gh* and the two adiabatics $h\alpha$ and $g\beta$. The diagram of course cannot be completed, and consequently the area cannot be measured; but when the equations to the isothermal and the adiabatics are known it can be computed. So computed, the area is found

to be $\frac{73}{100}$ of one of the three equal areas *abcd*, *cdfe*, and *efhg*. The absolute zero is consequently 273° C. below freezing-point. Further discussion of the absolute scale will be deferred till a comparison is made with the air-thermometer.

Spacing of Adiabatics. — Kelvin's graphical scale of temperature is clearly a method of spacing isothermals which depends only on our conceptions of thermodynamics and on the fundamental units of weight and length. Evidently the same method may be applied to spacing adiabatics, and thereby a new conception of great importance may be introduced into the theory of thermodynamics. On this conception is based the method for solving problems involving adiabatic expansion of steam, as will be explained in the discussion of that subject.

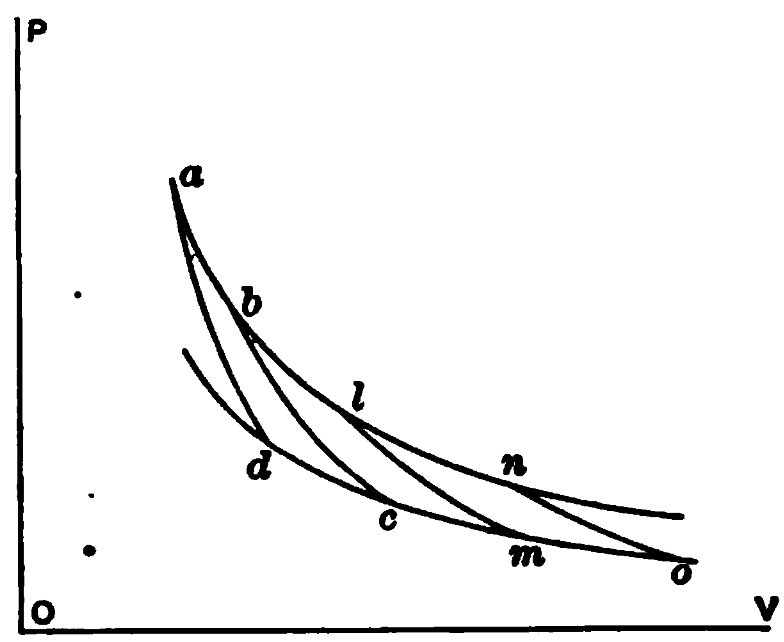

FIG. 19.

In Fig. 19 let *an* and *do* be two isothermals, and let *ad*, *bc*, *lm* and *no* be a series of adiabatics, so drawn that the areas of the figures *abcd*, *blmc*, and *lnom* are equal; then we have a series of adiabatics that are spaced in the same manner as are the isothermals in Figs. 17 and 18, and, as with those isothermals, the spacing depends only on our conceptions of thermodynamics and the fundamental units of weight and length.

In the discussion of Figs. 17 and 18 it was shown that the area of the strip between the initial isothermal *ab* and the two adiabatic lines must be treated as finite, and that in consequence the graphical process leads to an absolute zero of temperature. On the contrary, the area between the adiabatic *ad* and the two isothermals *an* and *do* if extended infinitely will be infinite, and it will be found that there is no limit to the number of adiabatics that can be drawn with the spacing indicated. A like result will follow if the isothermals are extended to the right and

upward, and if adiabatics are spaced off in the same manner. This conclusion comes from the fact pointed out on page 21, that the area under an isothermal curve which is extended without limit is infinite, because heat is continuously supplied, some part of which can be changed into work.

It is convenient to introduce a new function at this place which shall express the spacing of adiabatics as represented in Fig. 19, and which will be called entropy. From what precedes it is evident that entropy has the same relations to the adiabatics of Fig. 19 that temperature has to the isothermals of Figs. 17 and 18, but that there is this radical difference, that while there is a natural absolute zero of temperature, there is no zero of entropy. Consequently in problems we shall always deal with differences of entropy, and if we find it convenient to treat the entropy of a certain condition of a given substance as a zero point it is only that we may count up and down from that point.

If the adiabatic line *ad* in Fig. 19 should be extended to the right, it would clearly lie beneath the adiabatic *no*, which agrees with the tacit convention of that figure, i.e., that as spaced the adiabatics are to be numbered toward the right and that the entropy increases from *a* toward *n*.

The simplest and the most natural definition of entropy from the present considerations, is that entropy is that function which remains constant for any change represented by a reversible adiabatic expansion (or compression). With this definition in view, the adiabatic lines might be called isoentropic lines. It should be borne in mind that our present discussion is purposely limited to expansion in a non-conducting cylinder closed by a piston, or to like operations. More complex operations than that just mentioned may require an extension of the conception of entropy and lead to fuller definitions. Such extensions of the conception of entropy have been found very fruitful in certain physical investigations, and many writers on thermodynamics for engineers consider that they get like advantages from them. There is, however, an advantage in limiting the conception of a

new function, however simple that conception may be; and there is an added advantage in being able to return to a simple conception at will.

Efficiency of Reversible Engines. — Returning to equation (34) and replacing Carnot's function $f(t)$ by $\frac{1}{T}$, as agreed, we have for the differential equation of the efficiency of a reversible engine

$$\frac{dQ}{Q} = \frac{dt}{T},$$

or, integrating between limits,

$$\log_e \frac{Q'}{Q} = \log_e \frac{T'}{T}.$$

$$\therefore \frac{Q'}{Q} = \frac{T'}{T},$$

and the efficiency for the cycle becomes

$$\frac{Q - Q'}{Q} = \frac{T - T'}{T}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (35)$$

This result might have been obtained before (or without) the discussion of Kelvin's graphical method, and leads to the same conclusion, that the absolute temperature can be made to depend on the efficiency of Carnot's cycle, and may, therefore, be independent of any thermometric substance. As has already been said, this conception is more important on the physical side than on the engineering side, and its reiteration need not be considered to call for any speculation by the student at this time.

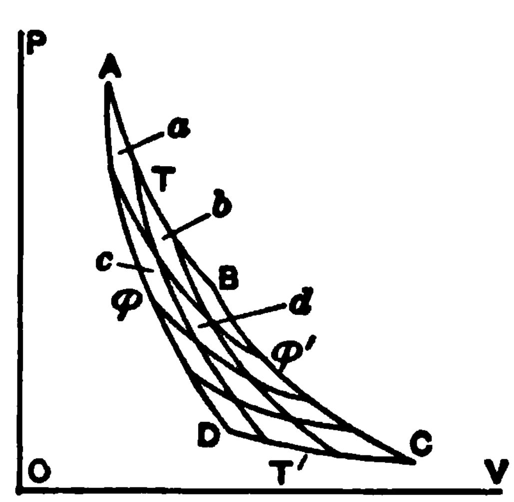

FIG. 20.

Graphical Representation of Efficiency. — Let Fig. 20 represent the cycle of a reversible heat-engine. For convenience it is supposed there are four degrees of temperature from the isothermal AB to the isothermal DC, and that there are three intervals or units of entropy between the adiabatics AD and

BC. First it will be shown that all the small areas into which the cycle is divided by drawing the intervening adiabatics and isothermals are equal. Thus we have to begin with $a = b$ and $a = c$ by construction. But engines working on the cycles a and b have the same efficiency and reject the same amounts of heat. These heats rejected are equal to the heats supplied to engines working on the cycles c and d, which consequently take in the same amounts of heat. But these engines work between the same limits of temperature and have the same efficiency, and consequently change the same amount of heat into work. Therefore the areas c and d are equal. In like manner all the small areas are equal, and each represents one thermal unit, or 778 foot-pounds of work.

It is evident that the heat changed into work is represented by

$$(T - T')\ (\phi' - \phi),$$

and, further, that the same expression would be obtained for a similar diagram, whatever number of degrees there might be between the isothermals, or intervals of entropy between the adiabatics, and that it is not invalidated by using fractions of degrees and fractions of units of entropy. It is consequently the general expression for the heat changed into work by an engine having a reversible cycle.

It is clear that the work done on such a cycle increases as the lower temperature T' decreases, and that it is a maximum when T' becomes zero, for which condition all of the heat applied is changed into work. Therefore the heat applied is represented by

$$Q = T\ (\phi' - \phi),$$

and the efficiency of the engine working on the cycle represented by Fig. 20 is

$$\frac{AW}{Q} = \frac{Q - Q'}{Q} = \frac{(T - T')\,(\phi' - \phi)}{T\ (\phi' - \phi)} = \frac{T - T'}{T},$$

as found by equation (35). The deduction of this equation by integration is more simple and direct, but the graphical method

is interesting and may give the student additional light on this subject.

Temperature-Entropy Diagram. — Thermal diagrams are commonly drawn with pressure and volume for the co-ordinates, but for some purposes it is convenient to use other properties as co-ordinates, in particular temperature and entropy. For example, Fig. 21 represents Carnot's cycle drawn with entropies for abscissæ and temperatures for ordinates, with the advantage that indefinite extensions of the lines are avoided, and the areas under consideration are evidently finite and measurable. With the exception that there appears now to be no necessity to show that the areas obtained by subdivision are all equal, the discussion for Fig. 20 drawn with pressures and volumes may be repeated with temperatures and entropies.

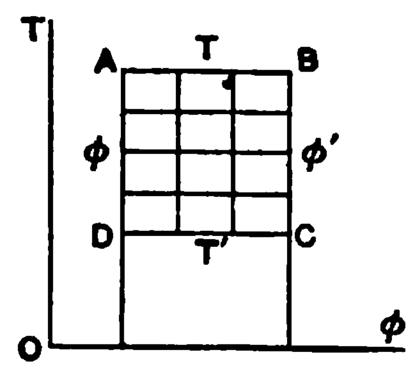

FIG. 21.

Expression for Entropy. — One advantage of using the temperature-entropy diagram is that it leads at once to a method for computing changes of entropy. Thus in Fig. 22 let AB represent an isothermal change, and let Aa and Bb be adiabatics drawn to the axis of ϕ; then the diagram $ABba$ may be considered to be the cycle for a Carnot's engine working between the temperature T and the absolute zero, and consequently having the efficiency unity. The heat changed into work may therefore be represented by

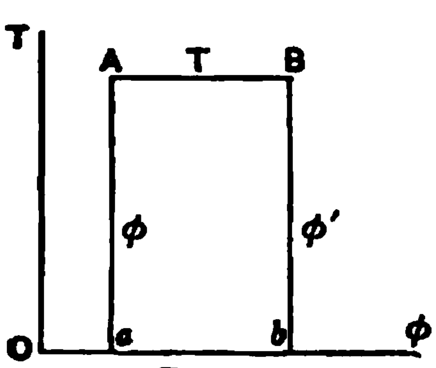

FIG. 22.

$$Q = T\,(\phi' - \phi) \quad . \quad . \quad . \quad . \quad . \quad . \quad (36)$$

If we are dealing with a change under any other condition than constant temperature, we may for an infinitesimal change, write the expression

$$d\phi = \frac{dQ}{T} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (37)$$

and for the entire change may express the change of entropy by

$$\phi' - \phi = \int \frac{dQ}{T},$$

which should for any particular case either be integrated between limits or else a constant of integration should be determined.

Attention should be called to the fact that the conception of the spacing of isothermals and adiabatics is based fundamentally on Carnot's cycle and the second law of thermodynamics, which has been applied only to reversible operations. The method of calculating changes of entropy applies in like manner to reversible operations; and when entropy is employed for calculations of operations that are not reversible, discretion must be used to avoid inconsistency and error.

On the other hand, the entropy of a unit weight of a given substance under certain conditions is a perfectly definite quantity and is independent of the previous history of the substance. This may be made evident by the consideration that any point on the line *no*, Fig. 19, page 31, has a certain number of units of entropy (for example, three) more than that of any point on the adiabatic *ad*.

Example. — There may be an advantage in giving a calculation of a change of entropy to emphasize the point that it can be represented by a number. Let it be required to find the change of entropy during an isothermal expansion of one pound air from four cubic feet to eight cubic.

The heat applied may be obtained by integrating the expression

$$d\phi = \frac{dQ}{T} = \frac{ldv}{T} = (c_p - c_v)\frac{p}{R}\frac{dv}{T},$$

the value of the latent heat having been taken from page 12. From the characteristic equation

$$pv = RT$$

the above expression may be reduced to

$$d\phi = (c_p - c_v)\frac{dv}{v}.$$

$$\therefore \phi' - \phi = (c_p - c_v) \log_e \frac{v}{v'}$$

or

$$\phi' - \phi = (0.2375 - 0.1690) \log_e \frac{8}{4} = 0.0475.$$

A problem for air is chosen because it can be readily worked out at this place; as a matter of fact, there are few occasions in practice where there is reason to refer to entropy of air.

Application to a Reversible Cycle. — A very important result is obtained by the application of equation (37) to the calculation of entropy during a reversible cycle. In the first place, it is clear that the entropy of a substance having its condition represented by the point a (Fig. 23), depends on the adiabatic line drawn through it; in other words, the entropy depends only on the condition of the substance. In this regard entropy is like intrinsic energy and differs from external work. Suppose now that the substance is made to pass through a cycle of operations represented by the point a tracing the diagram on Fig. 23; it is clear that the entropy will be the same at the end of the cycle as at the beginning, for the tracing-point will then be on the original adiabatic line. As the tracing-point moves toward the right from adiabatic to adiabatic the entropy increases, and as it moves to the left the entropy decreases, the algebraic sum of changes of entropy being zero for the entire cycle. This conclusion holds whether the cycle is reversible or non-reversible. The cycle represented by Fig. 23 is purposely drawn like a steam-engine indicator diagram (which is not reversible) to emphasize the fact that the change of entropy is zero in any case.

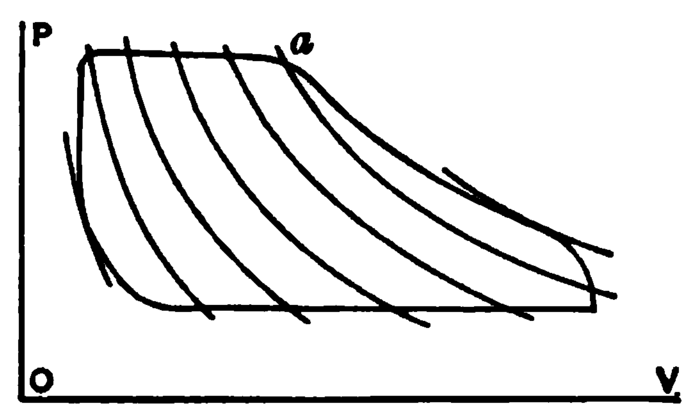

FIG. 23.

If the cycle is reversible, then equation (37) may be used for calculating the several changes of entropy, and for calculating the change for the entire cycle, giving for the cycle

$$\int \frac{dQ}{T} = 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (38)$$

This is a very important conclusion from the second law of thermodynamics, and is considered to represent that law. The second law is frequently applied by using this equation in connection with a general equation or a characteristic equation, in a manner to be explained later.

Though the discussion just given is simple and complete, there is some advantage in showing that equation (38) holds for certain simple and complex reversible cycles.

Thus for Carnot's cycle, represented by Fig. 20, the increase of entropy during isothermal expansion is

$$\phi' - \phi = \int \frac{dQ}{T} = \frac{1}{T}\int dQ = \frac{Q}{T},$$

because the temperature is constant. In like manner the decrease during isothermal compression is

$$\phi - \phi' = \frac{Q'}{T'},$$

so that the change of entropy for the cycle is

$$\frac{Q}{T} - \frac{Q'}{T'}.$$

But from the efficiency of the cycle we have

$$\frac{Q - Q'}{Q} = \frac{T - T'}{T}. \qquad \therefore \frac{Q'}{Q} = \frac{T'}{T}. \qquad \therefore \frac{Q}{T} - \frac{Q'}{T'} = 0.$$

A complex cycle like that represented by Fig. 24 may be broken up into two simple cycles $ABFG$ and $CDFE$, for each of which individually the same result will be obtained — that is, the increase of entropy from A to B is equal to the decrease from F to G, and the increase from C to D is equal to the decrease from E to F, so that the summation of changes for the entire cycle gives zero.

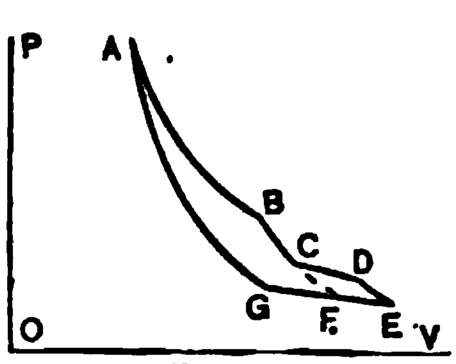

FIG. 24.

Fig. 25 represents the simplified ideal diagram of a hot-air engine, in which by the aid of a regenerator the adiabatic lines of Carnot's cycle are replaced by vertical lines without affecting the reversibility or the efficiency of the cycle. We may replace the actual diagram by a series of simple cycles made up of isothermals and adiabatics, so drawn that the perimeter of the complex cycle includes the same area and corresponds approximately with that of the actual diagram. The summation of the change of entropy for the complex cycle is clearly zero, as before. But by drawing the adiabatic lines near enough together we may make the perimeter approach that of the actual diagram as nearly as we please, and we may therefore conclude that the integration for the changes of entropy for that cycle is also zero.

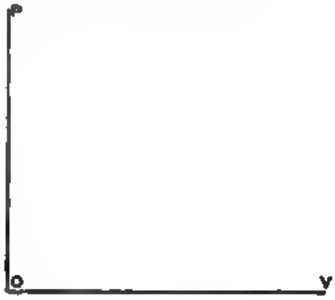

FIG. 25.

Maximum Efficiency. — In order that heat may be transformed into work with the greatest efficiency, all the heat should be applied at the highest practicable temperature, and the heat rejected should be given up at the lowest practicable temperature; this condition is found for Carnot's cycle, which serves as the ideal type to which we approach as nearly as practical conditions allow. Deviations from the ideal type are of two sorts, (1) commonly a different and inferior cycle is chosen as being practically more convenient, and (2) the material of which the working cylinder is made absorbs heat at high temperature and gives out heat at low temperature, thus interfering with the attainment of the cycle selected.

The principle just stated must be accepted as immediately evident; but there may be an advantage in giving an illustration. The complex cycle of Fig. 24 is made up of two simple Carnot cycles *ABFG* and *CDEF*; if two thirds of the heat is applied during the isothermal expansion *AB* at 500° C., and one third during the expansion *CD*, at 250° C., and if all the heat is re-

jected at 20° C., the combined efficiency of the diagram may be computed to be

$$\frac{2}{3} \times \frac{500 - 20}{500 + 273} + \frac{1}{3} \times \frac{250 - 20}{250 + 273} = 0.56;$$

had the heat been all applied at 500° C., the efficiency would have been

$$\frac{500 - 20}{500 + 273} = 0.62.$$

The loss in this case from applying part of the heat at lower temperature is, therefore,

$$\frac{0.62 - 0.56}{0.62} = 0.097.$$

Non-reversible Cycles. — If a process or a cycle is non-reversible, then the change of entropy cannot be calculated by equation (37), and equation (38) will not hold. The entropy will, indeed, be the same at the end as at the beginning of the cycle, but the integration of $\frac{dQ}{T}$ for the cycle will not give zero. On the contrary, it can be shown that the integration of $\frac{dQ}{T}$ for the entire cycle will give a negative quantity. Thus let the non-reversible engine A take the same amount of heat per stroke as the reversible engine R which works on Carnot's cycle, but let it have a less efficiency, so that

$$\frac{Q - Q_1'}{Q} < \frac{Q - Q'}{Q} \quad . \quad . \quad . \quad (39)$$

where Q_1' represents the heat rejected by the engine A. Then

$$Q - Q_1' < Q - Q' = (T - T')(\phi' - \phi) \quad . \quad . \quad (40)$$

Suppose now that T' approaches zero and that ϕ' approaches ϕ, then at the limit we shall have

$$dQ_1 < dQ = Td\phi,$$

or

$$\frac{dQ_1}{T} < d\phi.$$

Integrating for the entire cycle, we shall have

$$\int \frac{dQ_1}{T} < 0. \quad \therefore \int \frac{dQ_1}{T} = -N \quad . \quad . \quad . \quad (41)$$

where $-N$ represents a negative quantity. The absolute value of N will, of course, depend on the efficiency of the non-reversible engine. If the efficiency is small compared with that of a reversible engine, then the value of N will be large. If the efficiency approaches that of a reversible engine, then N approaches zero. It is scarcely necessary to point out that N cannot be positive, for that would infer that the non-reversible engine had a greater efficiency than a reversible engine working between the same temperatures.

Some non-reversible operations, like the flow of gas through an orifice, result in the development of kinetic energy of motion. In such case the equation representing the distribution of energy contains a fourth term K to represent the kinetic energy, and equation (15) becomes

$$dQ = A\ (dS + dI + dW + dK) \quad . \quad . \quad . \quad (42)$$

As before S represents vibration work, I represents disgregation work, and W represents external work. If the vibration and disgregation work cannot be separated, then we may write

$$dQ = A\ (dE + dW + dK) \quad . \quad . \quad . \quad . \quad . \quad (43)$$

If a non-reversible process like that just discussed takes place in apparatus or appliances that are made of non-conducting material, or if the action of the walls on the substance contained can be neglected, the operation may properly be called adiabatic; such a use is clearly an extension of the idea stated on page 32, and conclusions drawn from adiabatic expansion in a closed cylinder cannot be directly extended to this new application. Such a non-reversible operation is not likely to be isoentropic, and there is some advantage in drawing a distinction between operations which are isoentropic and those which are adiabatic.

A non-reversible operation in non-conducting receptacles may be isothermal, or may be with constant intrinsic energy, as will appear in the discussion of flow of air in pipes on page 380, and the discussion of the steam calorimeter, page 191. Any non-reversible process is likely to be accompanied by an increase of entropy; this will appear in special cases discussed in the chapter on flow of fluids.

Since the entropy of a pound of a given substance under given conditions, reckoned from an arbitrary zero, is a perfectly definite numerical quantity, it is possible to determine its entropy for any series of conditions, without regard to the method of passing from one condition to another. It is, therefore, always possible to represent any changes of a fixed weight of a substance, by a diagram drawn with temperatures and entropies for co-ordinates. If the diagram can be properly interpreted, conclusions from it will be valid. It is, however, to be borne in mind that thermodynamics is essentially an analytical mathematical treatment; the treatment, so far as it applies to engineering, is neither extensive nor difficult. But the student is cautioned not to consider that because he has drawn a diagram representing a given operation to the eye, he necessarily has a better conception of the operation. If any operation involves an increase (or decrease) of weight of the substance operated on, thermal diagrams are likely to be difficult to devise and liable to misinterpretation.

CHAPTER IV.

GENERAL THERMODYNAMIC METHOD.

In the three preceding chapters a discussion has been given of the three fundamental principles of thermodynamics, namely, (1) the assumption that the properties of any substance can be represented by an equation involving three variables; (2) the acceptance of the conservation of energy; and (3) the idea of Carnot's principle. In the ideal case each of these principles should be represented by an equation, and by the combination of the three several equations all the relations of the properties of a substance should be brought out so that unknown properties may be computed from known properties, and in particular advantage may be taken of opportunities to calculate such properties as cannot be readily determined by direct experiment from those which may be determined experimentally with precision.

Recent experiments have so far changed the condition of affairs that there is less occasion than formerly for such a general treatment. Of the three classes of substances that are interesting to engineers, namely, gases, saturated vapors, and superheated vapors, the conditions appear to be as follows. For gases there are sufficient experimental data to solve all problems without referring to the general method, though the ratio of the specific heats is probably best determined by that method. For saturated steam there is one property, namely, the specific volume, which is computed by aid of the general method, but there are experimental determinations of volume which are reliable though less extensive. The characteristic equation of superheated steam is now well determined, and the specific heat is determined with sufficient precision for engineering purposes, so that there is no difficulty in making the customary calculations.

The one class of substances for which the necessary properties must be computed by aid of the general method, are those volatile fluids like ammonia and sulphur dioxide, which are used for refrigerating machines.

On the whole, even with conditions as stated, it is desirable that the student should master the general thermodynamic method, given in this chapter. That method is neither long nor hard, and is so commonly accepted that students who have mastered it will have no difficulty in reading standard works and current literature involving thermodynamic discussions. Those cases remaining where the general method or its equivalent must be used, are best treated by that method, and in the case of volatile fluids can be treated only by that method.

The case having been presented as fairly as possible, discretion may be left with the student or his instructor whether he shall read the remainder of this chapter before proceeding, or whether the chapter shall be altogether omitted.

The following method of combining the three general principles of thermodynamics, which is due to Lord Kelvin, depends on the use of the expression

$$\frac{\delta^2 x}{\delta y \delta z} = \frac{\delta^2 x}{\delta z \delta y}$$

as the basis of an operation. This expression is generally used as a criterion to determine whether a certain differential is an exact differential that can be integrated directly, or whether some additional relation must be sought by aid of which the expression may be transformed so that it can be integrated.

Conversely, if we know, from the nature of a given property like intrinsic energy, that it can be always calculated for a given condition as represented by two variables like temperature and volume, then we are justified in concluding that the expression

$$\frac{\delta^2 E}{\delta v \delta t} = \frac{\delta^2 E}{\delta t \delta v} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (44)$$

must be true and that we can use it as the basis of an operation.

Now in laying out a general method it is impossible to select any particular characteristic equation, and for that reason, if no other, the form of the integral equation connecting E with t and v cannot be assigned. But the fact remains that the possibility of working out any method depends on the assumption of the ultimate possibility of writing such an equation, and that assumption carries with it the assumption that dE is an exact differential.

Application of the First Law. — The first general principle may be taken to be represented by equation (1),

$$dQ = c_v dt + l dv,$$

and the first law of thermodynamics by equation (16),

$$dQ = A\ (dE + dW) = A\ (dE + p dv).$$

Combining these equations gives

$$dE = \frac{c_v}{A}\ dt + \left(\frac{l}{A} - p\right) dv;$$

and comparing with the general form,

$$dE = \frac{\delta E}{\delta t}\ dt + \frac{\delta E}{\delta v}\ dv,$$

it is evident that

$$\frac{\delta E}{\delta t} = \frac{c_v}{A} \text{ and } \frac{\delta E}{\delta v} = \frac{l}{A} - p.$$

Now equation (44) is an abbreviated way of writing the expression for continued differentiation which may be expanded to

$$\frac{\delta \dfrac{\delta E}{\delta t}}{\delta v} = \frac{\delta \dfrac{\delta E}{\delta v}}{\delta t},$$

or replacing the first partial differential coefficients by their equivalents,

$$\frac{\delta}{\delta v}\left(\frac{c_v}{A}\right) = \frac{\delta}{\delta t}\left(\frac{l}{A} - p\right).$$

$$\therefore \quad \frac{1}{A}\left[\left(\frac{\delta l}{\delta t}\right)_v - \left(\frac{\delta c_v}{\delta v}\right)_t\right] = \frac{\delta p}{\delta t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (45)$$

the subscripts being written to avoid possible confusion with other partial differential coefficients to be deduced later.

From the first law of thermodynamics and equation (2) we have in like manner

$$dQ = A\ (dE + pdv) = c_p dt + mdp.$$

Since the differential dv is inconvenient, we may replace it by

$$dv = \frac{\delta v}{\delta p}dp + \frac{\delta v}{\delta t}dt,$$

so that

$$A\left(dE + p\frac{\delta v}{\delta p}dp + p\frac{\delta v}{\delta t}dt\right) = c_p dt + mdp.$$

$$\therefore dE = \left(\frac{c_p}{A} - p\frac{\delta v}{\delta t}\right)dt + \left(\frac{m}{A} - p\frac{\delta v}{\delta p}\right)dp.$$

Making use of the equation

$$\frac{\delta\frac{\delta E}{\delta t}}{\delta p} = \frac{\delta\frac{\delta E}{\delta p}}{\delta t}$$

gives

$$\frac{\delta}{\delta p}\left(\frac{c_p}{A} - p\frac{\delta v}{\delta t}\right) = \frac{\delta}{\delta t}\left(\frac{m}{A} - p\frac{\delta v}{\delta p}\right).$$

$$\therefore \frac{1}{A}\left(\frac{\delta c_p}{\delta p}\right)_t - \frac{\delta v}{\delta t} - p\frac{\delta^2 v}{\delta p \delta t} = \frac{1}{A}\left(\frac{\delta m}{\delta t}\right)_p - p\frac{\delta^2 v}{\delta t \delta p}.$$

But the assumption of a characteristic equation connecting p, v, and t carries with it the assumption that

$$\frac{\delta^2 v}{\delta p \delta t} = \frac{\delta^2 v}{\delta t \delta p},$$

so that

$$\frac{1}{A}\left[\left(\frac{\delta c_p}{\delta p}\right)_t - \left(\frac{\delta m}{\delta t}\right)_p\right] = \frac{\delta v}{\delta t} \quad . \quad . \quad . \quad . \quad . \quad (46)$$

Again, from equation (3) we may have

$$dQ = A\ (dE + pdv) = ndp + odv.$$

$$\therefore dE = \frac{n}{A} dp + \left(\frac{o}{A} - p\right) dv \quad . \quad . \quad . \quad . \quad . \quad . \quad (47)$$

or, making use of

$$\frac{\delta^2 E}{\delta v \delta p} = \frac{\delta^2 E}{\delta p \delta v},$$

$$\frac{1}{A}\left(\frac{\delta n}{\delta v}\right)_p = \frac{1}{A}\left(\frac{\delta o}{\delta p}\right)_v - 1.$$

$$\therefore \frac{1}{A}\left[\left(\frac{\delta o}{\delta p}\right)_v - \left(\frac{\delta n}{\delta v}\right)_p\right] = 1 \quad . \quad . \quad . \quad . \quad . \quad . \quad (48)$$

Application of the Second Law. — The second law of thermodynamics can be expressed by equation (38), page 37,

$$\int \frac{dQ}{T} = 0,$$

which makes $\frac{dQ}{T}$ or $d\phi$ an exact differential, so that we may write

$$\frac{\delta^2 \phi}{\delta v \delta t} = \frac{\delta^2 \phi}{\delta t \delta v}.$$

To prepare equation (1) for this transformation, we may write

$$d\phi = \frac{dQ}{T} = \frac{c_v}{T} dt + \frac{l}{T} dv,$$

so that the preceding equation gives

$$\frac{\delta}{\delta v}\left(\frac{c_v}{T}\right) = \frac{\delta}{\delta t}\left(\frac{l}{T}\right).$$

$$\therefore \frac{1}{T}\left(\frac{\delta c_v}{\delta v}\right)_t = \frac{T\left(\frac{\delta l}{\delta t}\right)_v - l}{T^2}$$

or

$$\left(\frac{\delta l}{\delta t}\right)_v - \left(\frac{\delta c_v}{\delta v}\right)_t = \frac{l}{T} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (49)$$

Performing a like operation on equation (2) we have

$$\frac{dQ}{T} = \frac{c_p}{T}dt + \frac{m}{T}dp,$$

$$\frac{\delta}{\delta p}\left(\frac{c_p}{T}\right) = \frac{\delta}{\delta t}\left(\frac{m}{T}\right).$$

$$\therefore \frac{1}{T}\left(\frac{\delta c_p}{\delta p}\right)_t = \frac{T\left(\frac{\delta m}{\delta t}\right)_p - m}{T^2}.$$

$$\therefore \left(\frac{\delta c_p}{\delta p}\right)_t - \left(\frac{\delta m}{\delta t}\right)_p = -\frac{m}{T} \quad . \quad . \quad . \quad . \quad . \quad . \quad (50)$$

Again, from equation (3) we have

$$\frac{dQ}{T} = \frac{n}{T}dp + \frac{o}{T}dv.$$

$$\therefore \frac{\delta}{\delta v}\left(\frac{n}{T}\right) = \frac{\delta}{\delta p}\left(\frac{o}{T}\right).$$

$$\therefore \frac{T\left(\frac{\delta n}{\delta v}\right)_p - n\frac{\delta t}{\delta v}}{T^2} = \frac{T\left(\frac{\delta o}{\delta p}\right)_n - o\frac{\delta t}{\delta p}}{T^2}.$$

$$\therefore \frac{1}{T}\left(o\frac{\delta t}{\delta p} - n\frac{\delta t}{\delta v}\right) = \left(\frac{\delta o}{\delta p}\right)_v - \left(\frac{\delta n}{\delta v}\right)_p \quad . \quad . \quad . \quad (51)$$

First and Second Laws Combined. — The result of applying both the first and the second laws of thermodynamics to the

equations (1), (2), and (3) may be obtained by combining the equations resulting from the application of the laws separately.

Thus equations (45) and (49) give

$$\frac{\delta p}{\delta t} = \frac{1}{A}\frac{l}{T} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (52)$$

Equations (46) and (50) give

$$\frac{\delta v}{\delta t} = -\frac{1}{A}\frac{m}{T} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (53)$$

And equations (48) and (51) give

$$A = \frac{1}{T}\left(o\frac{\delta t}{\delta p} - n\frac{\delta t}{\delta v}\right) \quad . \quad . \quad . \quad . \quad . \quad (54)$$

It is convenient to transform this last equation by taking values of n and o from page 12, yielding

$$c_p - c_v = AT\frac{1}{\dfrac{\delta t}{\delta v}\dfrac{\delta t}{\delta p}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (55)$$

The equations deduced in this chapter show the necessary relations among the thermal capacities if the laws of thermodynamics are accepted. Some of them, or equations deduced from them, have been used by writers on thermodynamics to establish relations or compute properties that cannot be readily obtained by direct experiments.

For the student familiarity with the processes is of more importance than any of the results.

Alternative Method. — Some writers on thermodynamics reserve the discussion of temperature until they are ready to define or assume an absolute scale independent of any substance and depending only on the fundamental units of length and weight. Of the three general equations (1), (2), and (3) they use at first only the latter:

$$dQ = ndp + odv.$$

Now from equation (16), representing the first law of thermodynamics,

$$dQ = A\ (dE + pdv),$$

it is evident that dQ is not an exact differential, since the equation cannot be integrated directly. The fact that in certain cases p may be expressed as a function of v, and the integral for external work can be deduced, does not affect this general statement. Suppose that there is some integrating factor, which may be represented by $\frac{1}{S}$, so that

$$\frac{dQ}{S} = \frac{n}{S}\,dp + \frac{o}{S}\,dv$$

may be integrated directly; we may then consider that we have a series of thermal lines represented by making

$$\frac{1}{S} = \text{const.},\qquad \frac{1}{S'} = \text{const.},\qquad \frac{1}{S''} = \text{const., etc.}$$

These lines with a series of adiabatic lines represented by

$$\phi = \text{const.},\qquad \phi' = \text{const.},\qquad \phi'' = \text{const., etc.,}$$

allow us to draw a simple cycle of operations represented by Fig. 25a, in which AB and CD are represented by the equations

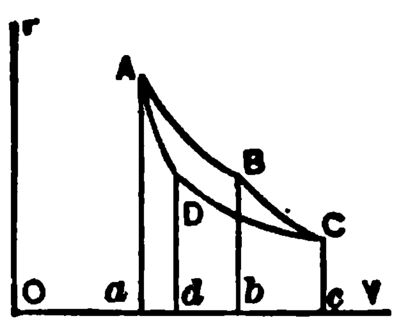

FIG. 25a.

$$\frac{1}{S} = C, \text{ and } \frac{1}{S'} = C',$$

while AD and BC are adiabatics. The efficiency of a reversible engine receiving the heat Q during the operation AB, and rejecting the heat Q' during the operation CD, will be

$$e = \frac{Q - Q'}{Q} = \frac{AW}{Q}.$$

But $\frac{dQ}{S}$ is an exact differential, and depends on the state of

the substance only, and consequently is the same at the end as at the beginning of the cycle, so that for the entire cycle

$$\int \frac{dQ}{S} = 0.$$

Now during the operations represented by the adiabatics AD and BC no heat is transmitted, and during the operations represented by the lines AB and CD, $\frac{1}{S}$ is constant; consequently the integration of the above equation for the cycle gives

$$\frac{Q}{S} - \frac{Q'}{S'} = 0.$$

$$\therefore \frac{Q - Q'}{Q} = \frac{S - S'}{S};$$

that is, the efficiency of an engine working on such a cycle depends on S and S', and on nothing else.

Zeuner's Equations. — A special form of thermodynamic equations has been developed by Zeuner and through his influence has been impressed to a large extent on German writings. These equations can be deduced from those already given in the following manner.

From the application of the first law of thermodynamics to equation (3) we have equation (47), page 47,

$$dE = \frac{n}{A}dp + \left(\frac{o}{A} - p\right)dv.$$

Now

$$dE = \frac{\delta E}{\delta p}dp + \frac{\delta E}{\delta v}dv,$$

so that

$$\frac{n}{A} = \frac{\delta E}{\delta p}, \qquad \frac{o}{A} = \frac{\delta E}{\delta v} + p.$$

These properties Zeuner writes

$$X = \frac{\delta E}{\delta p}, \qquad Y = p + \frac{\delta E}{\delta v}.$$

Solving equation (54) first for o and then for n,

$$o = \frac{AT + n\dfrac{\delta t}{\delta v}}{\dfrac{\delta t}{\delta p}}$$

$$-n = \frac{AT - o\dfrac{\delta t}{\delta p}}{\dfrac{\delta t}{\delta v}}.$$

In equation (3)

$$dQ = ndp + odv,$$

we may substitute the above values successively giving

$$dQ = \frac{1}{\dfrac{\delta t}{\delta p}}\left(n\frac{\delta t}{\delta p}dp + n\frac{\delta t}{\delta v}dv + ATdv\right).$$

$$\therefore\ dQ = \frac{1}{\dfrac{\delta t}{\delta p}}\left(ndt + ATdv\right)$$

because $\quad dt = \dfrac{\delta t}{\delta p}dp + \dfrac{\delta t}{\delta v}dv.$

And also

$$dQ = \frac{1}{\dfrac{\delta t}{\delta v}}\left(o\frac{\delta t}{\delta p}dp + o\frac{\delta t}{\delta v}dv - ATdp\right).$$

$$\therefore dQ = \frac{1}{\dfrac{\delta t}{\delta v}}\left(odt - ATdp\right).$$

Replacing o and n by their values in terms of X and Y,

$$dQ = A\ (Xdp + Ydv),$$

$$dQ = \frac{A}{\dfrac{\delta t}{\delta p}}\left[Xdt + \left(\frac{1}{\alpha} + t\right)dv\right],$$

$$dQ = \frac{A}{\dfrac{\delta t}{\delta v}}\left[Ydt + \left(\frac{1}{\alpha} + t\right)dp\right].$$

In these equations α is the coefficient of dilatation, or $\frac{1}{\alpha} + t$ is equal to T, and

$$X = \frac{1}{A} n = \frac{1}{A} \left(\frac{\delta Q}{\delta p}\right)_v; \quad Y = \frac{1}{A} o = \frac{1}{A} \left(\frac{\delta Q}{\delta v}\right)_p.$$

If this derivation of Zeuner's equations is borne in mind, the treatment of thermodynamics by many German writers may be readily recognized to be only a variant on that developed by Clausius and Kelvin.

CHAPTER V.

PERFECT GASES.

THE characteristic equation for a perfect gas is derived from a combination of the laws of Boyle and Gay-Lussac, which may be stated as follows:

Boyle's Law. — When a given weight of a perfect gas is compressed (or expanded) at a constant temperature the product of the pressure and the volume is a constant. This law is nearly true at ordinary temperatures and pressures for such gases as oxygen, hydrogen, and nitrogen. Gases which are readily liquefied by pressure at ordinary temperatures, such as ammonia and carbonic acid, show a notable deviation from this law. The law may be expressed by the equation

$$pv = p_1v_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (56)$$

in which p_1 and v_1 are the initial pressure and volume; p is any pressure and v is the corresponding volume.

Gay-Lussac's Law. — It was found by Gay-Lussac that any volume of gas at freezing-point increases about 0.003665 of its volume for each degree rise of temperature. Gases which are easily liquefied deviate from this law as well as from Boyle's law. In the deduction of this law temperatures were measured on or referred to the air-thermometer, and the law therefore asserts that the expansibility or the coefficient of dilatation of perfect gases is the same as that of air. Gay-Lussac's law may be expressed by the equation

$$v = v_0(1 + \alpha t) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (57)$$

in which v_0 is the original volume at freezing-point, α is the coefficient of dilatation or the increase of volume for one degree rise of temperature, and v is the volume corresponding to the temperature t measured from freezing-point.

Characteristic Equation. — From equation (57) we may calculate any special volume, such as v_1, getting

$$v_1 = v_0 (1 + \alpha t).$$

Assuming that p_1 in equation (56) is the normal pressure of the atmosphere p_0, and substituting the value just found for v_1, we have for the combination of the laws of Boyle and Gay-Lussac

$$pv = p_0 v_0 (1 + \alpha t) = p_0 v_0 \alpha \left(\frac{1}{\alpha} + t\right) \quad . \quad . \quad . \quad . \quad (58)$$

If it be assumed that a gas contracts α part of its volume at freezing-point for each degree of temperature below freezing then the absolute zero of the air-thermometer will be $\frac{1}{\alpha}$ degrees below freezing, and

$$\frac{1}{\alpha} + t$$

may be replaced by T, the absolute temperature by the air-thermometer.

The usual form of the characteristic equation for perfect gases may be derived from equation (58) by substituting T_0, the absolute temperature of freezing-point, for $\frac{1}{\alpha}$, giving

$$pv = \frac{p_0 v_0}{T_0} T = RT \quad . \quad . \quad . \quad . \quad . \quad . \quad (59)$$

where R is a constant representing the quantity

$$\frac{p_0 v_0}{T_0}.$$

For solution of examples it is more convenient to write equation (59) in the form

$$\frac{pv}{T} = \frac{p_0 v_0}{T_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (60)$$

Absolute Temperature. — Recent investigations of the properties of hydrogen by Professor Callender * indicate that the absolute zero is $273^\circ.1$ C. below freezing-point. This does not differ much from taking $\alpha = 0.003665$ as given by Regnault, for which the reciprocal is 272.8. In this work we shall take for the absolute temperature

$$T = t + 273^\circ \text{ centigrade scale.}$$
$$T = t + 459^\circ.5 \text{ Fahrenheit scale.}$$

These figures are convenient and sufficiently exact.

Relation of French and English Units. — For the purpose of conversion of units from the metric system (or vice versa) the following values may be used:

one metre = 39.37 inches.
one kilogram = 2.2046 pounds.

Specific Pressure. — The normal pressure of the atmosphere is assumed to be equivalent to that of a column of mercury, 760 mm. high at freezing-point. Now Regnault † gives for the weight of a litre, or one cubic decimetre, of mercury 13.5959 kilograms; consequently the specific pressure of the atmosphere under normal conditions is

$$p_0 = 10{,}333 \text{ kilograms per square metre.}$$

Using the conversion units given above for reducing this specific pressure to the English system of units gives

$$p_0 = 2116.32 \text{ pounds per square foot,}$$

which corresponds to

14.697 pounds per square inch,

or to

29.921 inches of mercury.

It is customary and sufficient to use for the pressure of the atmosphere 14.7 pounds to the square inch.

* *Phil. Mag.*, Jan., 1903.
† *Mémoires de l'Institut de France*, vol. xxi.

Specific Volumes of Gases. — From recent determinations of densities of gases by Leduc, Morley, and Raleigh it appears that the most probable values of the specific volume of the commoner gases are

VOLUMES IN CUBIC METRES OF ONE KILOGRAM.

Atmospheric air	0.7733
Nitrogen	0.7955
Oxygen	0.6996
Hydrogen	11.123

The corresponding quantities for English units are given in the next table.

VOLUMES IN CUBIC FEET OF ONE POUND.

Atmospheric air	12.39
Nitrogen	12.74
Oxygen	11.21
Hydrogen	178.2

To these may be added the value for carbon dioxide, 0.506 cubic metre per kilogram or 8.10 cubic feet per pound; but as the critical temperature for this substance is about 31° C., or 88° F., calculations by the equations for gases are liable to be affected by large errors.

Value of *R*. — The constant R which appears in the usual form of the characteristic equation for a gas represents the expression

$$\frac{p_0 v_0}{T_0}.$$

The values for R corresponding to the French and the English system of units may be calculated as follows:

$$\text{French units, } R = \frac{10333 \times 0.7733}{273} = 29.27 \quad . \quad . \quad (61)$$

$$\text{English units, } R = \frac{2116.3 \times 12.39}{491.5} = 53.35 \quad . \quad . \quad (62)$$

Value of R for other gases may be calculated in a like manner.

Solution of Problems. — Many problems involving the properties of air or other gases may be solved by the characteristic equation

$$pv = RT,$$

or by the equivalent equation

$$\frac{pv}{T} = \frac{p_0 v_0}{T_0},$$

which for general use is the more convenient.

In the first of these two equations the specific pressure and volume to be used for English measures are pounds per square foot, and the volume in cubic feet of one pound.

For example, let it be required to find the volume of 3 pounds of air at 60 pounds gauge-pressure and at 100° F. Assuming a barometric pressure of 14.7 pounds per square inch,

$$v = \frac{53.35\ (459.5 + 100)}{(14.7 + 60)\ 144} = 2.774 \text{ cubic feet}$$

is the volume of 1 pound of air under the given conditions, and 3 pounds will have a volume of

$$3 \times 2.774 = 8.322 \text{ cubic feet.}$$

The second equation has the advantage that any units may be used, and that it need not be restricted to one unit of weight.

For example, let the volume of a given weight of gas, at 100° C. and at atmospheric pressure, be 2 cubic yards; required the volume at 200° C. and at 10 atmospheres. Here

$$\frac{10\ v}{473} = \frac{1 \times 2}{373},$$

$$v = \frac{473 \times 2}{10 \times 373} = 0.2536 \text{ cubic yards.}$$

Specific Heat at Constant Pressure. — The specific heat for true gases is very nearly constant, and may be considered to be

so for thermodynamic equations. Regnault gives for the mean values for specific heat at constant pressure the following results:

Atmospheric air	0.2375
Nitrogen	0.2438
Oxygen	0.2175
Hydrogen	3.409

Ratio of the Specific Heats. — By a special experiment on the adiabatic expansion of air, Röntgen* determined for the ratio of the specific heats of air, at constant pressure and at constant volume,

$$\kappa = \frac{c_p}{c_v} = 1.405.$$

This value agrees well with a computation to follow, which depends on the application of the laws of thermodynamics to a perfect gas, and also with a determination from the theory of gases by Love† that the ratio for air should be 1.403. If the given value for this ratio be accepted the remainder of the work in this chapter follows without any reference to the laws of thermodynamics.

Application of the Laws of Thermodynamics. — The preceding statements concerning the specific heats of perfect gases and their ratio would be satisfactory were it definitely determined by experiment that the specific heat at constant volume is as nearly constant as is the specific heat at constant pressure. None of the experimental determinations (not even that by Joly ‡) can be considered as satisfactory as those for the specific heat at constant pressure; consequently there is considerable importance to be attached to the application of the laws of thermodynamics to the characteristic equation for a perfect gas, and, moreover, this application furnishes one of the most satisfactory determinations of the ratio of the specific heats.

* Poggendorff's *Annalen*, vol. cxlviii, p. 580.

† *Phil. Mag.*, July, 1899.

‡ *Proc. Royal Soc.*, vol. xli, p. 352, 1886.

It is convenient at this place to make the application of the laws of thermodynamics by aid of equation (55), page 49.

$$c_p - c_v = AT\frac{1}{\dfrac{\delta t}{\delta v}\dfrac{\delta t}{\delta p}}. \quad \ldots \ldots \quad (63)$$

From the equation

$$pv = RT,$$

we have

$$\frac{\delta t}{\delta v} = \frac{p}{R}; \frac{\delta t}{\delta p} = \frac{v}{R}.$$

$$\therefore c_p - c_v = AR \quad \ldots \ldots \quad (64)$$

This equation shows conclusively that if the characteristic equation is accepted the differences of the specific heats must be considered to be constant, and if one is treated as constant so also must the other. Conversely, the assumption of constant specific heats for any substance is in effect the assumption of the characteristic equation for a perfect gas.

The solution of equation (64) for the ratio of the specific heats gives

$$\kappa = \frac{c_p}{c_v} = \frac{1}{1 - \dfrac{AR}{c_p}}$$

$$\kappa = \frac{1}{1 - \dfrac{10333 \times 0.7733}{426.9 \times 273 \times 0.2375}} = 1.406.$$

For those who have not read Chapter IV, the following derivation of equation (64) may be interesting. In Fig. 26 let ab represent the change of volume at constant pressure due to the addition of heat $c_p\Delta t$ where Δt is the increase of temperature; and let cb represent the change of pressure due to the addition of heat $c_v\Delta t$; if ac is an isothermal, the latter change of temperature will be equal to the former, but the heat applied will be less on account of the external work $p\Delta v$ (approximtely). Consequently,

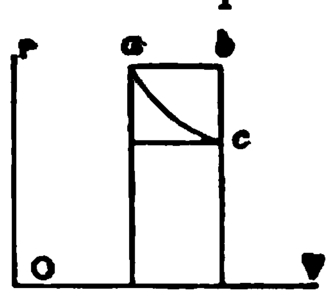

FIG. 26.

$$c_p - c_v = Ap\frac{\delta v}{\delta t} = AR,$$

the last transformation making use of the partial derivative

$$\frac{\delta v}{\delta t} = \frac{R}{p}.$$

Thermal Capacities. — The values of the several thermal capacities for a perfect gas were derived on page 12 and may be written

$$l = \frac{p}{R}(c_p - c_v) = \frac{T}{v}(c_p - c_v) \quad . \quad . \quad . \quad . \quad (66)$$

$$m = -\frac{v}{R}(c_p - c_v) = -\frac{T}{p}(c_p - c_v) \quad . \quad . \quad (67)$$

$$n = \frac{v}{R}c_v = \frac{T}{p}c_v \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (68)$$

$$o = \frac{p}{R}c_p = \frac{T}{v}c_p \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (69)$$

the transformations in equations (66) and (67) being made by aid of the characteristic equation.

General Equations. — To deduce the general equations for gases from equations (1), (2), and (3), it is only necessary to replace the letters l, m, n, and o by their values just obtained, giving

$$dQ = c_v dt + (c_p - c_v)\frac{T}{v}dv \quad . \quad . \quad . \quad . \quad . \quad (70)$$

$$dQ = c_p dt + (c_v - c_p)\frac{T}{p}dp \quad . \quad . \quad . \quad . \quad . \quad (71)$$

$$dQ = c_v \frac{T}{p}dp + c_p\frac{T}{v}dv \quad . \quad . \quad . \quad . \quad . \quad . \quad (72)$$

Isothermal Line. — The equation to the isothermal line for a gas is obtained by making T a constant in the characteristic equation, so that

$$pv = RT_1 = p_1v_1,$$

or

$$pv = p_1v_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad (73)$$

This equation will be recognized as the expression of Boyle's law. It is, of course, the equation to an equilateral hyperbola.

To obtain the external work during an isothermal expansion we may substitute for p in the expression

$$W = \int p dv$$

from the equation to the isothermal line just stated, using for limits the final and initial volumes, v_2 and v_1,

$$W = p_1 v_1 \int_{v_1}^{v_2} \frac{dv}{v} = p_1 v_1 \log_e \frac{v_2}{v_1} \quad . \quad . \quad . \quad . \quad (74)$$

If the problem in any case calls for the external work of one unit of weight of a gas, then v_1 and v_2 must be the initial and final specific volumes; but in many cases the initial and final volumes are given without any reference to a weight, and it is then sufficient to find the external work for the given expansion from the initial to the final volume without considering whether or not they are specific volumes.

The pressures must always be specific pressures; in the English system the pressures must be expressed in pounds on the square foot before using them in the equation for external work, or, for that matter, in any thermodynamic equation.

For example, the specific volume of air at freezing-point and at 14.7 pounds pressure per square inch is about 12.4 cubic feet; at the same temperature and at 29.4 pounds per square inch the specific volume is 6.2 cubic feet. The external work during an isothermal expansion of one pound of air from 6.2 to 12.4 cubic feet is

$$W = p_1 v_1 \int_{v_1}^{v_2} \frac{dv}{v} = p_1 v_1 \log_e \frac{v_2}{v_1}$$

$$= 29.4 \times 144 \times 6.2 \log_e \frac{12.4}{6.2} = 18,190 \text{ foot-pounds.}$$

For example, the external work of isothermal expansion from 3 cubic feet and 60 pounds pressure by the gauge to a volume of 7 cubic feet is

$$W = (60 + 14.7)\, 144 \times 3 \log_e \frac{7}{3} = 27,340 \text{ foot-pounds.}$$

In both problems the pressure per square inch is multiplied by 144 to reduce it to the square foot. In the first problem the pressures are absolute, that is, they are measured from zero pressure; in the second problem the pressure by the gauge is 60 pounds above the pressure of the atmosphere, which is here assumed to be 14.7 pounds per square inch, and is added to give the absolute pressure. In careful experimental work the pressure of the atmosphere is measured by a barometer and is added to the gauge-pressure.

Isoenergic Line. — The isothermal line for a perfect gas is also the isoenergic line, a fact that may be proved as follows: The heat applied during an isothermal expansion may be calculated by making T a constant in equation (70) and then integrating; thus:

$$Q = (c_p - c_v)\, T_1 \int_{v_1}^{v_2} \frac{dv}{v} = (c_p - c_v)\, T_1 \log_e \frac{v_2}{v_1},$$

or, substituting for $c_p - c_v$ from equation (64),

$$Q = ART_1 \log_e \frac{v_2}{v_1} = Ap_1v_1 \log_e \frac{v_2}{v_1} \quad . \quad . \quad . \quad (75)$$

A comparison of equation (75) with equation (74) shows that the heat applied during an isothermal expansion is equivalent to the external work, or we may say that all the heat applied is changed into external work, so that the intrinsic energy is not changed. This conclusion is based on the assumption that the properties are accurately represented by the characteristic equation and that the specific heats are constant. As both assumptions are approximate so also is the conclusion, as will appear in the discussion of flow through a porous plug.

An interesting corollary of the discussion just given is that an infinite isothermal expansion gives an infinite amount of work. Thus the area contained between the axis OV (Fig. 27), the ordinate ab, and the isothermal line $a\alpha$ extended without limit is

$$W = p_0v_0 \log_e \frac{\infty}{v_0} = \infty.$$

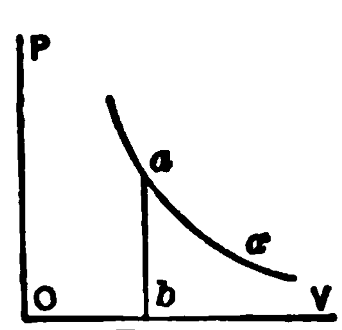

FIG. 27.

This may also be seen from the consideration that if heat be continually applied, and all changed into work, there will be a limitless supply of work.

Adiabatic Lines. — During an adiabatic change — for example, the expansion of a gas in a non-conducting cylinder — heat is not communicated to, nor abstracted from, the gas; consequently dQ in equations (70), (71), and (72) becomes zero.

From equation (72)

$$0 = dQ = c_v \frac{T}{p} dp + c_p \frac{T}{v} dv;$$

$$\therefore \quad \frac{c_p}{c_v}\frac{dv}{v} = -\frac{dp}{p};$$

$$\therefore \quad \log_e \left(\frac{v}{v_1}\right)^{\frac{c_p}{c_v}} = \log_e \left(\frac{p_1}{p}\right).$$

The ratio $\frac{c_p}{c_v}$ of the specific heats may be represented by κ, and the above equation may be written

$$\left(\frac{v}{v_1}\right)^{\kappa} = \frac{p_1}{p} \quad \ldots \ldots \ldots \quad (76)$$

$$\therefore \quad v^{\kappa} p = v_1^{\kappa} p_1 = \text{const.} \quad \ldots \ldots \quad (77)$$

This is the adiabatic equation for a perfect gas which is most frequently used. If adiabatic equations involving other variables, such as v_1 and T_1, are desired, they may be derived from equation (76) by aid of the characteristic equation, which for this purpose may be written

$$\frac{pv}{T} = \frac{p_1 v_1}{T_1},$$

so that

$$\frac{p_1}{p} = \frac{vT_1}{v_1 T},$$

and

$$\left(\frac{v}{v_1}\right)^{\kappa - 1} = \frac{T_1}{T} \quad \ldots \ldots \ldots \quad (78)$$

$$\therefore \quad Tv^{\kappa-1} = T_1 v_1^{\kappa-1} \quad \ldots \ldots \quad (79)$$

Or equations (78) and (79) may be deduced directly from equation (70) as equations (76) and (77) were from equation (72).

In like manner we may deduce from equation (71)

$$Tp^{\frac{1-\kappa}{\kappa}} = T_1 p_1^{\frac{1-\kappa}{\kappa}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (80)$$

or we may derive it from equation (76).

To find the external work the equation

$$W = \int p dv$$

may be used after substituting for p from equation (77)

$$W = \int_{v_1}^{v_2} p dv = v_1^{\kappa} p_1 \int_{v_1}^{v_2} \frac{dv}{v^{\kappa}} = - \frac{p_1 v_1^{\kappa}}{\kappa - 1} \left(\frac{1}{v_2^{\kappa-1}} - \frac{1}{v_1^{\kappa-1}} \right).$$

$$\therefore \; W = \frac{p_1 v_1}{\kappa - 1} \left\{ 1 - \left(\frac{v_1}{v_2} \right)^{\kappa-1} \right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (81)$$

In Fig. 28 the area between the axis OV, the ordinate ba, and the adiabatic line $a\alpha$ extended without limit, becomes

$$W_1 = \frac{p_1 v_1}{\kappa - 1},$$

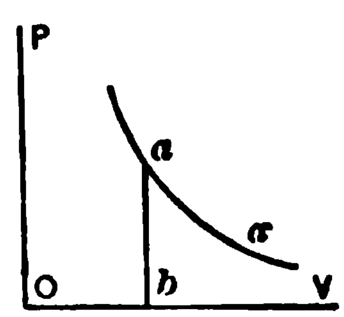

FIG. 28.

and not infinity, as is the case with the isothermal line.

Here, as with the calculation of external work during isothermal expansion, specific volumes should be used when the problem involves a unit of weight; but work may be calculated for any given initial and final volumes without considering whether they are specific volumes or not. The pressures are always pounds on the square foot for the English system.

For example, the external work of adiabatic expansion from 3 cubic feet and 60 pounds pressure by the gauge to the volume of 7 cubic feet is

$$W = \frac{74.7 \times 144 \times 3}{1.405 - 1} \left\{ 1 - \left(\frac{3}{7} \right)^{1.405-1} \right\} = 23{,}140 \text{ foot-pounds},$$

which is considerably less than the work for the corresponding isothermal expansion.

Attention should be called to the fact that calculations by this method are subject to a considerable error from the fact that the denominator of the coefficient contains the term $\kappa - 1$ equal to 0.405; if it be admitted that the last figure is uncertain to the extent of two units, the error of calculation becomes half a per cent.

Intrinsic Energy. — Since external work during an adiabatic expansion is done at the expense of the intrinsic energy, the work obtainable by an infinite expansion cannot be greater than the intrinsic energy. If it be admitted that such an expansion changes all of the intrinsic energy into external work we shall have

$$E_1 = W_1 = \frac{p_1 v_1}{\kappa - 1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (82)$$

which gives a ready way of calculating intrinsic energy. In practice we always deal with differences of intrinsic energy, so that even if there be a residual intrinsic energy after an infinite adiabatic expansion the error of our method will be eliminated from an equation having the form

$$E_1 - E_2 = \frac{p_1 v_1}{\kappa - 1} - \frac{p_2 v_2}{\kappa - 1} \quad . \quad . \quad . \quad . \quad . \quad (83)$$

Exponential Equation. — The expansions and compressions of air and other gases in practice are seldom exactly isothermal or adiabatic; more commonly an actual operation is intermediate between the two. It is convenient and usually sufficient to represent such expansions by an exponential equation,

$$pv^n = p_1 v_1^n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (84)$$

in which n has a value between unity and 1.405. The form of integration for external work is the same as for that of adiabatic expansion, and the amount of external work is intermediate between that for adiabatic and that for isothermal expansion.

Change of temperature during such an expansion may be calculated by the equations

$$Tv^{n-1} = T_1 v_1^{n-1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (85)$$

$$Tp^{\frac{1-n}{n}} = T_1 p_1^{\frac{1-n}{n}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (86)$$

which may be deduced from equation (84) by aid of the characteristic equation

$$pv = RT$$

as equation (79) is deduced from equation (76).

If it is desired to find the exponent of an equation representing a curve passing through two points, as a_1 and a_2 (Fig. 29), we may proceed by taking logarithms of both sides of the equation

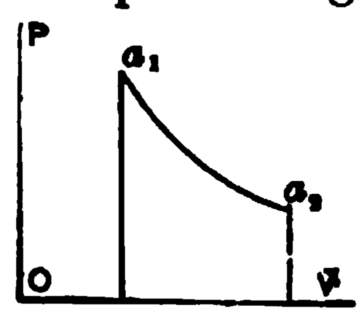

FIG. 29.

$$p_1 v_1^n = p_2 v_2^n,$$

giving

$$n \log v_1 + \log p_1 = n \log v_2 + \log p_2,$$

so that

$$n = \frac{\log p_1 - \log p_2}{\log v_2 - \log v_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (87)$$

For example, the exponent of an equation to a curve passing through the points

$$p_1 = 74.7, \quad v_1 = 3, \quad \text{and } p_2 = 30, \quad v_2 = 7,$$

is

$$n = \frac{\log 74.7 - \log 30}{\log 7 - \log 3} = 1.104.$$

It should be noted that as n approaches unity the probable error of calculation of external work is liable to be very large.

Entropy. — For any reversible process

$$d\phi = \frac{dQ}{T};$$

consequently from equations (70), (71), and (72) we have

$$d\phi = c_v \frac{dt}{T} + (c_p - c_v) \frac{dv}{v},$$

$$d\phi = c_p \frac{dt}{T} + (c_v - c_p) \frac{dp}{p},$$

$$d\phi = c_v \frac{dp}{p} + c_p \frac{dv}{v};$$

and, integrating between limits,

$$\phi_2 - \phi_1 = c_v \log_e \frac{T_2}{T_1} + (c_p - c_v) \log_e \frac{v_2}{v_1} \quad . \quad . \quad (88)$$

$$\phi_2 - \phi_1 = c_p \log_e \frac{T_2}{T_1} + (c_p - c_v) \log_e \frac{p_1}{p_2} \quad . \quad . \quad (89)$$

$$\phi_2 - \phi_1 = c_v \log_e \frac{p_2}{p_1} + c_p \log_e \frac{v_2}{v_1} \quad . \quad . \quad . \quad . \quad (90)$$

which give ready means of calculating changes of entropy. These equations give the entropy changes per pound, and are to be multiplied by the weight in pounds to give the change for the actual conditions.

For example, the change of entropy in passing from the pressure of 74.7 pounds absolute per square inch and the volume of 3 cubic feet to the pressure of 30 pounds absolute and the volume of 7 cubic feet is

$$\phi_2 - \phi_1 = \frac{0.2375}{1.405} \log_e \frac{30}{74.7} + 0.2375 \log_e \frac{7}{3} = 0.0454.$$

Since the pressures form the numerator and denominator of a fraction, there is no necessity to reduce them to the square foot. In this problem the pressures and volumes are taken at random; they correspond to a temperature of 146° F., at the initial condition. As has already been said, there is seldom occasion in practice for using the entropy of a gas.

Comparison of the Air-Thermometer with the Absolute Scale. — In connection with the isodynamic line it was shown that the intrinsic energy is a function of the temperature only. This conclusion is deduced from the characteristic equation on the assumption that the scale of the air-thermometer coincides with the thermodynamic scale, and it affords a delicate method of testing the truth of the characteristic equation, and of comparing the two scales.

The most complete experiments for this purpose were made by Joule and Lord Kelvin, who forced air slowly through a porous plug in a tube in such a manner that no heat was transmitted to or from the air during the process. Also the velocity both before and beyond the plug was so small that the work due to the change of velocity could be disregarded. All the work that would be developed in free expansion from the higher to the lower pressure was used in overcoming the resistance of friction in the plug, and so converted into heat, and as none of this heat escaped it was retained by the air itself, the plug remaining at a constant temperature. It therefore appears that the intrinsic energy remained the same, and that a change of temperature indicated a deviation from the assumptions of the theory of perfect gases.

In the discussion of results given by Joule and Lord Kelvin* in 1854 they gave for the absolute temperature of freezing-point 273°.7 C. As the result of later experiments† they stated that the cooling for a difference of pressure of 100 inches of mercury was represented on the centigrade scale by

$$0.^\circ 92 \left(\frac{273.7}{T}\right)^2.$$

From these experiments and from other considerations concerning the constant-volume hydrogen thermometer, Professor Callendar has determined that the most probable value for the absolute temperature of freezing-point is 273°.1 C., as already given, and gives a table of corrections to the hydrogen thermometer to obtain temperatures on the absolute scale. As the correction at any temperature between − 200° and + 450° C. is not more than $\frac{1}{100}$ of a degree this is interesting mainly to physicists. The corrections for the air-thermometer at constant pressure are somewhat larger, but approach $\frac{1}{10}$ of a degree only at 300° C.

* *Phil. Trans.* vol. cxliv, p. 349.

† *Ibid.* vol. clii, p. 579.

Deviation from Boyle's Law. — Early experiments on the permanent gases (as they were then known) indicated that there were small deviations evident to a physicist, but not of importance to engineers; but now that air is compressed to pressures as high as 2500 pounds per square inch, it becomes necessary to take account of such deviations in engineering practice.

Perhaps the best conception of this subject, and of the four recognized states of fluids, can be had from a consideration of Andrews' * experiments, which for the present purpose are conveniently represented by his isothermal curves, which are reproduced in Fig. 29a, together with the curves for air. The latter are approximate hyperbolæ referred to the proper axes, that for zero pressure being nearly the whole height of the diagram below the figure as it is drawn. At 48°.1 C., the isothermal for carbonic acid shows a marked deviation from the hyperbola, as may be seen by comparison with the curves for air, or better from the fact that a rectangular hyperbola through P will pass through Q. On the other hand, the isothermal for 13°.1 resembles that for steam, which is commonly known as a saturated vapor whose pressure is constant at constant temperature; the horizontal part of this line represents a mixture of liquid and vapor which at the left runs into the liquid curve, and as liquid carbonic acid has considerable compressibility, this curve becomes a straight line with an appreciable inclination to the axis of zero volume. At the right, the isothermal shows a decided break and slopes away as the volume becomes larger than that of the saturated vapor. The isothermal for 21°.5 shows similar characteristics, but the passages from one condition to another are more gradual. The dotted curve is drawn through the points of saturation and liquefaction, and its crest corresponds to the critical temperature.

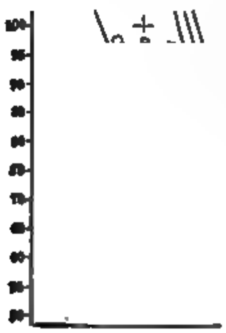

Fig. 29a.

* *Phil. Trans.*, 1869, part ii, p. 575, and 1876, part ii, p. 421.

The isothermal for 31.°1 is clearly above the critical temperature and does not indicate a liquefaction.

The several states of a fluid may be enumerated as

1. Liquid.
2. Saturated vapor, including mixtures of liquid and vapor.
3. Superheated vapor characterized by a larger volume than saturated vapor for a given temperature and pressure.
4. Gas; near the critical temperature the deviations from Boyle's law are very large, at higher temperature the deviations diminish and become unimportant.

Critical Temperatures. — The following table of critical temperatures and of boiling-points at atmospheric pressure is taken in part from Preston's "Theory of Heat," 1904.

	Boiling-Point.	Critical Temperature.
Hydrogen	−252.°7 C.	−234.°5 C.
Nitrogen	−194.4	−146
Oxygen	−182.2	−118.8
Air	−191.4	−140
Carbon monoxide	−190	−139.5
Carbon dioxide	−78.3	+31.35
Sulphur dioxide	−10	+157.0
Ether	34.5	175
Alcohol	78.4	248
Carbon bisulphide	43.3	254
Water	100	362

Density at High Pressure. — If the usual methods (given on page 58) for the solution of problems involving the properties of gases, are applied with very high pressure, errors amounting to two or three per cent are liable to be incurred, owing to the deviation from Boyle's law. In some cases, this error may be ignored in engineering practice; in some cases the error may be included in a practical factor, as will be indicated in the design of air compressors; and in other cases allowances must be made from the experimental information furnished by Armagat, and which may be found in Landolt and Börnstein's Tables.

Röntgen's Experiments. — Direct experiments to determine κ may be made as follows. Suppose that a vessel is filled with air at a pressure p_1, while the pressure of the atmosphere is p_a. Let a communication be opened with the atmosphere sufficient to suddenly equalize the pressure; then let it be closed, and let the pressure p_2 be observed after the air has again attained the temperature of the atmosphere. If the first operation is sufficiently rapid it may be assumed to be adiabatic, and we may use equation (77), from which

$$\kappa = \frac{\log p_1 - \log p_a}{\log v_a - \log v_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (91)$$

The second operation is at constant volume; consequently the specific volume is the same at the final state as after the adiabatic expansion of the first operation. But the initial and final temperatures are the same; consequently

$$v_1 p_1 = v_a p_2;$$

$$\therefore \log v_a - \log v_1 = \log p_1 - \log p_2,$$

which substituted in equation (91) gives

$$\kappa = \frac{\log p_1 - \log p_a}{\log p_1 - \log p_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (92)$$

The same experiment may be made by rarefying the air in the vessel, in which case the sign of the second term changes.

Röntgen* employed this method, using a vessel containing 70 litres, and measuring the pressure with a gauge made on the same principle as the aneroid barometer. Instead of assuming the pressure p_a at the instant of closing the stop-cock to be that of the atmosphere, he measured it with the same instrument. A mean of ten experiments on air gave

$$\kappa = 1.4053.$$

* *Poggendorff's Annalen*, vol. cxlviii, p. 580.

EXAMPLES.

1. Find the weight of 4 cubic metres of hydrogen at 30° C., and under the pressure of 800 mm. of mercury. Ans. 0.341 kg.

2. Find the volume of 3 pounds of nitrogen at a pressure of 45 pounds to the square inch by the gauge and at 80° F. Ans. 10.33.

3. Find the temperature at which one kilogram of air will occupy one cubic metre when at a pressure of 20,000 kilograms per square metre. Ans. 410° C.

4. Oxygen and hydrogen are to be stored in tanks 10 inches in diameter and 35 inches long. At a maximum temperature of 110° F., the pressure must not exceed 250 pounds gauge. What weight of oxygen can be stored in one tank? What of hydrogen? Ans. Oxygen 2.21 pounds. Hydrogen 0.138 pound.

5. A balloon of 12,000 cubic feet capacity, weighing with car, occupant, etc., 665 pounds, is inflated with 9500 cubic feet hydrogen at 60° F., the barometer reading 30 inches. Find the weight of the hydrogen and the pull on the anchor rope; find also the amount that the balloon must be lightened to reach a height where the barometer reads 20 inches, and the temperature is 10° below zero Fahrenheit. Ans. Weight hydrogen 50.4 pounds; pull on rope 12 pounds; balloon lightened 7.5 pounds.

6. A gas-receiver holds 14 ounces of nitrogen at 20° C., and under a pressure of 29.6 inches of mercury. How many will it hold at 32° F., and at the normal pressure of 760 mm.? Ans. 15.18 ounces.

7. A gas-receiver having the volume of 3 cubic feet contains half a pound of oxygen at 70° F. What is the pressure? Ans. 29.6 pounds per square inch.

8. Two cubic feet of air expand at 50° F. from a pressure of 80 pounds to a pressure of 60 pounds by the gauge. What is the external work? Ans. 6464 foot-pounds.

9. What would have been the external work had the air expanded adiabatically? Ans. 4450 foot-pounds.

10. Find the external work of 2 pounds of air which expand adiabatically until the volume is doubled, the initial pressure being 100 pounds absolute and the initial temperature 100° F. Ans. 36,100 foot-pounds.

11. Find the external work of one kilogram of hydrogen, which, starting with the pressure of 4 atmospheres and with the temperature of 500° C., expands adiabatically till the temperature becomes 30° C. Ans. 489,000 m.-kg.

12. Find the exponent for an exponential curve passing through the points $p = 30$, $v = 1.9$, and $p_1 = 15$, $v_1 = 9.6$. Ans. 0.4279.

13. Find the exponent for a curve to pass through the points $p = 40$, $v = 2$, and $p_1 = 12$, $v_1 = 6$. Ans. 1.0959.

14. In examples 12 and 13 let p be the pressure in pounds on the square inch and v the volume in cubic feet. Find the external work of expansion in each case. Ans. 21,900 and 12,010 foot-pounds.

15. Find the intrinsic energy of one pound of nitrogen under the standard pressure of one atmosphere and at freezing-point of water. Ans. 66,500 foot-pounds.

16. A cubic foot of air at 492.7° F. exerts 14.7 pounds gauge pressure per square inch. How much do its internal energy and its entropy exceed those of the same air under standard conditions? Ans. 5052 foot-pounds; .00912 units of entropy.

17. Find the increase in entropy of 2 pounds of a perfect gas during isothermal expansion at 100° F. from an initial pressure of 84.3 pounds gauge and a volume of 20 cubic feet to a final volume of 40 cubic feet. Ans. 0.453.

18. A kilogram of oxygen at the pressure of 6 atmospheres and at 100° C. expands isothermally till it doubles its volume. Find the change of entropy. Ans. 0.0434.

19. Twenty pounds of air are heated at a constant pressure of 200 pounds absolute per square inch until the volume increases from 20 cubic feet to 40 cubic feet. Find the initial and final temperatures, the change in internal energy and the increase in entropy. How much heat is added? Ans. 80° and 620°;

increase of intrinsic energy 1,420,000 foot-pounds; increase in entropy 3.29; heat 2570 B.T.U.

20. Suppose a hot-air engine, in which the maximum pressure is 100 pounds absolute, and the maximum temperature is 600° F., to work on a Carnot cycle. Let the initial volume be 2 cubic feet, let the volume after isothermal expansion be 5 cubic feet, and the volume after adiabatic expansion be 8 cubic feet. Find the horse-power if the engine is double-acting and makes 30 revolutions per minute. Ans. 8.3 horse-power.

CHAPTER VI.

SATURATED VAPOR.

FOR engineering purposes steam is generated in a boiler which is partially filled with water and arranged to receive heat from the fire in the furnace. The ebullition is usually energetic, and more or less water is mingled with the steam; but if there is a fair allowance of steam space over the water, and if proper arrangements are provided for withdrawing the steam, it will be found when tested to contain a small amount of water, usually between half a per cent and a per cent and a half. Steam which contains a considerable percentage of water is passed through a separator which removes almost all the water. Such steam is considered to be approximately dry.

If the steam is quite free from water it is said to be dry and saturated; steam from a boiler with a large steam space and which is making steam very slowly is nearly if not quite dry.

Steam which is withdrawn from the boiler may be heated to a higher temperature than that found in the boiler, and is then said to be superheated.

The physical properties of both saturated and superheated steam have now been determined by methods susceptible of certainty and precision so that computations based on them show satisfactory concordance. The results of these investigations will be quoted directly from the original authorities, together with their estimate of the degree of precision to be attributed to their results. This matter should be read with care, so that each one may determine for himself the confidence he can have in the tables based upon it and the accuracy of computation made by their aid.

Saturated Steam. — The essential properties of saturated steam are heat of the liquid, heat of vaporization, specific pressure and specific volume; other properties dependent on these are heat

equivalent of external work, heat equivalent of internal work, entropy of the liquid and entropy of vaporization. All these properties depend on the temperature only, and may conveniently be determined and tabulated for use in solving engineering problems. The author's *Tables of the Properties of Steam*, etc., have been prepared for this purpose.

Standard Temperature. — It is customary to refer all calculations for gases to the standard conditions of the pressure of the atmosphere (760 mm. of mercury) and to the freezing-point of water. Formerly the freezing-point was taken as the standard temperature for water and steam as even now it is the initial point for tables of the properties of saturated vapors. But the investigation of the mechanical equivalent of heat by Rowland resulted in a determination of the specific heat of water with much greater delicacy than is possible by the method of earlier experimenters, and showed that the freezing-point is not well adapted for the standard temperature for water. It is the habit of many physicists to take 15° C. as the standard temperature, and this corresponds substantially with 62° F., at which the English units of measure are standard.

Unit of Heat. — The unit for the measurement of heat is the amount of heat required to raise one unit of weight of water one degree from the standard temperature.

The calorie is the amount of heat required to raise the temperature of one kilogram of water from 15° to 16° C.

The British Thermal Unit is the amount of heat required to raise the temperature of one pound of water from 62° to 63° F.

These two definitions lead to a discrepancy of 0.03 of one per cent, which is insignificant for engineering purposes; in the author's tables the B.T.U. is taken as the standard, and the discrepancy noted is ignored.

Some physicists prefer to use for the unit of heat, one hundredth part of the heat required to raise a kilogram of water from freezing-point to boiling-point. Such a mean calorie is greater than those defined above, by 0.2 of one per cent. It requires also that a different value shall be assigned to the mechanical equivalent of heat than that given in the following section.

Mechanical Equivalent of Heat.—If mechanical energy or work is transformed into heat and applied to heating water, it will be found that 778 foot-pounds of work will be required to heat one pound of water from 62° to 63° F.; in other words, one B.T.U. is equivalent to that number of foot-pounds. This is known as the mechanical equivalent of heat. The most authoritative determination of this important constant appears to be that by Rowland,* who gives the value quoted, namely,

778 foot-pounds.

This is equivalent to

427 metre kilograms

in the metric system. Since his experiments were made, this important physical constant has been investigated by several experimenters, and also a recomputation of his results has been made after a recomparison of his thermometers. The conclusion appears to be that his results may be a little small, but the differences are not important, and it is not certain that the conclusion is valid. There seems, therefore, no sufficient reason for changing the accepted values given above.

Specific Heat is the number of thermal units required to raise a unit of weight of a given substance one degree of temperature. The specific heat of water at standard temperature is unity, and any specific heat is essentially a ratio.

Specific Heat of Water. — The most reliable determination of the specific heat of water is that by Dr. Barnes,† who used an electrical method devised by Professor Callendar and himself, and who extended the method to and below freezing-point by carefully cooling water without the formation of ice to − 5° C. This method gives relative results with great refinement, and gives also a good confirmation of Rowland's determination of the mechanical equivalent of heat. Dr. Barnes reports values of the specific heat of water up to 95° C.

For temperatures above boiling-point values of the specific heat of water have been determined by the author from Regnault's ‡

* *Proc. Am. Acad.*, vol. xv (N. S. vii), 1879.

† *Physical Review*, vol. xv, p. 71, 1902.

‡ *Mémoires de l' Institut de France*, etc., tome xxvi.

experiments on the heat of the liquid, allowing for the correct specific heat of the water in his calorimeter from Barnes's work. The probable error of the heats of the liquid thus obtained, appears to be one-fourth of a per cent. But the heat of the liquid for temperatures above boiling-point is habitually associated with the heat of vaporization, and the above error is less than one-tenth per cent of their sum.

In the following table Barnes's results are quoted directly from 0° to 55° C.; from 55 to 95 degrees his results have been slightly increased to join with results determined by recomputing Regnault's experiments on the heat of the liquid for water by allowing for the true specific heat at low temperature from Dr. Barnes's experiments. The maximum effect of modifying Dr. Barnes's results is to increase the heat of the liquid at 95 degrees by one-tenth of one per cent.

Temperature.		Specific Heat.	Temperature.		Specific Heat.	Temperature.		Specific Heat.
C.	F.		C.	F.		C.	F.	
0	32	1.0094	45	113	0.99760	90	194	1.00705
5	41	1.00530	50	122	0.99800	95	103	1.00855
10	50	1.00230	55	131	0.99850	100	212	1.01010
15	59	1.00030	60	140	0.99940	120	248	1.01620
20	68	0.99895	65	149	1.00040	140	284	1.02230
25	77	0.99806	70	158	1.00150	160	320	1.02850
30	86	0.99759	75	167	1.00275	180	356	1.03475
35	95	0.99735	80	176	1.00415	200	392	1.04100
40	104	0.99735	85	188	1.00557	220	428	1.04760

The specific heats of water at high temperatures have been determined by Dieterici * using a method which does not appear to have the certainty of Barnes's method. His results appear to be systematically larger than Barnes's results, the discrepancy at 95° C. being four-tenths of a per cent. Should his specific heats be used to determine the heat of the liquid at 200° C., the results would appear to be four-tenths of a per cent larger than the values of the heat of the liquid at 200° C., in the author's tables. Even so if this be compared with the sum of the heat of the liquid and

* *Annalen der Physik*, vol. 16, part 4, p. 593, 1905.

the heat of vaporization, the discrepancy becomes about one-tenth of a per cent.

Heat of the Liquid. — The heat required to raise one unit of weight of any liquid from freezing-point to a given temperature is called the heat of the liquid at that temperature.

If the specific heat of water were constant the heat of the liquid would be found by multiplying the increase of temperature by the specific heat. An approximate result can be obtained by using the mean specific heat. For example, the mean specific heat from 0° to 25° C. may be taken to be $\frac{1}{5}(\frac{1}{2} \times 1.0094 + 1.00530 + 1.00230 + 1.00030 + 0.99895 + \frac{1}{2} \times 0.99806) = 1.00212$,

and $$25 \times 1.00212 = 25.05,$$

which in this case corresponds exactly with the value in the author's tables.

The integral calculus gives for a varying specific heat the expression

$$q = \int c\, dt$$

for the heat of the liquid. An equivalent of the operation represented by this equation is to draw a curve with temperatures and specific heats as coördinates and to measure the area under that curve. The fact that the specific heat does not vary much from unity suggests the following method:

Let $$c = 1 + k$$

where k is the difference between the specific heat and unity; it may be positive or negative as the case may be. Then

$$q = t + \int k dt,$$

which leads to a convenient graphical method since k is always small, and the diagram may be drawn with a large scale for ordinates, and accurate results can be obtained. The values for the heat of the liquid in the tables were obtained in this way.

The following table gives equations for heats of the liquid for various substances as determined by Regnault:*

* *Mémoires de l'Institut de France*, etc., tome xxvi.

HEAT OF THE LIQUID.

Alcohol .... $q = 0.54754t + 0.0011218t^2 + 0.0000022206t^3$
Ether............. $q = 0.52901t + 0.0002959t^2$
Chloroform.........$q = 0.23235t + 0.0000507t^2$
Carbon bisulphide...$q = 0.23523t + 0.0000815t^2$
Carbon tetrachloride $q = 0.19798t + 0.0000906t^2$
Aceton .... $q = 0.50643t + 0.0003965t^2$

Heat of Vaporization. — If a unit of weight of a liquid be at a certain temperature and subject to the corresponding pressure, then the amount of heat required to entirely vaporize it into dry saturated vapor at that temperature and against that pressure, is called the heat of vaporization. Henning* gives the following formula for the heat of vaporization of a kilogram of water in calories,

$$r = 94.210\ (365 - t)^{0.31249} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (93)$$

He gives as the probable error of this equation one-tenth of one per cent. Other experiments by Dieterici,† Griffiths,‡ and A. C. Smith§ are represented by this equation with a like degree of precision.

The heat of vaporization of one pound of water in B.T.U. is given by the following equation, obtained by transforming equation (93).

$$r = 141.124\ (689 - t)^{0.31249} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (94)$$

Both of the above equations are applicable from freezing to boiling-point; equation (93) from 0° to 100° C., and equation (94) from 32° to 212° F.

Total Heat. — The amount of heat required to raise a unit of weight of a liquid from freezing-point to a given temperature and to vaporize it into dry saturated vapor against the corresponding pressure is called the total heat.

The quantity is clearly equal to the sum of the heat of the liquid and the heat of vaporization; if the first is represented by

* *Annalen der Physik*, vol. 21, part 4, p. 849, 1906.
† *Annalen der Physik*, vol. 16, part 4, p. 912, 1905.
‡ *Phil. Trans.* 186, p. 261, 1895; p. 593, 1905.
§ *Physical Review*, vol. xxv, 1907.

q and the latter by r, then H, the total heat, is given by the following equation,

$$H = r + q \quad \ldots \ldots \ldots \ldots \quad (95)$$

Conversely, if H and q are known, the preceding equation will give r.

From an investigation of certain experiments on the superheating of steam by throttling, Dr. Harvey N. Davis* gives for the total heat of steam in B.T.U. per pound,

$$H = H_{212} + 0.3745\,(t - 212) - 0.000550\,(t - 212)^2, \quad (96)$$

in which H_{212} is the total heat at boiling-point. Equation (94) gives for the heat of vaporization at boiling-point 969.7, and the method on page 80, for finding the heat of the liquid, gives 180.3 at that temperature, consequently the above equation may be written, for English units,

$$H = 1150 + 0.3745\,(t - 212) - 0.000550\,(t - 212)^2. \quad (97)$$

For French units the equation takes the form

$$H = 638.9 + 0.3745\,(t - 100) - 0.00099\,(t - 100)^2. \quad (98)$$

Davis gives one-tenth of one per cent for the probable error of this equation.

For other liquids the heats of vaporization are given by Regnault.

Ether................	$H = 94 + 0.45t - 0.00055556t^2$
Chloroform..........	$H = 67 + 0.1375\,t$
Carbon bisulphide.....	$H = 90 + 0.14601t - 0.0004123t^2$
Carbon tetrachloride...	$H = 52 + 0.14625t - 0.000172t^2$
Aceton..............	$H = 140.5 + 0.36644t - 0.000516t^2$

Specific Pressure. — It is customary to develop theoretical thermodynamic equations with the specific pressure expressed in pounds per square foot, for English units. Engineers habitually express pressures in pounds per square inch.

For French units, specific pressures are expressed in kilograms per square meter. Engineers use kilograms per square centimeter, and on the other hand physicists commonly express pressure in millimeters of mercury.

One cubic decimeter (or one liter) of mercury weighs 13.5959 kilograms, and a cubic decimeter is one-thousandth of a cubic

* *Trans. Am. Soc. Mech. Eng.*, 1908.

meter, consequently the pressure of a column of mercury one millimeter high, on a base one meter square, is 13.5959 kilograms.

The normal pressure of the atmosphere is taken to be 760 mm. of mercury (at 0° C.), which is equivalent to 10,333 kilograms per square meter. The normal pressure of the atmosphere is, therefore, 1.0333 kilograms per square centimeter. It was formerly the custom to graduate pressure gauges in atmospheres, for use in countries using the metric system. There is a tendency to confusion of units that are roughly approximate, and in some cases it is necessary to determine whether a pressure is intended to be in atmospheres or in kilograms per square centimeter.

Taking the meter to be equivalent to 39.37 inches, and the kilogram to weigh 2.20462 pounds, then one millimeter of mercury will be equivalent to

$$\frac{13.5959 \times 2.20462}{\overline{39.37}^2} = .019338$$

of a pound per square inch. The normal pressure of the atmosphere is 760 times this, or 14.696 pounds per square inch. The corresponding specific pressure is 2116 pounds per square foot.

Pressure of Saturated Steam. — Recent determinations of the pressure of saturated steam have been made by Holborn and Henning* with all the resources of modern physical methods including the platinum thermometer. Their results reduced to the thermometric scale are set down in Table III of the author's tables, exactly as given in their original report. Their own tests covered the range of temperature from 50° C. to 200° C., but they extend their results to 205° C. The results which they give from freezing-point to 50° C. were deduced by them from experiments of Thiesen and Scheel. In Table III the pressures from 205° to 220° C. are extrapolated by the author by aid of a curve of corrections for Regnault's equation for the range 100° to 220° C.

Holborn and Henning attribute to their own experiments a precision of $\frac{1}{100}$ of a degree Centigrade; this is far beyond technical requirements for direct application, but is needed in the computation of specific volumes, as will appear later. Thiesen

* *Annalen der Physik*, vol. 26, part 4, p. 833, 1908.

and Scheel's experiments had a less degree of precision; and the extrapolation from 205° to 220° C. is open to some doubt.

Pressures of Other Vapors. — Regnault determined the pressures of various vapors and deduced for all of them equations having the form

$$\log p = a + b\alpha^n + c\beta^n. \qquad (99)$$

The following table gives the special forms of the equation and the constants for several vapors:

	$\log \alpha$.	a.	b.	c.
Alcohol	$a - b\alpha^n + c\beta^n$	5.4562028	4.9809960	0.0485397
Ether	$a + b\alpha^n - c\beta^n$	5.0286298	0.0002284	3.1906390
Chloroform	$a - b\alpha^n - c\beta^n$	5.2253893	2.9531281	0.0668673
Carbon bisulphide	$a - b\alpha^n - c\beta^n$	5.4011662	3.4405663	0.2857386
Carbon tetrachloride	$a - b\alpha^n - c\beta^n$	12.0962331	9.1375180	1.9674890

	$\log \alpha$.	$\log \beta$.	n.	Limits.
Alcohol	9.99708557 − 10	9.9409485 − 10	$t + 20$	− 20°, + 150° C.
Ether	0.0145775 − 10	9.996877 − 10	$t + 20$	− 20°, + 120° C.
Chloroform	9.9974144 − 10	9.9868176 − 10	$t - 20$	+ 20°, + 164° C.
Carbon bisulphide	9.9977628 − 10	9.9911997 − 10	$t + 20$	− 20°, + 140° C.
Carbon tetrachloride	9.9997120 − 10	9.9949780 − 10	$t + 20$	− 20°, + 188° C.

Zeuner* gives the following equation for aceton based on Regnault's work:

$$\log p = a - b\alpha^n + c\beta^n;$$
$$a = 5.3085419;$$
$$\log b\alpha^n = + 0.5312766 - 0.0026148\,t;$$
$$\log c\beta^n = - 0.9645222 - 0.0215592\,t.$$

Specific Volume of Liquids. — The coefficient of expansion of most liquids is large as compared with that of solids, but it is small as compared with that of gases or vapors. Again, the specific volume of a vapor is large compared with that of the liquid from which it is formed. Consequently the error of neglecting the increase of volume of a liquid with the rise of temperature is small in equations relating to the thermodynamics of a saturated vapor, or of a mixture of a liquid and its vapor when a considerable part by weight of the mixture is vapor. It

* Mechanische Wärmetheorie.

is, therefore, customary to consider the specific volume of a liquid to be constant.

The following table gives the specific gravities and specific volumes of liquids.

SPECIFIC GRAVITIES AND SPECIFIC VOLUMES OF LIQUIDS.

	Specific Gravity compared with Water at 4° C.	Specific Volume.	
		Cubic Metres.	Cubic Feet.
Alcohol	0.80625	0.001240	
Ether	0.736	0.001350	
Chloroform	1.527	0.000655	
Carbon bisulphide	1.2922	0.000774	
Carbon tetrachloride	1.6320	0.00613	
Aceton	0.81	0.00123	
Sulphur dioxide	1.4336	0.0006981	0.0112
Ammonia	0.6364	0.001571	0.0252
Water	1	0.001	0.01602

Volumes of Liquids. — The volumes of liquids at high temperatures, compared with the volume at freezing-point, are represented by the following equations given by Hirn:* —

		Logs.
Water 100° C. to 200° C. (vol. at 4° C. = unity)	$v = 1 + 0.00010867875t$	6.0361445 − 10
	$+0.0000030073653t^2$	4.4781862 − 10
	$+0.000000028730422t^3$	1.4583419 − 10
	$-0.0000000000066457031t^4$	8.8225409 − 20
Alcohol 30° C. to 160° C. (vol. at 0° C. = unity)	$v = 1 + 0.00073892265t$	6.8685991 − 10
	$+0.00001055235t^2$	3.0233492 − 10
	$-0.000000092480842t^3$	2.9660517 − 10
	$+0.0000000040413567t^4$	0.6065278 − 10
Ether 30° C. to 130° C. (vol. at 0° C. = unity)	$v = 1 + 0.0013489059t$	7.1299817 − 10
	$+0.0000065537t^2$	4.8164866 − 10
	$-0.000000034490756t^3$	2.5377028 − 10
	$+0.0000000033772062t^4$	0.5285571 − 10
Carbon bisulphide 30° to 160° C. (vol. at 0° C. = unity)	$v = 1 + 0.0011680559t$	7.0674636 − 10
	$+0.0000016489598t^2$	4.2172103 − 10
	$-0.0000000081119062t^3$	0.9091229 − 10
	$+0.000000000060946589t^4$	8.7849494 − 20
Carbon tetrahcloride 30° to 160° C. (vol. at 0° C. = unity)	$v = 1 + 0.0010671883t$	7.0282409 − 10
	$+0.0000035651378t^2$	4.5520763 − 10
	$-0.000000014949281t^3$	2.1746202 − 10
	$+0.000000000085182318t^4$	9.9303494 − 20

* *Annales de Chimie et de Physique*, 1867.

Internal and External Latent Heat. — The heat of vaporization overcomes external pressure, and changes the state from liquid to vapor at constant temperature and pressure. Let the specific volume of the saturated vapor be s, and that of the liquid be σ, then the change of volume is $s - \sigma = u$, on passing from the liquid to the vaporous state. The external work is

$$p\,(s - \sigma) = pu,$$

and the corresponding amount of heat, or the external latent heat, is

$$Ap\,(s - \sigma) = Apu,$$

A being the reciprocal of the mechanical equivalent of heat.

That part of the heat of vaporization which is not used in doing external work is considered to be used in changing the state from liquid to vapor. This work required to change the molecular arrangement is called disgregation work. The heat required to do the disgregation work is represented by

$$\rho = r - Apu. \tag{100}$$

Quality or Dryness Factor. — All the properties of saturated steam, such as pressure, volume, and heat of vaporization, depend on the temperature only, and are determinable either by direct experiment or by computation, and are commonly taken from tables like the *Tables of the Properties of Steam, etc.*

Many of the problems met in engineering deal with mixtures of liquid and vapor, such as water and steam. In such problems it is convenient to represent the proportions of water and steam by a variable known as the quality or the dryness factor; this factor, x, is defined as that portion of each pound of the mixture which is steam; the remnant, $1 - x$, is consequently water.

Specific Volume of Wet Steam. — If a pound of a homogeneous mixture of water and steam is x part steam, then the specific volume may be represented by

$$v = xs + (1 - x)\,\sigma = xu + \sigma, \tag{101}$$

where u is the increase of volume due to vaporization.

General Equation. — In order to apply the general thermodynamic method to a mixture of a liquid and its vapor, it is

customary to write a differential equation involving the temperature t, the quality x, the specific heats of water and steam c and h, and the heat of vaporization r; these last three properties are functions of the temperature only.

The principal result of the application of the general method to such an equation is a formula for calculating the specific volume s, as will appear later. In addition to the general method, a special derivation of the formula for s will be given which may be preferred by some readers.

When a mixture of liquid and its vapor receives heat there is in general an increase in the temperature of the portion x of vapor and in the portion $1 - x$ of liquid, and there is a vaporization of part of the liquid. Taking c for the specific heat of the liquid and h for the specific heat of the vapor, while r is the heat of vaporization, we shall have for an infinitesimal change,

$$dQ = hxdt + c\,(1 - x)\,dt + rdx. \qquad (103)$$

Application of the First Law. — The first law of thermodynamics is applied to equation (103) by combining it with equation (16), so that

$$dQ = A\,(dE + pdv) = hxdt + c\,(1 - x)\,dt + rdx;$$

$$\therefore\quad dE = \frac{1}{A}\,[hx + c\,(1 - x)]\,dt + \frac{r}{A}\,dx - pdv.$$

Now v is a function of both t and x, as is evident from equation (101), in which u is a function of t; consequently,

$$dv = \frac{\partial v}{\partial t}\,dt + \frac{\partial v}{\partial x}\,dx.$$

$$\therefore\quad dE = \left\{ \frac{1}{A}[hx + c\,(1 - x)] - p\,\frac{\partial v}{\partial t} \right\} dt + \left[\frac{r}{A} - p\,\frac{\partial v}{\partial x} \right] dx.$$

But E being expressed in terms of t and x gives

$$\frac{\partial^2 E}{\partial t\,\partial x} = \frac{\partial^2 E}{\partial x\,\partial t},$$

so that $\dfrac{\partial}{\partial x}\left\{ \dfrac{1}{A}\,[hx + c(1 - x)] - p\,\dfrac{\partial v}{\partial t} \right\} = \dfrac{\partial}{\partial t}\left(\dfrac{r}{A} - p\,\dfrac{\partial v}{\partial x} \right).$

Bearing in mind that all the functions but x and v are functions of t only, the differentiation gives

$$\frac{1}{A}(h - c) - p\frac{\delta^2 v}{\delta t\, \delta x} = \frac{1}{A}\frac{dr}{dt} - \frac{dp}{dt}\frac{\delta v}{\delta x} - p\frac{\delta^2 v}{\delta x\, \delta t}.$$

Equation (101) gives

$$\frac{\delta v}{\delta x} = u,$$

and

$$\frac{\delta^2 v}{\delta t\, \delta x} = \frac{\delta^2 v}{\delta x\, \delta t};$$

so that the above equation reduces to

$$\frac{dr}{dt} + c - h = Au\frac{dp}{dt} \quad . \quad . \quad . \quad . \quad . \quad (104)$$

Application of the Second Law. — The second law of thermodynamics makes

$$\frac{dQ}{T} = d\phi$$

for a reversible process, so that the general equation (103) may be reduced to

$$\frac{dQ}{T} = \frac{hx + c\,(1 - x)}{T}\,dt + \frac{r}{T}\,dx.$$

But

$$\frac{\delta^2 \phi}{\delta x\, \delta t} = \frac{\delta^2 \phi}{\delta t\, \delta x}.$$

$$\therefore \frac{\delta}{\delta x}\frac{hx + c\,(1 - x)}{T} = \frac{\delta}{\delta t}\frac{r}{T}.$$

$$\therefore \frac{h - c}{T} = \frac{T\dfrac{dr}{dt} - r}{T^2}.$$

$$\therefore \frac{dr}{dt} + c - h = \frac{r}{T} \quad . \quad . \quad . \quad . \quad . \quad (105)$$

First and Second Laws Combined. — The combination of equations (104) and (105) gives

$$r = AuT\frac{dp}{dt} \quad \text{. . . . . (106)}$$

Special Method. — The preceding equation may be obtained by a special method making use of the diagram *abcd* in Fig. 30, which represents Carnot's cycle for a mixture of a liquid and its vapor, the change of temperature ΔT being very small. Let *a* represent the volume of one pound of water at the temperature T, and *b* the volume of one pound of steam at the same temperature and pressure. The line *ab* therefore represents the vaporization of one pound of water at constant temperature, involving the application of the heat of vaporization r, and the increase of volume

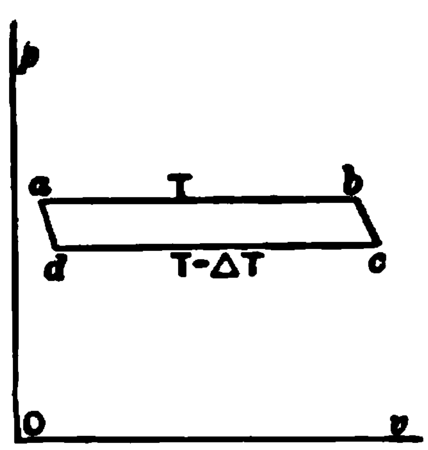

FIG. 30.

$$u = s - \sigma,$$

where s and σ are the specific volumes of steam and water. By the second law of thermodynamics the efficiency of this cycle will be

$$\frac{T-(T-\Delta T)}{T} = \frac{\Delta T}{T},$$

so that the heat changed into work will be

$$\frac{r\Delta T}{T}.$$

But by the first law of thermodynamics this heat is equivalent to the external work, which in this case is approximately equal to the increase of volume u multiplied by the change of pressure Δp; consequently,

$$\frac{r\Delta T}{AT} = u\Delta p,$$

or, at the limit as ΔT approaches zero,

$$r = AuT\frac{dp}{dt}.$$

Specific Volume of Saturated Vapor. — From the extreme difficulty of direct experimental determinations of the specific volume of saturated vapor it has been customary to compute this property by aid of the equation

$$s = u + \sigma = \frac{r}{AT}\frac{1}{\frac{dp}{dt}} + \sigma, \qquad (107)$$

where s is the volume of the vapor and σ is the volume of the liquid; the other quantities are r the heat of vaporization, T the absolute temperature, $\frac{1}{A}$ the mechanical equivalent of heat, and $\frac{dp}{dt}$ the differential coefficient of the pressure with regard to temperature. A close approximation to the differential coefficient may be had by the following process: choosing a temperature (for example 100° C.), take the pressure at two degrees higher (102° C.) and at two degrees lower (98° C.) and divide by 4. The pressures must be in kilograms per square meter. From Table III we deduce

$$\frac{\Delta p}{\Delta t} = \frac{1109.3 - 961.6}{4} = 36.92.$$

The pressures are 1000 times the tabular pressures in kilograms per square centimeter. The expression $\frac{\Delta p}{\Delta t}$ is taken to represent an operation of the nature explained above.

Equation (107) must be used for all other vapors than steam, and for steam at temperatures less than 100° C.; it probably gives the best values for the specific volume of steam at temperatures higher than 100° C., as will appear in the discussion of experimental results.

Specific Volume of Saturated Steam. — The relation of the pressure of saturated steam to the temperature is given by Holborn and Henning in the form of a table of results which are quoted directly in Table III, the pressure being expressed in

millimeters of mercury. It is considered that the best way of dealing with the differential coefficient $\frac{dp}{dt}$ is to replace it by the ratio $\frac{\Delta p}{\Delta t}$, using 4° C. for the interval of temperatures Δt.

A number of elements enter into this consideration. If the relation of the pressure to the temperature could be represented by a second degree curve, that is, if such a curve were a parabola with its axis vertical, the ratio $\frac{\Delta p}{\Delta t}$ for any interval would be precisely equal to $\frac{dp}{dt}$. A table that could be represented by such a curve would have constant second differences; by second differences are meant the differences of the tabular differences as in Table III. An examination of second differences derived from Table III shows that they increase slowly, but that the increase is not perceptible for four degrees. For a six-degree interval the increase is barely perceptible, and for ten degrees it is very apparent. Now the precision claimed for the measurement of temperature is $\frac{1}{100}$ of a degree, so that a four-degree interval appears to give a precision of computation of $\frac{1}{400}$ for a single value of $\frac{\Delta p}{\Delta t}$. It may be noted in passing that the precision of observation of the height of the mercury column is better than the temperature determinations and therefore does not contribute to the probable error.

In order to diminish the effect of local variations of the nature of accidental errors, the values of the ratio $\frac{\Delta p}{\Delta t}$ were computed for each degree of temperature from 0° C. to 220° C. The first and second differences were then computed, and the computed values of $\frac{\Delta p}{\Delta t}$ were changed when necessary to the amount of $\frac{1}{1000}$ in order to make the second differences regular. This process is equivalent to drawing a smooth or fair curve to represent physical properties obtained by observation.

Having values of the ratio $\frac{\Delta p}{\Delta t}$ for each degree of temperature, the specific volumes were computed by equation (107). These values were then tested for fairness by taking second differences, and again the computed values were varied when necessary to the extent of $\frac{1}{1000}$ to make the second differences regular. The combined effect of both fairings is estimated not to exceed $\frac{1}{500}$, and it is believed that the probable error of the specific volumes thus determined is not greater than that amount for the range of temperature 50° C. to 200° C. covered by Holborn and Henning's experiments. This estimate carries with it the assumption that the methods of fairing give somewhat greater mean precision than can be attributed to a single computation of $\frac{\Delta p}{\Delta t}$.

For the range of temperature from 0° C. to 50° C., and especially for temperatures less than 30° C. (86° F.), so small a probable error cannot be claimed for the specific volumes; but that range has less interest for engineers. For temperatures less than 30° C. the specific volumes were derived in the following way. In the first place the values Apu given in Table III were computed from the specific volumes, and a curve was drawn to represent them; above 30° C. the computed values varied from the curve less than $\frac{1}{500}$; in only a few cases was the variation greater than $\frac{1}{1000}$. Below 30° C. it was considered more correct to take values of Apu from the curve which was there appreciably straight, and values of the specific volume were obtained for Table III by inversion of the method of computing Apu. In passing it may be said that all values of Apu in Tables I and III were derived from the curve mentioned, which gave a greater degree of precision than needed for that purpose.

Since the pressures corresponding to temperatures above 200° C. are extrapolated, the specific volumes computed from them are affected by the same degree of uncertainty that attaches to the pressures.

Specific Volumes of Other Vapors. — In order to apply equation (107) to the computation of vapors for which Regnault's equations

are given on page 83, we may derive the differential coefficient in the form

$$\frac{1}{p}\frac{dp}{dt} = A\alpha^n + B\beta^n.$$

The following table gives values to be used for the factors that appear in that equation.

	Sign. $A\alpha^n$	Sign. $B\beta^n$	Log $(A\alpha^n)$.	Log $(B\beta^n)$.
Alcohol	+	−	−1.1720041−0.0029143 t	−2.9992701−0.0590515 t
Ether	+	+	−1.3396624−0.0031223 t	−4.4616396+0.0145775 t
Chloroform	+	+	−1.3410130−0.0025856 t	−2.0667124−0.0131824 t
Carbon bisulphide	+	+	−1.4339778−0.0022372 t	−2.0511078−0.0088003 t
Carbon tetrachloride	+	+	−1.8611078−0.0002880 t	−1.3812195−0.0050220 t
Aceton	+	+	−1.3268535−0.0026148 t	−1.9064582−0.0215592 t
			t, temperature C.	

Experimental Determinations of Specific Volumes. — By far the best direct determinations of the specific volumes of saturated steam are those reported by Knoblauch,* Linde, and Klebe in connection with their determinations of the properties of superheated steam. These experiments determined the pressures and temperature at constant volume, and the results are so treated as to give the volume at saturation by extrapolation with great certainty. In their report they claim for their results, including volumes at saturation, a probable error not greater than $\frac{1}{500}$.

COMPARISON OF EXPERIMENTAL AND COMPUTED VALUES OF THE SPECIFIC VOLUME OF SATURATED STEAM.

Temperature.	Volume Cu. M.			Temperature.	Volume Cu. M.		
	Experimental.	Computed.	Per Cent Deviation.		Experimental.	Computed.	Per Cent Deviation.
100	1.674	1.671	+0.18	145	0.4458	0.4457	+0.02
105	1.421	1.419	+0.14	150	0.3927	0.3921	+0.15
110	1.211	1.209	+0.17	155	0.3466	0.3463	+0.09
115	1.036	1.036	0	160	0.3069	0.3063	+0.20
120	0.8894	0.8910	−0.18	165	0.2724	0.2729	+0.18
125	0.7688	0.7698	−0.13	170	0.2426	0.2423	+0.12
130	0.6670	0.6677	−0.10	175	0.2168	0.2164	+0.19
135	0.5809	0.5812	−0.05	180	0.1940	0.1941	−0.05
140	0.5080	0.5081	−0.02	......	....	....	....

* *Mitteilungen über Forschungsarbeiten*, etc., Heft 21, S. 33, 1905.

These experimenters give 32 determinations of the volume of saturated steam. In order to make a comparison of these experimental values with computations in Table III, a large plot was made with temperatures for abscissæ and logarithms of volumes for ordinates, and a fair curve was drawn; from this curve the experimental values set down in the preceding table were deduced; the computed values are taken from Table III.

The greatest deviation is 0.2 of one per cent, which is the probable error assigned by the experimenters to their work. It may therefore be concluded that the claim of a probable error not in excess of $\frac{1}{500}$ for the computed values of the specific volume of saturated steam, and of a similar degree of precision for the experimental values, is warranted.

Now equation (107) includes explicitly the heat of vaporization, the absolute temperature and the mechanical equivalent of heat as well as the differential coefficient $\frac{dp}{dt}$. It also includes the heat of the liquid implicitly, since the heat of vaporization is derived from the total heat. Consequently the claim of a precision of $\frac{1}{500}$ for the specific volume attributes a like degree of precision to the first three named properties, and the same effective certainty to the heat of the liquid. It is true that we may independently attribute a greater precision to the three first properties named. Thus a probable error of $\frac{1}{1000}$ is claimed for the total heat by Davis, and Callendar* claims a probable error of $\frac{1}{5000}$ or better for the absolute temperature; the real value of the mechanical equivalent is even now slightly in question, but the value assigned is probably in error less than $\frac{1}{1000}$.

The conclusion appears to be that our knowledge of the properties of saturated steam is sufficient for engineering purposes, and that tables computed with available data will not require change.

Nature of the Specific Heats. —In the application of the general thermodynamic method on page 86 the term h is introduced to represent the specific heat of saturated steam, and there is some interest in the determination of the true nature of this

* *Phil. Mag.*, Jan., 1903.

property, which clearly cannot be a specific heat at constant pressure, nor a specific heat at constant volume, since both pressure and volume change with the temperature. The specific heat of the liquid c properly is affected by the same consideration, but as the increase of volume is small and is neglected in thermodynamic discussions, the importance of the consideration is much less. The specific heat h of saturated vapor is the amount of heat necessary to raise the temperature of one pound of the vapor one degree, under the condition that the pressure shall increase with the temperature, according to the law for saturated vapor.

Equation (105) gives a ready way of calculating the specific heat for a vapor, for from it

$$h = c + \frac{dr}{dt} - \frac{r}{T}.$$

Above boiling-point the total heat of water may be expressed by equation (98), page 81, so that we may write

$$r = H - q = H - \int c dt$$

and

$$\frac{dr}{dt} = \frac{dH}{dt} - c:$$

consequently

$$h = \frac{dH}{dt} - \frac{r}{T} = .3745 - 0.00198\,(t - 100) - \frac{r}{T}.$$

The following tables give a few values of h.

SPECIFIC HEAT OF SATURATED STEAM.

Temperature C.	100	125	150	175	200
Temperature F.	212	257	302	347	392
Pressure, pounds per sq. in.	14.7	33.7	69.0	129.4	225.2
Specific heat h	− 1.070	− 0.986	− 0.915	− 0.853	− 0.804

The negative sign shows that heat must be abstracted from saturated steam when the temperature and pressure are increased, otherwise it will become superheated. On the other hand, steam, when it suddenly expands with a loss of temperature and pressure, suffers condensation, and the heat thus liberated supplies that required by the uncondensed portion.

Hirn* verified this conclusion by suddenly expanding steam in a cylinder with glass sides, whereupon the clear saturated steam suffered partial condensation, as indicated by the formation of a cloud of mist. The reverse of this experiment showed that steam does not condense with sudden compression, as shown by Cazin.

Ether has a positive value for h. As the theory indicates, a cloud is formed during sudden compression, but not during sudden expansion.

The table of values of h for steam shows a notable decrease for higher temperatures, which indicates a point of inversion at which h is zero and above which h is positive, but the temperature of that point cannot be determined from our experimental knowledge. For chloroform the point of inversion was calculated by Cazin† to be 123°.48, and determined experimentally by him to be between 125° and 129°. The discrepancy is mostly due to the imperfection of the apparatus used, which substituted finite changes of considerable magnitude for the indefinitely small changes required by the theory.

Isothermal Lines. — Since the pressure of saturated vapor is a function of the temperature only, the isothermal line of a mixture of a liquid and its vapor is a line of constant pressure, parallel to the axis of volumes. Steam expanding from the boiler into the cylinder of an engine follows such a line; that is, the steam-line of an automatic cut-off engine with ample ports is nearly parallel to the atmospheric line.

The heat required for an increase of volume at constant pressure is

$$Q = r(x_2 - x_1) \quad . \quad . \quad . \quad . \quad . \quad (108)$$

in which r is the heat required to vaporize one pound of liquid and x_1 and x_2 are the initial and final qualities, so that $x_2 - x_1$ is the weight of liquid vaporized.

The external work done during an isothermal expansion is

$$W = p\,(v_2 - v_1) = pu\,(x_2 - x_1) \quad . \quad . \quad . \quad (109)$$

* *Bulletin de la Société Ind. de Mulhouse*, cxxxiii.
† *Comptes rendus de l'Académie des Sciences*, lxii.

Intrinsic Energy. — Of the heat required to raise a pound of any liquid from freezing-point to a given temperature and to completely vaporize it at that temperature, a part q is required to increase the temperature, another part ρ is required to change the state or do disgregation work, and a third part Apu is required to do the external work of vaporization. Consequently for complete vaporization we may have,

$$Q = A\ (S + I + W) = q + \rho + Apu = H.$$

For partial vaporization the heat required to do the disgregation work will be $x\rho$, and the heat required to do the external work will be $Apxu$. Therefore the heat required to raise a pound of a liquid from freezing-point to a given temperature and to vaporize x part of it will be

$$Q = q + x\rho + Apxu = A(E + W)$$

where E is the increase of intrinsic energy from freezing-point. It is customary to consider that

$$E = \frac{1}{A}(x\rho + q) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (110)$$

represents the intrinsic energy of one unit of weight of a mixture of a liquid and its vapor.

Isoenergic or Isodynamic Lines. — If a change of a mixture of a liquid and its vapor takes place at constant intrinsic energy, the value of E will be the same at the initial and final conditions, and

$$q_2 - q_1 + x_2\rho_2 - x_1\rho_1 = 0 \quad . \quad . \quad . \quad . \quad (111)$$

which equation, with the formulæ

$$v_2 = x_2u_2 + \sigma; \quad v_1 = x_1u_1 + \sigma \quad . \quad . \quad . \quad . \quad (112)$$

enable us to compute the initial and final volumes. If desired, intermediate volume corresponding to intermediate temperature can be computed in the same way, and a curve can be drawn in the usual way with pressures and volumes for the coordinates.

For example, if a mixture of $\frac{9}{10}$ steam and $\frac{1}{10}$ water expands

isoenergically from 100 pounds absolute to 15 pounds absolute, the final condition will be

$$x_2 = \frac{q_1 - q_2 + x_1\rho_1}{\rho_2} = \frac{298.5 - 181.3 + 0.9 \times 805.7}{896.2} = 0.9399.$$

The initial and final specific volumes are

$$v_1 = x_1 u_1 + \sigma = 0.9\ (4.432 - 0.016) + 0.016 = 3.990;$$

$$v_2 = x_2 u_2 + \sigma = 0.9399\ (26.28 - 0.016) + 0.016 = 24.70.$$

The converse problem requiring the pressure corresponding to a given volume cannot be solved directly. The only method of solving such a problem is to assume a probable final pressure and find the corresponding volume; then, if necessary, assume a new final pressure larger or smaller as may be required, and solve for the volumc again; and so on until the desired degree of accuracy is obtained.

This method does not give an explicit equation connecting the pressures and volumes, but it will be found on trial that a curve, drawn by the process given above can be represented fairly well by an exponential equation, for which the exponent can be determined by the method on page 66.

Having given or determined the initial and final volumes, the exponential equation may be determined, and then the external work may be calculated by the equation

$$W = \int pdv = \frac{p_1 v_1}{n - 1}\left\{ 1 - \left(\frac{v_1}{v_2}\right)^{n-1} \right\}$$

For example, the exponent for the equation representing the expansion of the above problem is

$$n = \frac{\log p_1 - \log p_2}{\log v_2 - \log v_1} = \frac{\log 100 - \log 15}{\log 24.70 - \log 3.990} = 1.041,$$

and the external work of expansion is

$$W = \frac{100 \times 144 \times 3.990}{1.041 - 1}\left\{ 1 - \left(\frac{3.990}{24.70}\right)^{0.041} \right\} = 100,800 \text{ ft.-lbs.}$$

Since there is no change in the intrinsic energy during an isoenergic expansion, the external work is equivalent to the heat applied. Thus in the example just solved the heat applied is equal to

$$100{,}000 \div 778 = 130 \text{ B.T.U.}$$

There is little occasion for the use of the method just given, which is fortunate, as it is not convenient.

Entropy of the Liquid. — Suppose that a unit of weight of a liquid is intimately mingled with its vapor, so that its temperature is always the same as that of the vapor; then if the pressure of the vapor is increased the liquid will be heated, and if the vapor expands the liquid will be cooled. So far as the unit of weight of the liquid under consideration is concerned, the processes are reversible, for it will always be at the temperature of the substance from which it receives or to which it imparts heat, i.e., it is always at the temperature of its vapor.

The change of entropy of the liquid can therefore be calculated by equation (37),

$$d\phi = \frac{dQ}{T},$$

which may here be written

$$\theta = \int \frac{dq}{T} = \int \frac{cdt}{T} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (113)$$

On page 83 it is suggested that the specific heat of water for temperature Centigrade may be expressed as follows:

$$c = 1 + k$$

where k is a small corrective term that may be positive or negative as the case may be. Using this correction, equation (113) may be written

$$\theta = \int \frac{dt}{T} + \int \frac{kdt}{T} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (114)$$

The first term can readily be integrated and computed, and the second term, which is small, can be determined graphically, so that the expression for entropy of water becomes

$$\theta = \log_e \frac{T}{T_0} + \int_{t_0}^{t} k \frac{dt}{T} \quad . \quad . \quad . \quad . \quad . \quad . \quad (115)$$

The columns of entropy of water in the tables were determined in this manner.

In the discussion of entropy on page 31 it was pointed out that there is no natural zero of entropy corresponding to the absolute zero of temperature. It is customary to treat the freezing-point of water as the zero of entropy both for that liquid and for other volatile liquids; some liquids therefore have negative entropies at temperatures below freezing-point of water in the appropriate tables of their properties.

For a liquid like ether which has the heat of the liquid represented by an empirical equation,

$$q = 0.52901\, t + 0.0002959\, t^2,$$

the specific heat is first obtained by differentiation, giving

$$c = 0.52901 + 0.0005918\, t.$$

Then the increase of entropy above that for the freezing-point of water may be obtained by aid of equation (113), which gives for ether with the French system of units,

$$\theta = \int_{273}^{T} \left\{ 0.52901 + 0.0005918\ (T - 273) \right\} \frac{dt}{T};$$

$$\therefore \quad \theta = \int_{273}^{T} \left(0.3670 \frac{dt}{T} + 0.0005918\, dt \right);$$

$$\therefore \quad \theta = 0.0005918\ (T - 273) + 0.3670 \log_e \frac{T}{273}:$$

$$\therefore \quad \theta = 0.0005918\, t + 0.3670 \log_e \frac{T}{273} \quad . \quad . \quad . \quad . \quad . \quad (116)$$

For temperatures below the freezing-point of water, equation (116) gives negative numerical results.

Other liquids for which equations for the heat of the liquid are given on page 83, may be treated in a similar method.

Entropy due to Vaporization. — When a unit of weight of a liquid is vaporized r thermal units, equal to the heat of vaporization, must be applied at constant temperature. Treating such a vaporization as a reversible process, the change of entropy may be calculated by the equation

$$\phi - \phi_0 = \frac{r}{T}$$

This property is given in the "Tables for Saturated Steam," but not in general for other liquids.

Entropy of a Mixture of a Liquid and its Vapor. — The increase in entropy due to heating a unit of weight of a liquid from freezing-point to the temperature t and then vaporizing x portion of it is

$$\theta + \frac{xr}{T},$$

where θ is the entropy of the liquid, r is the heat of vaporization, and T is the absolute temperature. For steam $\frac{r}{T}$ may be taken from the tables; for other vapors it must usually be calculated.

For any other state determined by x_1 and t_1 we shall have, for the increase of entropy above that of liquid at freezing-point,

$$\frac{x_1 r_1}{T_1} + \theta_1.$$

The change of entropy in passing from one state to another is

$$\phi - \phi_1 = \frac{xr}{T} + \theta - \frac{x_1 r_1}{T_1} - \theta_1 \quad . \quad . \quad . \quad (117)$$

When the condition of the mixture of a liquid and its vapor is given by the pressure and value of x, then a table giving the properties at each *pound* may be conveniently used for this work.

Adiabatic Equation for a Liquid and its Vapor. — During an adiabatic change the entropy is constant, so that equation (117) gives

$$\frac{x_1 r_1}{T_1} + \theta_1 = \frac{x_2 r_2}{T_2} + \theta_2 \quad . \quad . \quad . \quad . \quad . \quad (118)$$

When the initial state, determined by x_1 and t_1 or p_1, is known and the final temperature t_2, or the final pressure p_2, the final value x_2 may be found by equation (118). The initial and final volumes may be calculated by the equations

$$v_1 = x_1 u_1 + \sigma \quad \text{and} \quad v_2 = x_2 u_2 + \sigma \quad . \quad . \quad . \quad (119)$$

Tables of the properties of saturated vapor commonly give the specific volume s_1 but

$$s = u + \sigma.$$

The value of σ for water is 0.016, and for other liquids will be found on page 85.

For example, one pound of dry steam at 100 pounds absolute pressure will have the values

$$\frac{r_1}{T_1} = 1.1273, \quad \theta_1 = 0.4748, \quad x_1 = 1.$$

If the final pressure is 15 pounds absolute, we have

$$\frac{r_2}{T_2} = 1.4409, \quad \theta_2 = 0.3143;$$

whence

$$1.1273 + 0.4748 = 1.4409\, x_2 + 0.3143;$$

$$\therefore\ x_2 = 0.894.$$

The initial and final volumes are

$$v_1 = s_1 = 4.43$$

$$v_2 = x_2 u_2 + \sigma = 23.5.$$

Problems in which the initial condition and the final temperature or pressure are given may be solved directly by aid of the preceding equations. Those giving the final volume instead

of the temperature or pressure can be solved only by approximations. An equation to an adiabatic curve in terms of p and v cannot be given, but such a curve for any particular case may be constructed point by point.

Clausius and Rankine independently and at about the same time deduced equations identical with equations (117) and (118), but by methods each of which differed from that given here.

Rankine called the function

$$\theta + \frac{xr}{T}$$

the *thermodynamic function*; Clausius called it entropy.

In the discussion of the specific heat h of a saturated vapor, it appeared that the expansion of dry saturated steam in a non-conducting cylinder would be accompanied by partial condensation. The same fact may be brought out more clearly by the above problem.

On the other hand, h is positive for ether, and partial condensation takes place during compression in a non-conducting cylinder.

For example, let the initial condition for ether be

$$t_1 = 10^\circ \text{ C.}, \quad r_1 = 93.12, \quad \theta = 0.0191, \quad x_1 = 1,$$

and let the final conditions be

$$t_2 = 120^\circ \text{ C.}, \quad r_2 = 72.26, \quad \theta_2 = 0.2045;$$

then
$$\frac{93.12}{283} + 0.0191 = \frac{72.26x_2}{393} + 0.2045,$$

and
$$x_2 = 0.724.$$

Equation (118) applies to all possible mixtures of a liquid and its vapor, including the case of $x_1 = 0$ or the case of liquid without vapor, but at the pressure corresponding to the temperature according to the law of saturated vapor. When applied to hot water, this equation shows that an expansion in a non-conducting cylinder is accompanied by a partial vaporization.

There is some initial state of the mixture such that the value of x shall be the same at the beginning and at the end, though it may vary at intermediate states. To find that value make $x_2 = x_1$ in equation (118) and solve for x_1, which gives

$$x_1 = \frac{\theta_1 - \theta_2}{\frac{r_2}{T_2} - \frac{r_1}{T_1}}.$$

The value of x_1 for steam to fulfil the conditions given varies with the initial and final temperatures chosen, but in any case it will not be much different from one half. It may therefore be generally stated that a mixture of steam and water, when expanded in a non-conducting cylinder, will show partial condensation if more than half is steam, and partial evaporation if more than half water. If the mixture is nearly half water and half steam, the change must be investigated to determine whether evaporation or condensation will occur; but in any case the action will be small.

External Work during Adiabatic Expansion. — Since no heat is transmitted during an adiabatic expansion, all of the intrinsic energy lost is changed into external work, so that, by equation (110),

$$W = E_1 - E_2 = \frac{1}{A}(q_1 - q_2 + x_1\rho_1 - x_2\rho_2) \quad . \quad . \ (120)$$

For example, the external work of one pound of dry steam in expanding adiabatically from 100 pounds to 15 pounds absolute is

$$W = 778\ (298.5 - 181.3 + 1 \times 805.7 - 0.894 \times 896.2)$$

$$W = 121.7 \times 778 = 94{,}700 \text{ foot-pounds.}$$

Attention should be called to the unavoidable defect of this method of calculation of external work during adiabatic expansion, in that it depends on taking the difference of quantities which are of the same order of magnitude. For example, the above calculation appears to give four places of significant figures,

while, as a matter of fact, the total heat H from which ρ is derived is affected by a probable error of $\frac{1}{1000}$. Both the quantities

$$q_1 + x_1\rho_1 \text{ and } q_2 + x_2\rho_2$$

have a numerical value somewhere near 1000, and an error of $\frac{1}{1000}$ is nearly equivalent to one thermal unit, so that the probable error of the above calculation is nearly one per cent. For a wider range of temperature the error is less, and for a narrower range it is of course larger. This matter should be borne in mind in considering the use of approximate methods of calculations; for example, the temperature-entropy diagram to be discussed later.

The adiabatic curve cannot be well represented by an exponential equation; for if an exponent be determined for such a curve passing through points representing the initial and final states, it will be found that the exponent will vary widely with different ranges of pressure, and still more with different initial values of x; and that, further, the intermediate points will not be well represented by such an exponential curve even though it passes through the initial and final points.

This fact was first pointed out by Zeuner, who found that the most important element in determining n was x_1, the initial condition of the mixture. He gives the following empirical formula for determining n, which gives a fair approximation for ordinary ranges of temperature:

$$n = 1.035 + 0.100x_1.$$

There does not appear to be any good reason for using an exponential equation in this connection, for all problems can be solved by the method given, and the action of a lagged steam-engine cylinder is far from being adiabatic. An adiabatic line drawn on an indicator-diagram is instructive, since it shows to the eye the difference between the expansion in an actual engine and that of an ideal non-conducting cylinder; but it can

be intelligently drawn only after an elaborate engine test. For general purposes the hyperbola is the best curve for comparison with the expansion curve of an indicator-diagram, for the reason that it is the conventional curve, and is near enough to the curve of the diagrams from good engines to allow a practical engineer to guess at the probable behavior of an engine, from the diagram alone. It cannot in any sense be considered as the theoretical curve.

Temperature-Entropy Diagram. — If the entropies of the liquid and the entropies of vaporization for steam are plotted with temperature for ordinates we get a diagram like 30a; very commonly absolute temperatures are taken in drawing the diagram in order to emphasize the rôle played by absolute temperatures in the determination of the efficiency of Carnot's cycle. It would seem better to take the temperature by the centigrade or the Fahrenheit thermometer, as they are the basis of steam-tables, and the temperature-entropy diagram is the equivalent of such a table.

FIG. 30a.

Now the entropy of a mixture containing x part steam is

$$\theta + x\frac{r}{T};$$

so that the entropy of a mixture containing x part of steam can be determined by dividing the line such as de (which represents the entropy of vaporization) in the proper ratio.

$$\frac{dc}{de} = x.$$

It is convenient to divide the several lines like ab and de into tenths and hundredths, and then, if an adiabatic expansion is

represented by a vertical line like bc, the entropy at c may be determined by inspection of the diagram. *Conversely*, by noting the temperature at which a given line of constant entropy crosses a line of given quality we may determine the temperature to which it is necessary to expand to attain that quality, a determination which cannot be made directly by the equation.

When a temperature-entropy diagram is used as a substitute for a "Table of the Properties of Saturated Steam," it is customary to draw the lines of constant quality or dryness factor, and other lines like constant volume lines and lines of constant heat contents or values of the expression

$$xr + q,$$

the use of which will appear in the discussion of steam-engines and steam-turbines.

To get a series of constant volume lines we may compute the volume for each quality $x_1 = .1_1$, $x_2 = .2$, $x = .3$, etc., by the equation

$$v = xu + \sigma,$$

and since the volume increases proportionally to the increase in x, we may readily determine the points on that line for which the volume shall be whole units, such as 2 cubic feet, 3 cubic feet, etc. Points for which the volumes are equal may now be connected by fair curves, so that for any temperature and entropy the volume may readily be estimated.

Curves of equal heat contents can be constructed in a similar way.

If desired, a curve of temperatures and pressures can be drawn so that many problems can be solved approximately by aid of the compound diagram.

At the back of this book a temperature-entropy diagram will be found which gives the properties of saturated and superheated steam. It is provided with a scale of temperatures at either side, and a scale of entropies at the bottom, while there is a scale of pressure at the right.

To solve a problem like that on page 100, i.e., to find the quality after an adiabatic expansion from 100 pounds absolute to 15 pounds absolute, and the specific volume at the initial and final states, proceed as follows:

From the curve of temperatures and pressures, select the temperature line which corresponds to 100 pounds and note where it cuts the saturation curve, because it is assumed that the steam is initially dry. The diagram gives the entropy as approximately 1.61. Note the temperature line which cuts the temperature-pressure curve at 15 pounds, and estimate the value of x from its intersection with the entropy line 1.61; by this method the value of x is found to be about 0.89. In like manner the volume may be estimated to be about 23.4 cubic feet.

Temperature-Entropy Table. — Now that the computation of isoentropic changes has ceased to be the diversion of students of theoretical investigations and has become the necessity of engineers who are engaged in such matters as the design of steam-turbines, the somewhat inconvenient methods which were incapable of inverse solutions, have become somewhat burdensome. A remedy has been sought in the use of temperature-entropy diagrams just described. Such a diagram to be really useful in practice must be drawn on so large a scale as to be very inconvenient, and even then is liable to lack accuracy. To meet this condition of affairs a temperature-entropy *table* has been computed and added to the "Tables of the Properties of Saturated Steam." In this table each degree Fahrenheit from 180° to 430° is entered together with the corresponding pressure. There have been computed and entered in the proper columns the following quantities, namely, *quality* x, *heat contents* $xr + q$, and *specific volume* v, for each hundredth of a unit of entropy.

In the use of this table it is recommended to take the nearest degree of temperature corresponding to the absolute pressure if pressures are given. Following the line across the table select that column of entropy which corresponds most nearly with the initial condition; the corresponding initial volume may be read directly. Follow down the entropy column to the lower temper-

ature and then find the value of x and the specific volume. The external work for adiabatic expansion may now readily be found by aid of equation (120), page 102. As will appear later, the problems that arise in practice usually require the heat contents and not the intrinsic energy, so that property has been chosen in making up the table.

For example, the nearest temperature to 100 pounds per square inch is 328° F.; the entropy column 1.59 gives for x, 0.995, which indicates half of one per cent of moisture in the steam. The corresponding volume is 4.39 cubic feet. The nearest temperature to 15 pounds absolute is 213° F., and at 1.59 entropy the quality is 0.888 and the specific volume corresponding is 23.2 cubic feet.

If greater accuracy is desired we must resort to interpolation. Usually it will be sufficient to interpolate between the lines for temperature in a given column of entropy, because the quantity that must be determined accurately is usually the *difference*

$$x_1r_1 + q_1 - (x_2r_2 + q_2)$$

and this difference for two given temperatures t_1 and t_2 is very nearly the same if taken out of two adjacent entropy columns. A similar result will be found for the difference

$$x_1\rho_1 + q_1 - (x_2\rho_2 + q_2),$$

if computed for values of x found in adjacent columns.

Another way of looking at this matter is that one hundredth of a unit of entropy at 330 pounds corresponds to one per cent of moisture.

Evidently this table can be used to solve problems in which the final volumes are given, or, as will appear later, to determine intermediate pressures for steam-turbines.

EXAMPLES.

1. Water at 100° F. is fed to a boiler in which the pressure is 120 pounds absolute per square inch. How much heat must be supplied to evaporate each pound? Ans. 1121.2 B.T.U.

2. One pound wet steam at 150 pounds absolute occupies 2.5 cubic feet. What per cent of moisture is present? What is the "quality" of the steam? Ans. 17.05 per cent of moisture $x = 0.8295$.

3. A pound of steam and water at 150 pounds pressure is 0.6 steam. What is the increase of entropy above that of water at 32° F.? Ans. 1.1473.

4. A kilogram of chloroform at 100° C. is 0.8 vapor. What is the increase of entropy above that of the liquid at 0° C.? Ans. 0.1959.

5. The initial condition of a mixture of water and steam is $t = 320°$ F., $x = 0.8$. What is the final condition after adiabatic expansion to 212° F.? Ans. 0.74.

6. The initial condition of a mixture of steam and water is $p = 3000$ mm., $x = 0.9$. Find the condition after an adiabatic expansion to 600 mm. Ans. 0.830.

7. A cubic foot of a mixture of water and steam, $x = 0.8$, is under the pressure of 60 pounds by the gauge. Find its volume after it expands adiabatically till the pressure is reduced to 10 pounds by the gauge; also the external work of expansion. Ans. 2.69 cubic feet and 10,000 foot-pounds.

8. Three pounds of a mixture of steam and water at 120 pounds absolute pressure occupy 4.5 cubic feet. How much heat must be added to double the volume at the same pressure, and what is the change of intrinsic energy? Ans. 1065 B.T.U.; 750,100 foot-pounds.

9. Find the intrinsic energy, heat contents and volume of 5 pounds of a mixture of water and steam which is 80 per cent steam, the pressure being 150 pounds absolute. Ans. Intrinsic energy, 3,709,000; heat contents, 5102 B.T.U.; volume, 12.1 cubic feet.

10. Three pounds of water are heated from 60° F. and evaporated under 135.3 pounds gauge pressure. Find the heat added, and the changes in volume, and intrinsic energy. Ans. Heat added, 3498 B.T.U.; increase in volume, 9.00 cubic feet; intrinsic energy, 2,530,000.

11. A pound of steam at 337°.7 F. and 100 pounds gauge pressure occupies 3 cubic feet. Find its intrinsic energy and its entropy above 32° F. Ans. Intrinsic energy, 718,000; entropy, 1.338.

12. Two pipes deliver water into a third. One supplies 300 gallons per minute at 70° F.; the other, 90 gallons per minute at 200° F. What is the temperature of the water after the two streams unite? Ans. 100°.1 F.

13. A test of an engine with the cut-off at 0.106 of the stroke, and the release at 0.98 of the stroke, and with 4.5 per cent clearance, gave for the pressure at cut-off 62.2 pounds by the indicator, and at release 6.2 pounds; the mixture in the cylinder at cut-off was 0.465 steam, and at release 0.921 steam. Find (1) condition of the mixture in the cylinder at release on the assumption of adiabatic expansion to release; (2) condition of mixture on the assumption of hyperbolic expansion, or that $pv = p_1v_1$; (3) the exponent of an exponential curve passing through points of cut-off and release; (4) exponent of a curve passing through the initial and final points on the assumption of adiabatic expansion; (5) the piston displacement was 0.7 cubic feet, find the external work under exponential curve passing through the points of cut-off and release; also under the adiabatic curve. Ans. (1) 0.473; (2) 0.530; (3) $n = 0.6802$; (4) $n = 1.0565$; (5) 3093 and 2130 foot-pounds.

CHAPTER VII.

SUPERHEATED VAPORS.

A DRY and saturated vapor, not in contact with the liquid from which it is formed, may be heated to a temperature greater than that corresponding to the given pressure for the same vapor when saturated; such a vapor is said to be superheated. When far removed from the temperature of saturation, such a vapor follows the laws of perfect gases very nearly, but near the temperature of saturation the departure from those laws is too great to allow of calculations by them for engineering purposes.

All the characteristic equations that have been proposed, have been derived from the equation

$$pv = RT,$$

which is very nearly true for the so-called perfect gases at moderate temperatures and pressures; it is, however, well known that the equation does not give satisfactory results at very high pressures or very low temperatures. To adapt this equation to represent superheated steam, a corrective term is added to the right-hand side, which may most conveniently be assumed to be a function of the temperature and pressure, so that calculations by it may be made to join on to those for saturated steam.

The most satisfactory characteristic equation of this sort is that given by Knoblauch,* Linde, and Klebe,

$$pv = BT - p\,(1 + ap)\left[C\left(\frac{373}{T}\right)^3 - D\right] \quad . \quad . \quad (121)$$

in it the pressure is in kilograms per square metre, v is in cubic metres, and T is the absolute temperature by the

* *Mitteilungen über Forschungsarbeiten*, etc., Heft 21, S. 33, 1905.

centigrade thermometer. The constants have the following values:

$$B = 47.10, \quad a = 0.000002, \quad C = 0.031, \quad D = 0.0052.$$

In the English system of units, the pressures being in pounds per square foot, the volumes in cubic feet per pound, and the temperatures on the Fahrenheit scale, we have

$$pv = 85.85\,T - p(1 + 0.00000976\,p)\left(\frac{150,300,000}{T^3} - 0.0833\right) \quad (122)$$

The following equation may be used with the pressure in pounds per square *inch:*

$$pv = 0.5962\,T - p\,(1 + 0.0014\,p)\left(\frac{150,300,000}{T^3} - 0.0833\right) \quad (123)$$

The labor of calculation is principally in reducing the corrective term, and especially in the computation of the factor containing the temperature. A table on page 112 gives values of this factor for each five degrees from 100° to 600° F.; the maximum variation in the calculation of volume by aid of the table without interpolation is about 0.4 of one per cent at 336 pounds pressure and 428° F.; that is at the upper limit of our table for saturated steam. At 150 pounds and 358° F., which is about the middle range of our table for saturated steam, the variation is not more than 0.2 of one per cent, which is not greater than the probable error of the equation itself under those conditions. At lower pressures and at higher temperatures the error tends to diminish. Exact results can be had at all temperatures by interpolation in the table.

Knoblauch attributes to his equation a probable error of 0.2 of a per cent within the range of his experiments which extends from 100° C. to 180° C., and to about 50° C. of superheating. It has been shown that a special treatment of his experimental values extrapolated to saturation shows at no place a greater discrepancy from the tabular values of Table III than 0.2 of a per cent. His equation is nearly as good, the maximum discrepancy within his

range being one-third of a per cent at 160° C. Below boiling-point the greatest discrepancy of his equation is half a per cent at 50° C.; toward freezing-point the decrepancy decreases to zero.

TABLE I.

Values of the factor $\frac{150,300,000}{T^{3}} - 0.0833$.

Temperature.		Value of Factor.	Temperature.		Value of Factor.	Temperature.		Value of Factor.	Temperature.		Value of Factor.
Fahr.	Abs.		Fahr.	Abs.		Fahr.	Abs.		Fahr.	Abs.	
200	659.5	0.441	300	759.5	0.260	400	859.5	0.153	500	959.5	0.087
205	664.5	0.429	305	764.5	0.253	405	864.5	0.149	505	964.5	0.084
210	669.5	0.417	310	769.5	0.247	410	869.5	0.145	510	969.5	0.083
215	674.5	0.405	315	774.5	0.240	415	874.5	0.141	515	974.5	0.079
220	679.5	0.395	320	779.5	0.234	420	879.5	0.138	520	979.5	0.077
225	684.5	0.385	325	784.5	0.228	425	884.5	0.134	525	984.5	0.074
230	689.5	0.375	330	789.5	0.222	430	889.5	0.131	530	989.5	0.072
235	694.5	0.365	335	794.5	0.216	435	894.5	0.127	535	994.5	0.070
240	699.5	0.356	340	799.5	0.211	440	899.5	0.123	540	999.5	0.067
245	704.5	0.347	345	804.5	0.205	445	904.5	0.120	545	1004.5	0.065
250	709.5	0.338	350	809.5	0.200	450	909.5	0.117	550	1009.5	0.063
255	714.5	0.329	355	814.5	0.195	455	914.5	0.113	555	1014.5	0.061
260	719.5	0.320	360	819.5	0.190	460	919.5	0.110	560	1019.5	0.059
265	724.5	0.312	365	824.5	0.185	465	924.5	0.107	565	1024.5	0.057
270	729.5	0.304	370	829.5	0.180	470	929.5	0.104	570	1029.5	0.055
275	734.5	0.296	375	834.5	0.175	475	934.5	0.101	575	1034.5	0.053
280	739.5	0.288	380	839.5	0.171	480	939.5	0.098	580	1039.5	0.051
585	744.5	0.281	385	844.5	0.166	485	944.5	0.095	585	1044.5	0.049
290	749.5	0.274	390	849.5	0.162	490	949.5	0.092	590	1049.5	0.047
295	754.5	0.267	395	854.5	0.158	495	954.5	0.090	595	1054.5	0.045

Specific Heat. — Two investigations have been made of the specific heat of superheated steam at constant pressure, one by Professor Knoblauch* and Dr. Jakob and the other by Professor Thomas and Mr. Short;† the results of the latter's investigation have been communicated for use in this book in anticipation of the publication of the completed report.

* *Mitteilungen über Forschungsarbeiten*, Heft 36, p. 109.

† Thesis by Mr. Short, Cornell University

Professor Knoblauch's report gives the results of the investigations made under his direction in the form of a table giving specific heats at various temperatures and pressures and in a diagram, which can be found in the original memoir, and he also gives a table of mean specific heats from the temperature of saturation to various temperatures at several pressures. This latter table is given here in both the metric system and in the English system of units.

SPECIFIC HEAT OF SUPERHEATED STEAM.

Knoblauch and Jakob

p Kg per Sq Cm		1	2	4	6	8	10	12	14	16	18	20
p Lbs per Sq In.		14.2	28.4	56.9	85.3	113.8	142.2	170.6	199.1	227.5	156.0	284.4
t_s Cent.		99°	120°	143°	158°	169°	179°	187°	194°	200°	206°	211°
t_s Fahr.		210°	248°	289°	316°	336°	350°	368°	381°	392°	403°	412°
Fahr.	Cent.											
212°	100°	0.463	...	...	...	...	...	...	...	...	...	...
302°	150°	0.462	0.478	0.515	...	...	...	...	...	...	...	...
392°	200°	0.462	0.475	0.502	0.530	0.560	0.597	0.635	0.677	...	...	...
482°	250°	0.463	0.474	0.495	0.514	0.532	0.552	0.570	0.588	0.609	0.635	0.664
572°	300°	0.464	0.475	0.492	0.505	0.517	0.530	0.541	0.550	0.561	0.572	0.585
662°	350°	0.468	0.477	0.492	0.503	0.512	0.522	0.529	0.536	0.543	0.550	0.557
752°	400°	0.473	0.481	0.494	0.504	0.512	0.520	0.526	0.531	0.537	0.542	0.547

The construction of this table is readily understood from the following example: — *Required* the heat needed to superheat a kilogram of steam at 4 kilograms per square centimetre from saturation to 300° C. The saturation temperature (to the nearest degree) is 143° C.; so that the steam at 300° is superheated 157°, and for this is required the heat

$$157 \times 0.492 = 77.2 \text{ calories.}$$

The experiments of Professor Knoblauch were made at 2, 4, 6, and 8 kilograms per square centimetre; the remainder of the table was obtained from the diagram which was extended by aid of a diagram to the extent indicated. Within the limits of the experimental work the table may be used with confidence; the greatest error being probably not more than one-third of one per cent.

The following table gives the mean specific heat of superheated steam as measured on a facsimile of Professor Thomas's original diagram without exterpolation.

SPECIFIC HEAT OF SUPERHEATED STEAM

Thomas and Short.

Degrees of Superheat Fahr.	Pressure Lbs. per Sq. In. (Absolute.)						
	6	15	30	50	100	200	400
20°	0.536	0.547	0.558	0.571	0.593	0.621	0.649
50°	0.522	0.532	0.542	0.555	0.575	0.600	0.621
100°	0.503	0.512	0.524	0.537	0.557	0.581	0.599
150°	0.486	0.496	0.508	0.522	0.544	0.567	0.585
200°	0.471	0.480	0.494	0.509	0.533	0.556	0.574
250°	0.456	0.466	0.481	0.496	0.522	0.546	0.564
300°	0.442	0.453	0.468	0.484	0.511	0.537	0.554

Here again the arrangement of the table can be made evident by an example: — *Required* the heat needed to superheat steam 100 degrees at 200 pounds per square inch absolute. The mean specific heat from saturation is 0.581, so that the heat required is 58.1 thermal units.

Total Heat. — In the solution of problems that arise in engineering it is convenient to use the total amount of heat required to raise one pound of water from freezing-point to the temperature of saturated steam at the given pressure and to vaporize it and to superheat it at that pressure to the given temperature. This total heat may be represented by the expression

$$H_{Sup.} = q + r + c_p\,(t - t_s)$$

where t is the superheated temperature of the superheated steam, t_s is the temperature of saturated steam at the given pressure p, and q and r are the corresponding heat of the liquid and heat of vaporization. The mean specific heat c_p may usually be selected from one of the given tables without inter-

polation, as a small variation does not have a very large effect.

The total heat or heat contents of superheated steam in the temperature-entropy table were obtained by the following method. From Professor Thomas's diagram giving mean specific heats, curves of specific heats at various temperatures and at a given pressure were obtained, and the curves thus obtained were faired after a comparison with curves constructed with Professor Knoblauch's specific heats at those temperatures. These curves were then integrated graphically and the results checked by comparison with his mean specific heats.

Entropy. — By the entropy of superheated steam is meant the increase of entropy due to heating water from freezing-point to the temperature of saturated steam at the given pressure, to the vaporization and to the superheating at that pressure. This operation may be represented as follows:

$$\theta + \frac{r}{T_s} + \int_{T_s}^{T} \frac{c_p dt}{T}$$

in which T is the absolute temperature of the superheated steam, and T_s is the temperature of the saturated steam at the given pressure; θ and $\frac{r}{T_s}$ may be taken from the "Tables of Saturated Steam." The last term was obtained for the temperature-entropy table by graphical integration of curves plotted with values of $\frac{c_p}{T}$ derived from the curves of specific heats at various temperatures just described under the previous section.

If the temperature-entropy table is not at hand, the last term of the above expression may be obtained approximately by dividing the heat of superheating, by the mean absolute temperature of superheating.

This may be expressed as follows:

$$\frac{c_p \ (t - t_s)}{\frac{1}{2} \ (t + t_s) + 459.5}$$

where t is the temperature of the superheated steam, t_s is the temperature of saturated steam at the given pressure, and c_p is the mean specific heat of superheated steam.

If this method is considered to be too crude, the computation can be broken into two or more parts. Thus if t_1 is an intermediate temperature, the increase of entropy due to superheating may be computed as follows:

$$\frac{c_p'\ (t_1 - t_s)}{\frac{1}{2}\ (t_1 + t_s) + 459.5} + \frac{c_p''\ (t - t_s) - c_p'\ (t_1 - t_s)}{\frac{1}{2}\ (t + t_1) + 459.5}$$

where c_p' is the mean specific heat between t_s and t_1, and c_p'' is the specific heat between t_s and t. This method may evidently be extended to take in two intermediate temperatures and give three terms.

Adiabatic Expansion. — The treatment of superheated steam in this chapter resembles that for saturated steam in that it does not yield an explicit equation for the adiabatic line. If the steam were strongly superheated during the whole operation it is probable that the adiabatic line would be well represented by an exponential equation, and for such case a mean value of the exponent could be determined that would suffice for engineering work. But even with strongly superheated steam at the initial condition the final condition is likely to show moisture in the steam after adiabatic expansion, or, for that matter, after expansion of the steam in the cylinder of an engine or in a steam-turbine.

Problems involving adiabatic expansion of steam which is initially superheated can be solved by an extension of the method for saturated steam, and this method applies with equal facility to problems in which the steam becomes moist during the expansion. The most ready method of solution is by aid of the temperature-entropy table, which may be entered at the proper pressure (or the corresponding temperature of saturated steam) and the proper superheated temperature, it being in practice sufficient to take the line for the nearest tabular pressure and the column

showing the nearest degree of superheating. Following down the column for entropy to the final pressure, the properties for the final condition will be found; these will be the heat contents, specific volume, and either the temperature of superheated steam or the quality x, depending on whether the steam remains superheated during the expansion or becomes moist.

If the external work of adiabatic expansion of steam initially superheated is desired, it can be had by taking the difference of the intrinsic energies. The heat equivalent of intrinsic energy of moist steam is

$$x\rho + q = x\,(r - Apu) + q = xr + q - Apxu,$$

and of this expression the quantity $xr + q$ may be taken from the temperature-entropy table, and the quantity $Ap\,x\,u$ can be determined by aid of the steam table. In like manner the heat contents of superheated steam

$$q + r + \int c_p dt$$

which is computed and set down in the temperature-entropy table may be made to yield the heat equivalent of the intrinsic energy by subtracting the heat equivalent of the external work of vaporizing and superheating the steam

$$Ap\,(v - \sigma),$$

where v is the specific volume of the superheated steam. This method is subject to some criticism, especially when the steam is not highly superheated, because some heat will be required to do the disgregation work of superheating. Fortunately the greater part of problems arising in engineering involve the heat contents, so that this question is avoided.

Properties of Sulphur Dioxide. — One of the most interesting and important applications of the theory of superheated vapors is found in the approximate calculation of properties of certain volatile liquids which are used in refrigerating-machines, and for which we have not sufficient experimental data to construct tables in the manner explained in the chapter on saturated vapors.

For example, Regnault made experiments on the pressures of saturated sulphur dioxide and ammonia, but did not determine the heat of the liquid nor the total heat. He did, however, determine some of the properties of these substances in the gaseous or superheated condition, from which it is possible to construct the characteristic equations for the superheated vapors. These equations can then be used to make approximate calculations of the saturated vapors, for such equations are assumed to be applicable down to the saturated condition. Of course such calculations are subject to a considerable unknown error, since the experimental data are barely sufficient to establish the equations for the superheated vapors.

The specific heat of gaseous sulphur dioxide is given by Regnault * as 0.15438, and the coefficient of dilatation as 0.0039028. The theoretical specific gravity compared with air, calculated from the chemical composition, is given by Landolt and Börnstein † as 2.21295. Gmelin ‡ gives the following experimental determinations: by Thomson, 2.222; by Berzelius, 2.247. The figure 2.23 will be assumed in this work, which gives for the specific volume at freezing-point and at atmospheric pressure

$$v_0 = \frac{0.7733}{2.23} = 0.347 \text{ cubic metres.}$$

The corresponding pressure and temperature are 10,333 and 273° C.

At this stage it is necessary to assign a probable form for the characteristic equation, and for that purpose the form

$$pv = BT - Cp^a \quad . \quad . \quad . \quad . \quad . \quad . \quad (125)$$

proposed by Zeuner has commonly been used, and it is convenient to admit that it may take the form

$$pv = \frac{c_p}{A} aT - Cp^a \quad . \quad . \quad . \quad . \quad . \quad . \quad (126)$$

* *Mémoires de l'Institut de France*, tome xxi, xxvi.

† *Physikalische-chemische Tabellen.*

‡ Watt's translation, p. 280.

The value of the arbitrary constant a may be determined from the coefficient of dilatation as follows. The coefficient of dilatation is the ratio of the increase of volume at constant pressure, for one degree increase of temperature, to the original volume; so that the preceding equation applied at 0° C. and at 1° C. gives

$$p_0 v_0 = \frac{c_p}{A} a T_0 - C p_0^a;$$

$$p_0 v_1 = \frac{c_p}{A} a T_1 - C p_0^a;$$

$$\therefore \quad \frac{v_1 - v_0}{v_0} = \frac{c_p}{A} \frac{a}{p_0 v_0}.$$

The value of a obtained by substituting known values in the above equation is 0.212. Now as a appears in both the first and the last terms of the right-hand side of equation (126), a considerable change in a has but little effect on the computations by aid of that equation. As will appear later an assumption of a value 0.22 for a will make equation (126) agree well with certain experiments on the compressibility of sulphur dioxide, and it will consequently be chosen. If now we reverse the process by which a was calculated from the coefficient of dilatation, the latter constant will appear to have a computed value of 0.004, which is but little different from the experimental value.

To compute C we have

$$0.15438 \times 426.9 \times 0.22 = 14.5,$$

and the coefficient of p^a is

$$\frac{14.5 \times 273 - 10333 \times 0.347}{10333^{0.22}} = 48 \text{ nearly};$$

so that the equation becomes

$$pv = 14.5\, T - 48\, p^{0.22} \quad . \quad . \quad . \quad . \quad . \quad (127)$$

Regnault found for the pressures

$$p_1 = 697.83 \text{ mm. of mercury},$$

$$p_2 = 1341.58 \text{ mm. of mercury},$$

and at 7°.7 C. the ratio

$$\frac{p_1 v_1}{p_2 v_2} = 1.02088.$$

Reducing the given pressures to kilograms on the square metre, and the temperature to the absolute scale, and applying to equation (127), we obtain 1.016 instead of the experimental value for the above ratio.

Regnault gives for the pressure of saturated sulphur dioxide, in mm. of mercury, the equation

$$\log p = a - b\alpha^n - c\beta^n;$$
$$a = 5.6663790;$$
$$\log b = 0.4792425;$$
$$\log c = 9.1659562 - 10;$$
$$\log \alpha = 9.9972989 - 10;$$
$$\log \beta = 9.98729002 - 10;$$
$$n = t + 28^\circ \text{ C.}$$

Applying equation (95), page 76, to this case,

$$\frac{1}{p}\frac{dp}{dt} = A\alpha^n + B\beta^n;$$
$$\log \alpha = 9.9972989;$$
$$\log \beta = 9.98729002;$$
$$\log A = 8.6352146;$$
$$\log B = 7.9945332;$$
$$n = t + 28^\circ \text{ C.}$$

The specific volume of saturated sulphur dioxide may be calculated by inserting in equation (127) for the superheated vapor the pressures calculated by aid of the above equation. The results at several temperatures are as follows:

t	-30° C.	0	$+30^\circ$ C.
s	0.8292	0.2256	0.0825

Andréeff * gives for the specific gravity of fluid sulphur dioxide 1.4336; consequently the specific volume of the liquid is

$$\sigma = 0.0007.$$

* *Ann. Chem. Pharm.*, 1859.

The value of r, the heat of vaporization, may now be calculated at the given temperatures by equation (106), page 89,

$$r = AuT\frac{dp}{dt},$$

in which $$u = s - \sigma.$$

The results are

t	$-30°$ C.	0	$+30°$ C.
r	106.9	97.60	90.54

Within the limits of error of our method of calculation, the value of r may be found by the equation

$$r = 98 - 0.27\,t \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (128)$$

The specific heat of the liquid is derived by the following device. First assume that the entropy of the superheated vapor may be calculated by the equation

$$d\phi = c_p\frac{dt}{T} + (c_v - c_p)\ \frac{dp}{p}$$

given on page 67 for perfect gases. This may be transformed into

$$d\phi = c_p\left(\frac{dt}{T} - \frac{\kappa - 1}{\kappa}\,\frac{1}{p}\,dp\right) \quad . \quad . \quad . \quad . \quad (129)$$

But if we introduce into the equation for a perfect gas

$$pv = RT,$$

the value of R from the equation

$$c_p - c_v = AR,$$

the characteristic equation may take the form

$$pv = \frac{c_p}{A}\,\frac{\kappa - 1}{\kappa}\,T.$$

Comparison of this equation with equation (126) suggests replacing the term $\frac{\kappa - 1}{\kappa}$ in equation (129) by the arbitrary factor a, so that it may read

$$d\phi = c_p\left(\frac{dt}{T} - a\frac{1}{p}\,dp\right) \quad . \quad . \quad . \quad . \quad . \quad (130)$$

The expression for the entropy of a liquid and its vapor is

$$\frac{xr}{T} + \theta \text{ or } \frac{r}{T} + \int c dt$$

if the vapor is dry. When differentiated this yields

$$d\phi = \frac{1}{T}\left(cdt + dr - \frac{r}{T}dt\right) \quad . \quad . \quad . \quad . \quad (131)$$

If it be assumed that equations (130) and (131) may both be applied at saturation we have

$$c_p\left(1 - a\frac{T}{p}\frac{dp}{dt}\right) = c + \frac{dr}{dt} - \frac{r}{T} \quad . \quad . \quad . \quad (132)$$

If it be admitted further that the differential coefficient $\frac{dp}{dt}$ can be computed by the equation on page 120, the above equation affords a means of estimating the specific heat of the liquid. At 0° C., this method gives for the specific heat

$$c = 0.4.$$

In English units we have for superheated sulphur dioxide

$$pv = 26.4\, T - 184\, p^{0.22} \quad . \quad . \quad . \quad . \quad . \quad (133)$$

the pressures being in pounds on the square foot, the volumes in cubic feet, and the temperatures in Fahrenheit degrees absolute.

For pressures in pounds on the square inch at temperatures on the Fahrenheit scale,

$$\log p = a - b\alpha^n - c\beta^n;$$
$$a = 3.95278847;$$
$$\log b = 0.4792425;$$
$$\log c = 9.1659562 - 10;$$
$$\log \alpha = 9.9984994 - 10;$$
$$\log \beta = 9.99293890 - 10;$$
$$n = t + 18°.4 \text{ F.}$$

For the heat of vaporization

$$r = 176 - 0.27\,(t - 32) \quad . \quad . \quad . \quad . \quad . \quad (134)$$

and for the specific heat of the liquid

$$c = 0.4.$$

In applying these equations to the calculation of a table of the properties of saturated sulphur dioxide the pressures corresponding to the temperatures are calculated as usual. Then the heat of the liquid is calculated by aid of the constant specific heat. The heat of vaporization is calculated by aid of equation (134). Next the specific volume is calculated by inserting the given temperature and the corresponding pressure for the saturated vapor in the characteristic equation (133). Having the specific volume of the vapor and that of the liquid, the heat equivalent (Apu) of the external work is readily found. Finally, the entropy of the liquid is calculated by the equation

$$\theta = c \log_e \frac{T}{T_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (135)$$

If the reader should object that this method is tortuous and full of doubtful approximations and assumptions, he must bear in mind that any method that can give approximations is better than none, and that all the computations for refrigerating-machines, that use volatile fluids, depend on results so obtained. And further, much of the waste and disappointment of earlier refrigerating-machines could have been avoided if tables as good as those computed by this method were then available.

Properties of Ammonia. — The specific heat of gaseous ammonia, determined by Regnault, is 0.50836. The theoretical specific gravity compared with air, calculated from the chemical composition, is given by Landolt and Bornstein as 0.58890. Gmelin gives the following experimental determinations: by Thomson, 0.5931; by Biot and Arago, 0.5967. For this work the figure 0.597 will be assumed, which gives for the specific volume at freezing-point and at atmospheric pressure

$$v_0 = \frac{0.7733}{0.597} = 1.30 \text{ cubic metres.}$$

The coefficient of dilatation has not been determined, and consequently cannot be used to determine the value of a in equation (126). It, however, appears that consistent results are obtained if a is assumed to be $\frac{1}{4}$. The coefficient of T then becomes

$$0.50836 \times 426.9 \times \tfrac{1}{4} = 54.3,$$

and the coefficient of $p^{\frac{1}{4}}$ is

$$\frac{54.3 \times 273 - 10333 \times 1.30}{10333^{\frac{1}{4}}} = 142;$$

so that the equation becomes

$$pv = 54.3\,T - 142\,p^{\frac{1}{4}} \quad . \quad . \quad . \quad . \quad . \quad (136)$$

The coefficient of dilatation, calculated by the same process as was used in determining a for sulphur dioxide, is 0.00404, which may be compared with that for sulphur dioxide.

Regnault found for the pressures

$$p_1 = 703.50 \text{ mm. of mercury,}$$
$$p_2 = 1435.3 \text{ mm. of mercury,}$$

and at 8°.1 C. the ratio

$$\frac{p_1 v_1}{p_2 v_2} = 1.0188,$$

while equation (136) gives under the same conditions 1.0200.

For saturated ammonia Regnault gives the equation

$$\log p = a - b\alpha^n - c\beta^n;$$
$$a = 11.5043330;$$
$$\log b = 0.8721769;$$
$$\log c = 9.9777087 - 10;$$
$$\log \alpha = 9.9996014 - 10;$$
$$\log \beta = 9.9939729 - 10;$$
$$n = t + 22^\circ \text{ C.};$$

by aid of which the pressures in mm. of mercury may be calculated for temperatures on the centigrade scale. The differential coefficient may be calculated by aid of the equation

$$\frac{1}{p}\frac{dp}{dt} = A\alpha^n + B\beta^n;$$

$$\begin{aligned} \log A &= 8.1635170 - 10; \\ \log B &= 8.4822485 - 10; \\ \log \alpha &= 9.9996014 - 10; \\ \log \beta &= 9.9939729 - 10; \\ n &= t + 22^\circ \text{ C.} \end{aligned}$$

The specific volume of saturated ammonia calculated by equation (136) at several temperatures are

t	-30° C.	0	$+30^\circ$ C.
s	0.9982	0.2961	0.1167

Andréeff gives for the specific gravity of liquid ammonia at 0° C. 0.6364, so that the specific volume of the liquid is

$$\sigma = 0.0016.$$

The values of r at the several given temperatures, calculated by equation (128), are

t	-30° C.	0	$+30^\circ$ C.
r	325.7	300.15	277.5

which may be represented by the equation

$$r = 300 - 0.8\,t.$$

The specific heat of the liquid, calculated by aid of equation (132), is

$$c = 1.1.$$

In English units the properties of superheated or gaseous ammonia may be represented by the equation

$$pv = 99\,T - 710\,p^{\frac{1}{4}},$$

in which the pressures are taken in pounds on the square foot and volumes in cubic feet, while T represents the absolute temperature in Fahrenheit degrees.

The pressure in pounds on the square inch may be calculated by the equation

$$\log p = a - b\alpha^n - c\beta^n;$$
$$a = 9.7907380;$$
$$\log b = 0.8721769 - 10;$$
$$\log c = 9.9777087 - 10;$$
$$\log \alpha = 9.9997786 - 10;$$
$$\log \beta = 9.9966516 - 10;$$
$$n = t + 7^\circ.6 \text{ F.}$$

The heat of vaporization may be calculated by the equation

$$r = 540 - 0.8\,(t - 32),$$

and the specific heat of the liquid is

$$c = 1.1.$$

EXAMPLES.

1. What is the weight of one cubic foot of superheated steam at 500° F. and at 60 pounds pressure absolute? Knoblauch's equation. Ans. 0.106 pounds.

2. At 129.3 pounds gauge pressure 2 pounds of steam occupy 7 cubic feet. Find its temperature. Assume value of T for entering Table 1, page 112, and solve by trial. Ans. 424° F.

3. What is the volume of 5 pounds of steam at 129.3 pounds gauge pressure and at 359°.5 F.? Ans. 15.8.

4. Superheated steam at 50 pounds absolute has three-fourths the density of saturated steam at the same pressure. What is the temperature? Ans. 482° F.

5. A cubic foot of steam at 140 pounds absolute weighs 0.30 pounds. What is its temperature? Ans. 374° F.

6. Two pounds of steam and water at 129.3 pounds pressure above the atmosphere occupy 6 cubic feet. Heat is added and the pressure kept constant till the volume is 8.5 cubic feet. Find the final condition, and the external work done in expanding. Ans. Temperature 681° F.; work 51800.

7. Saturated steam at 150 pounds gauge, containing 2 per cent of water, passes through a superheater on its way to an engine. Its final temperature is 400° F. Find the increase in volume and the heat added per pound. Ans. 0.222 cubic feet; 39 B.T.U.

8. Let the initial temperature of superheated steam be 380° F. at the pressure of 150 pounds absolute. Find the condition after an adiabatic expansion to 20 pounds absolute. Determine also the initial and final volumes. Ans. (1) 0.895; (2) 3.09 cubic feet; (3) 17.8 cubic feet.

9. In example 14, page 109, suppose that the steam at cut-off were superheated 10° F. above the temperature of saturated steam at the given pressure, and solve the example. Ans. (1) 0.887; (2) 87° superheating; (3) same as before; (4) $n =$ 1.137; (5) 1972 and 1950 foot-pounds.

CHAPTER VIII.

THE STEAM-ENGINE.

THE steam-engine is still the most important heat-engine, though its supremacy is threatened on one hand by the steam-turbine and on the other by the gas-engine. When of large size and properly designed and managed its economy is excellent and can be excelled only by the largest and best gas-engines, and in many cases these engines (even with the advantage of a more favorable range of temperature) depend for their commercial success on the utilization of by-products.

It can be controlled, regulated, and reversed easily and positively — properties which are not possessed in like degree by other heat-engines. It is interesting to know that the theory of thermodynamics was developed mainly to account for the action and to provide methods of designing steam-engines; though neither object is entirely accomplished, on account of the fact that the engine-cylinder must be made of some metal to be hard and strong enough to endure service, for all metals are good conductors of heat, and seriously affect the action of a condensable fluid like steam.

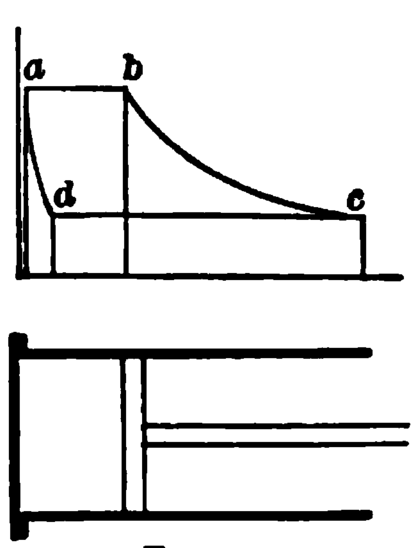

FIG. 31.

Carnot's Cycle for a steam-engine is represented by Fig. 31, in which *ab* and *cd* are isothermal lines, representing the application and rejection of heat at constant temperature and at constant pressure. *bc* and *da* are adiabatic lines, representing change of temperature and pressure, without transmission of heat through the walls of the cylinder. The diagram representing Carnot's cycle has an external resemblance to the indicator-diagram from some actual engines, but it differs in essential particulars.

In the condition represented by the point a the cylinder contains a mixture of water and steam at the temperature t_1 and the pressure p_1. If connection is made with a source of heat at the temperature t_1, and heat is added, some of the water will be vaporized and the volume will increase at constant pressure as represented by ab. If thermal communication is now interrupted, adiabatic expansion may take place as represented by bc till the temperature is reduced to t_2, the temperature of the refrigerator, with which thermal communication may now be established. If the piston is forced toward the closed end of the cylinder some of the steam in it will be condensed, and the volume will be reduced at constant pressure as represented by cd. The cycle is completed by an adiabatic compression represented by da.

If the absolute temperature of the source of heat is T_1, and if that of the refrigerator is T_2, then the efficiency is

$$e = \frac{T_1 - T_2}{T_1}$$

whatever may be the working fluid.

For example, if the pressure of the steam during isothermal expansion is 100 pounds above the atmosphere, and if the pressure during isothermal compression is equal to that of the atmosphere, then the temperatures of the source of heat and of the refrigerator are 337°.6 F. and 212° F., or 797.1 and 671.5 absolute, so that the efficiency is

$$\frac{797.1 - 671.5}{797.1} = 0.157.$$

The following table gives the efficiencies worked out in a similar way, for various steam-pressures, — both for t_2 equal to 212° F., corresponding to atmospheric pressure, and for t_2, equal to 116° F., corresponding to an absolute pressure of 1.5 pounds to the square inch:

EFFICIENCY OF CARNOT'S CYCLE FOR STEAM-ENGINES.

Initial Pressure by the Gauge, above the Atmosphere.	Atmospheric Pressure.	1.5 Pounds Absolute.
15	0.053	0.189
30	0.084	0.215
60	0.124	0.249
100	0.157	0.278
150	0.186	0.302
200	0.207	0.320
300	0.238	0.347

The column for atmospheric pressure may be used as a standard of comparison for non-condensing engines, and the column for 1.5 pounds absolute may be used for condensing engines.

It is interesting to consider the condition of the fluid in the cylinder at the different points of the diagram for Carnot's cycle. Thus if the fluid at the condition represented by b in Fig. 31 is made up of x_b part steam and $1-x_b$ part water, then from equation (118) the condition at the point c is given by

$$x_c = \frac{T_2}{r_2}\left(\frac{r_1}{T_1}x_b + \theta_1 - \theta_2\right) \quad . \quad . \quad . \quad . \quad (137)$$

In like manner the condition of the mixture at the point d is

$$x_d = \frac{T_2}{r_2}\left(\frac{r_1}{T_1}x_a + \theta_1 - \theta_2\right) \quad . \quad . \quad . \quad . \quad (138)$$

It is interesting to note that if x_b is larger than one-half, that is, if there is more steam than water in the cylinder at b, then the adiabatic expansion is accompanied by condensation. Again, if x_a is less than one-half, then the adiabatic compression is also accompanied by condensation. Very commonly it is assumed that x_b is unity, so that there is dry saturated steam in the cylinder at b; and that x_a is zero, so that there is water only in the

cylinder at a; but there is no necessity for such assumptions, and they in no way affect the efficiency.

The temperature-entropy diagram for Carnot's cycle for a steam-engine is shown by Fig. 32, on which are drawn also the lines for entropy of the liquid ma, and the entropy of saturated vapor be, as well as the lines which represent the value of x, the dryness factor. This diagram represents to the eye the vaporization during the isothermal expansion ab, the partial condensation during the adiabatic expansion bc,

FIG. 32.

the isothermal condensation along cd, and the condensation during the adiabatic compression da. In the diagram the working substance is shown as water at a and as dry steam at b; the efficiency would clearly be the same for a cycle $a'\,b'\,c'\,d'$, which contains a varying mixture of water and steam under all conditions.

If the cylinder contains M pounds of steam and water, the heat absorbed by the working substance during isothermal expansion is

$$Q_1 = Mr_1 (x_b - x_a) \quad . \quad . \quad . \quad . \quad . \quad . \quad (139)$$

and the heat rejected during isothermal compression is

$$Q_2 = Mr_2 (x_c - x_d),$$

so that the heat changed into work during the cycle is

$$Q_1 - Q_2 = M\{r_1 \ (x_b - x_a) - r_2 (x_c - x_d)\}$$

But from equations (137) and (138)

$$r_2 (x_c - x_d) = \frac{T_2}{T_1} r_1 (x_b - x_a),$$

and the expression for the heat changed into work becomes

$$Q_1 - Q_2 = Mr_1 (x_b - x_a) \frac{T_1 - T_2}{T_1}. \quad . \quad . \quad (140)$$

This equation is deduced because it is convenient for making comparisons of various other volatile liquids and their vapors, with steam, for use in heat-engines. It is of course apparent that

$$e = \frac{Q_1 - Q_2}{Q_1} = \frac{T_1 - T_2}{T_1},$$

from equations (139) and (140), a conclusion which is known independently, and indeed is necessary in the development of the theory of the adiabatic expansion of steam.

In the discussion thus far it has been assumed that the working fluid is steam, or a mixture of steam and water. But a mixture of any volatile liquid and its vapor will give similar results, and the equations deduced can be applied directly. The principal difference will be due to the properties of the vapor considered, especially its specific pressures and specific volumes for the temperatures of the source of heat and the refrigerator.

For example, the efficiency of Carnot's cycle for a fluid working between the temperatures 160° C. and 40° C. is

$$\frac{160 - 40}{160 + 273} = 0.277.$$

The properties of steam and of chloroform at these temperatures are

	Steam.		Chloroform.	
	40° C.	160° C.	40° C.	160° C.
Pressure, mm. mercury . .	55.13	4633.	369.26	8734.2
Volume, cubic meters . . .	19.57	0.3063	0.4449	0.0243
Heat of vaporization, r . .	574.2	496.5	63.13	50.53
Entropy of liquid, θ	0.1368	0.4644	0.03196	0.11041

For simplicity, we may assume that one kilogram of the fluid is used in the cylinder for Carnot's cycle, and that x_b is unity while x_a is zero, so that from equation (140)

$$Q_1 - Q_2 = r_1 \frac{T_1 - T_2}{T_1},$$

and for steam

$$Q_1 - Q_2 = 496.5 \times 0.277 = 137 \text{ calories,}$$

while for chloroform

$$Q_1 - Q_2 = 50.53 \times 0.277 = 14 \text{ calories.}$$

After adiabatic expansion the qualities of the fluid will be, from equation (137), for steam

$$x_c = \frac{40 + 273}{574.2}\left(\frac{496.5}{160 + 273} + 0.4644 - 0.1368\right) = 0.804,$$

and for chloroform

$$x_c = \frac{40 + 273}{63.13}\left(\frac{50.53}{160 + 273} + 0.11041 - 0.03196\right) = 0.969.$$

The specific volumes after adiabatic expansion are, consequently, for steam

$$v_c = x_c u_2 + \sigma = 0.804\,(19.57 - 0.001) + 0.001 = 16.4,$$

and for chloroform

$$v_c = x_c u_2 + \sigma = 0.969\,(0.4449 - 0.000655) + 0.000655 = 0.431.$$

These values for v_c just calculated are the volumes in the cylinder at the extreme displacement of the piston, on the assumption that one kilogram of the working fluid is vaporized during isothermal expansion. A better idea of the relative advantages of the two fluids will be obtained by finding the heat changed into work for each cubic metre of maximum piston-displacement, or for a cylinder having the volume of one cubic metre. This is obtained by dividing $Q_1 - Q_2$, the heat changed into work for each kilogram by v_c. For steam the result is

$$(Q_1 - Q_2) \div v_c = 137 \div 16.4 = 8.3,$$

and for chloroform it is

$$(Q_1 - Q_2) \div v_c = 14 \div 0.431 = 34;$$

from which it appears that for the same volume chloroform can produce more than three and a half times as much power.

Even if we consider that the difference of pressure for chloroform,

$$8734.2 - 369.3 = 8364.9 \text{ mm.},$$

is nearly twice that for steam, which has only

$$4633 - 55 = 4578 \text{ mm.}$$

difference of pressure, the advantage appears to be in favor of chloroform. If, however, the difference of pressures given for chloroform is allowable also for steam, giving

$$8364.9 + 55.1 = 8420 \text{ mm.}$$

for the superior pressure, then the initial temperature for steam becomes 185° C., and the efficiency becomes

$$\frac{185 - 40}{185 + 273} = 0.318,$$

instead of 0.277. On the whole, steam is the more desirable fluid, even if we do not consider the inflammable and poisonous nature of chloroform. Similar calculations will show that on the whole steam is at least as well adapted for use in heat-engines as any other saturated fluid; in practice, the cheapness and incombustibility of steam indicate that it is the preferable fluid for such uses.

Non-conducting Engine. *Rankine Cycle.* — The conditions required for alternate isothermal expansion and adiabatic expansion cannot be fulfilled for Carnot's cycle with steam any more than they could be for air. The diagram for the cycle with steam, however, is well adapted to production of power; the contrary is the case with air, as has already been shown.

In practice steam from a boiler is admitted to the cylinder of the steam-engine during that part of the cycle which corresponds to the isothermal expansion of Carnot's cycle, thus transferring the isothermal expansion to the boiler, where steam is formed under constant pressure. In like manner the isothermal compression is replaced by an exhaust at constant pressure, during which steam may be condensed in a separate condenser,

cooled by cold water. The cylinder is commonly made of cast iron, and is always some kind of metal; there is consequently considerable interference due to the conductivity of the walls of the cylinder, and the expansion and compression are never adiabatic. There is an advantage, however, in discussing first an engine with a cylinder made of some non-conducting material, although no such material proper for making cylinders is now known.

The diagram representing the operations in a non-conducting cylinder for a steam-engine (sometimes called the Rankine cycle) can be represented by Fig. 33. *ab* represents the admission of dry saturated steam from the boiler; *bc* is an adiabatic expansion to the exhaust pressure; *cd* represents the exhaust; and *da* is an adiabatic compression to the initial pressure. It is assumed that the small volume, represented by *a*, between the piston and the head of the cylinder is filled with dry steam, and that the steam remains homogeneous during exhaust so that the quality is the same at *d* as at *c*. These conditions are consistent and necessary, since the change of condition due to adiabatic expansion (or compression) depends only on the initial condition and the initial and final pressures; so that an adiabatic expansion from *a* to *d* would give the same quality at *d* as that found at *c* after adiabatic expansion from *b*, and conversely adiabatic compression from *d* to *a* gives dry steam at *a* as required.

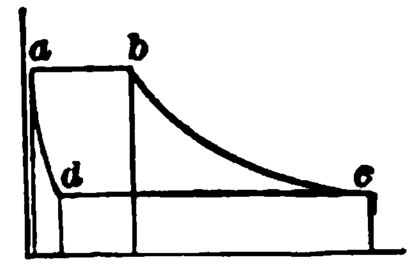

FIG. 33.

The cycle represented by Fig. 33 differs most notably from Carnot's cycle (Fig. 32) in that *ab* represents admission of steam and *cd* represents exhaust of steam, as has already been pointed out. It also differs in that the compression *da* gives dry steam instead of wet steam. The compression line *da* is therefore steeper than for Carnot's cycle, and the area of the figure is slightly larger on this account. This curious fact does not indicate that the cycle has a higher efficiency; on the contrary, the efficiency is less, and the cycle is irreversible.

If the pressure during admission (equal to the pressure in

the boiler) is p_1, and if the pressure during exhaust is p_2, then the heat required to raise the water resulting from the condensation of the exhaust-steam is

$$q_1 - q_2,$$

where q_1 is the heat of the liquid at the pressure p_1, and q_2 is the heat of the liquid at the pressure p_2. The heat of vaporization at the pressure p_1 is r_1, so that the heat required to raise the feed-water from the temperature of the exhaust to the temperature in the boiler and evaporate it into dry steam is

$$Q_1 = r_1 + q_1 - q_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (141)$$

and this is the quantity of heat supplied to the cylinder per pound of steam.

The steam exhausted from the cylinder has the quality x_2, calculated by aid of the equation

$$x_2 = \frac{T_2}{r_2}\left(\frac{r_1}{T_1} + \theta_1 - \theta_2\right),$$

and the heat that must be withdrawn when it is condensed is

$$Q_2 = x_2 r_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (142)$$

this is the heat rejected from the engine. The heat changed into work per pound of steam is

$$Q_1 - Q_2 = r_1 + q_1 - q_2 - x_2 r_2 \quad . \quad . \quad . \quad . \quad (143)$$

The efficiency of the cycle is

$$e = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{x_2 r_2}{r_1 + q_1 - q_2} \quad . \quad . \quad . \quad (144)$$

If values are assigned to p_1 and p_2 and the proper numerical calculations are made, it will appear that the efficiency for a non-conducting engine is always less than the efficiency for Carnot's cycle between the corresponding temperatures.

It should be remarked that the efficiency is not affected by the clearance or space between the piston and the head of the cylinder and the space in the steam-passages of the cylinder, provided that the clearance is filled with dry saturated steam as

indicated in Fig. 33. This is evident from the fact that no term representing the clearance, or volume at *a*, Fig. 33, appears in equation (144). Or, again, we may consider that the steam in the cylinder at the beginning of the stroke, occupying the volume represented by *a*, expands during the adiabatic expansion and is compressed again during compression, so that one operation is equivalent to and counterbalances the other, and so does not affect the efficiency of the cycle.

Use of the Temperature-Entropy Diagram. — The Rankine cycle is drawn with a varying quantity of steam in the cylinder, beginning at *a*, Fig. 33, with the steam caught in the clearance and finishing at *b*, with that weight plus the weight drawn from the boiler; consequently a proper temperature-entropy diagram, which represents the changes of one pound of the working substance, cannot be drawn.

We may, however, use the temperature-entropy diagram (like Fig. 30, page 104, or the plate at the end of the book) for solving problems connected with that cycle instead of equations (143) and (144).

In the first place we have by equation (96), page 83,

$$q = \int c dt,$$

and by equation (113), page 97,

$$\theta = \int \frac{c dt}{T}$$

for a volatile liquid. From the latter we have

$$c dt = T d\theta;$$

therefore

$$q = \int T d\theta.$$

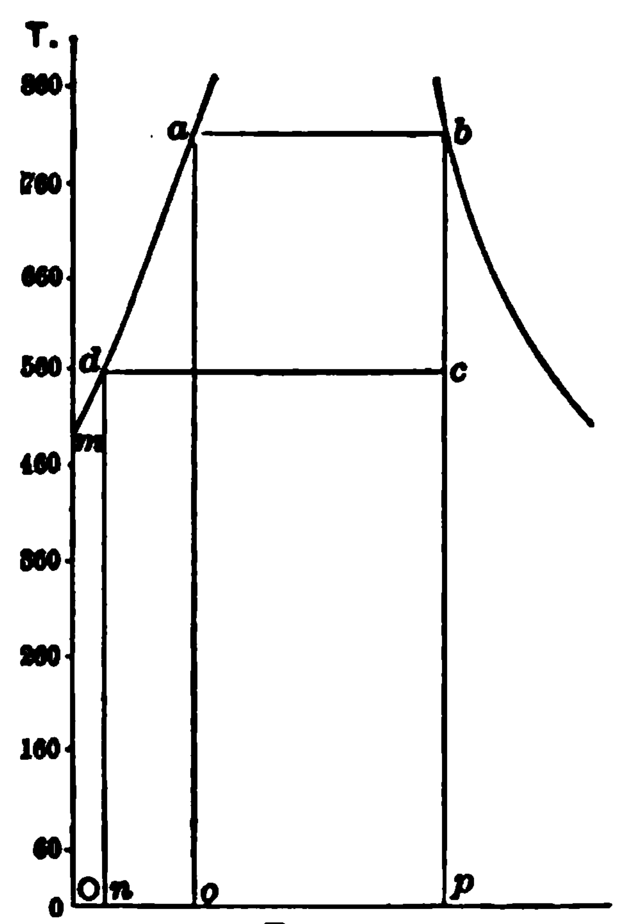

FIG. 34.

From this last equation it is evident that the heat of the liquid q_1 for water represented by the point *a* in Fig. 34, is measured by

the area *Omao*. In like manner the heat of the liquid q_2 corresponding to the point *d*, is represented by the area *Omdn*. Again, the heat added during the vaporization represented by *ab*, is r_1, while the increase of entropy is $\frac{r_1}{T_1}$. Therefore the heat of vaporization can be represented by the area *oabp*. In like manner the partial vaporization x_2r_2 can be represented by the area *ndcp*. Therefore the heat changed into work for the cycle in Fig. 33, which has been represented by

$$r_1 + q_1 - (x_2r_2 + q_2),$$

can equally well be represented by the area

$$\begin{aligned} abcd = & \text{ area } Omao + \text{area } oabp \\ & - (\text{area } Omdn + \text{area } ndcp). \end{aligned}$$

It will consequently be sufficient to measure the area *abcd* by any means, for example, by aid of a planimeter, in order to determine the heat changed into work during the operation of the non-conducting engine working on the Rankine cycle. If the planimeter determines the area in square inches, the scale of the drawing for Fig. 34 should be one inch per degree, and one inch per unit of entropy, or, if other and more convenient scales are to be used, proper reductions must be made to allow for those scales.

It must be firmly fixed in mind that the use of a diagram like Fig. 34 is justified because it has been proved that the area *abcd* (drawn to the proper scale) is numerically equal to the heat changed into work as computed by equation (143), and that the diagram *does not represent the operations of the cycle*. This is entirely different from the case of the diagram, Fig. 32, which correctly represents the operations of Carnot's cycle.

The illustration of the use of the temperature-entropy diagram for this purpose is chosen for convenience with dry saturated steam at *b*, Fig. 34. It is evident that it could (with equal propriety) be applied to an engine supplied with moist steam if r_1 is replaced by x_1r_1 in equation (143) and if *b* is located at the proper place between *a* and *b*.

The actual measurement of areas by a planimeter is seldom

if ever applied, but the diagram is used effectively in the discussion of certain problems of non-reversible flow of steam in nozzles and turbines, with allowance for friction.

It further suggests an approximation that may sometimes be useful, especially if the change of pressure (and temperature) is small. Thus the area *abcd* may be approximately represented by the expression

$$\frac{1}{2}(ab + dc)\,bc = \frac{1}{2}\left(\frac{r_1}{T_1} + \frac{x_2 r_2}{T_2}\right)(t_1 - t_2),$$

so that in place of equation (143) we may have

$$Q_1 - Q_2 = \frac{1}{2}\left(\frac{r_1}{T_1} + \frac{x_2 r_2}{T_2}\right)(t_1 - t_2) \quad . \quad . \quad . \quad . \quad (145)$$

for the heat changed into work by Rankine's cycle.

This approximation depends on treating *ad* as a straight line, and this assumption is more correct as the difference of temperature is less; that is for those cases in which equation (143) deals with the difference of quantities of about the same magnitude, and may consequently be affected by a large relative error.

Temperature-Entropy Table. — The temperature-entropy table which has been described on page 106 was computed for solution of problems of this nature, more especially in turbine design, and enables us to determine the heat changed into work directly with sufficient accuracy for engineering work, without interpolation; it also gives the quality x and the specific volume.

Incomplete Cycle. — The cycle for a non-conducting engine may be incomplete because the expansion is not carried far enough to reduce the pressure to that of the back-pressure line, as is shown in Fig. 35. Such an incomplete cycle has less efficiency than a complete cycle, but in practice the advantage of using a smaller cylinder and of reducing friction is sufficient compensation for the small loss of efficiency due to a moderate drop at the end of the stroke, as shown in Fig. 35.

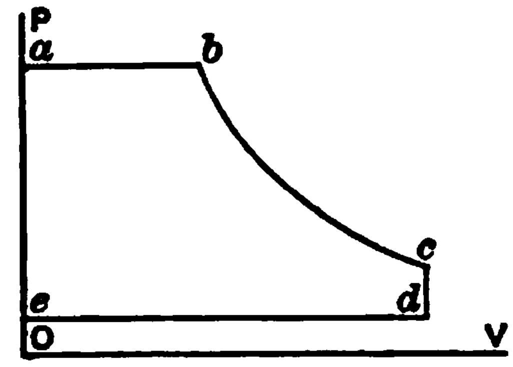

FIG. 35.

The discussion of the incomplete cycle is simplified by assuming that there is no clearance and no compression as is indicated by Fig. 35. It will be shown later that the efficiency will be the same with a clearance, provided the compression is complete.

The most ready way of finding the efficiency for this cycle is to determine the work of the cycle. Thus the work during admission is

$$p_1 (u_1 + \sigma),$$

where u_1 is the increase of volume due to vaporization of a pound of steam, and σ is the specific volume of water. The work during expansion is

$$E_b - E_c = \frac{1}{A} (\rho_1 + q_1 - x_c\rho_c - q_c),$$

where q_1 and ρ_1 are the heat of the liquid and the heat-equivalent of the internal work during vaporization at the pressure p_1, while q_c and ρ_c are corresponding quantities for the pressure at c. x_c is to be calculated by the equation

$$x_c = \frac{T}{r_c} \left(\frac{r_1}{T_1} + \theta_1 - \theta_c \right).$$

The work done by the piston on the steam during exhaust is

$$p_2 (x_c u_c + \sigma).$$

The total work of the cycle is obtained by adding the work during admission and expansion and subtracting the work during exhaust, giving

$$\frac{1}{A} (\rho_1 + A p_1 u_1 - x_c\rho_c - A p_2 x_c u_c + q_1 - q_c) + (p_1 - p_2)\, \sigma. \quad (146)$$

The last term is small, and may be neglected. Adding and subtracting $A p_c x_c u_c$ and multiplying by A, we get for the heat-equivalent of the work of the cycle

$$Q_1 - Q_2 = r_1 - x_c r_c + A (p_c - p_2)\, u_c x_c + q_1 - q_c \quad (147)$$

which is equal to the difference between the heat supplied and the heat rejected as indicated. The heat supplied is

$$Q_1 = r_1 + q_1 - q_2,$$

as was deduced for the complete cycle; the cost of making the steam remains the same, whether or not it is used efficiently. Finally, the efficiency of the cycle is

$$e = \frac{Q_1 - Q_2}{Q_1} = \frac{r_1 + q_1 - x_c r_c - q_c + A\,(p_c - p_2)\,x_c u_c}{r_1 + q_1 - q_2}$$

$$\therefore\ e = 1 - \frac{x_c r_c + q_c - q_2 - A\,(p_c - p_2)\,x_c u}{r_1 + q_1 - q_2} \quad . \quad . \quad . \quad . \quad (148)$$

If p_c is made equal to p_2 in the preceding equation, it will be reduced to the same form as equation (144), because the expansion in such case becomes complete.

Steam-Consumption of Non-conducting Engine. — A horse-power is 33000 foot-pounds per minute or 60 × 33000 foot-pounds per hour. But the heat changed into work per pound of steam by a non-conducting engine with complete expansion is, by equation (143),

$$r_1 + q_1 - q_2 - x_2 r_2,$$

so that the steam required per horse-power per hour is

$$\frac{60 \times 33000}{778\,(r_1 + q_1 - q_2 - x_2 r_2)}.$$

Similarly, the steam per horse-power per hour for an engine with incomplete expansion, by aid of expression (146), is

$$\frac{60 \times 33000}{778\,(\rho_1 + Ap_1 u_1 - x_c \rho_c - Ap_2 x_c u_c + q_1 - q_c)}.$$

The value of x_2 or x_c is to be calculated by the general equation

$$x = \frac{T}{r}\left(\frac{r_1}{T_1} + \theta_1 - \theta\right).$$

The denominator in either of the above expressions for the steam per horse-power per hour is of course the work done per pound of steam, and the parenthesis without the mechanical

equivalent 778 is equal to $Q_1 - Q_2$. If then we multiply and divide by

$$Q_1 = r_1 + q_1 - q_2,$$

that is, by the heat brought from the boiler by one pound of steam, we shall have in either case for the steam consumption in pounds per hour

$$\frac{60 \times 33000 \times Q_1}{778 (Q_1 - Q_2) Q_1} = \frac{60 \times 33000}{778e (r_1 + q_1 - q_2)} \quad . \quad . \quad (149)$$

where

$$e = \frac{Q_1 - Q_2}{Q_1}$$

is the efficiency for the cycle.

Actual Steam-Engine. — The indicator-diagram from an actual steam-engine differs from the cycle for a non-conducting engine in two ways: there are losses of pressure between the boiler and the cylinder and between the cylinder and the condenser, due to the resistance to the flow of steam through pipes, valves, and passages; and there is considerable interference of the metal of the cylinder with the action of the steam in the cylinder. The losses of pressure may be minimized for a slow-moving engine by making the valves and passages direct and large. The interference of the walls of the cylinder cannot be prevented, but may be ameliorated by using superheated steam or by steam-jacketing.

When steam enters the cylinder of an engine, some of it is condensed on the walls which were cooled by contact with exhaust-steam, thereby heating them up nearly to the temperature of the steam. After cut-off the pressure of the steam is reduced by expansion and some of the water on the walls of the cylinder vaporizes. At release the pressure falls rapidly to the back-pressure, and the water remaining on the walls is nearly if not all vaporized. It is at once evident that so much of the heat as remains in the walls until release and is thrown out during exhaust is a direct loss; and again, the heat which is restored during expansion does work with less efficiency,

because it is reëvaporated at less than the temperature in the boiler or in the cylinder during admission. A complete statement of the action of the walls of the cylinder of an engine, with quantitative results from tests on engines, was first given by Hirn. His analysis of engine tests, showing the interchanges of heat between the walls of the cylinder and the steam, will be given later. It is sufficient to know now that a failure to consider the action of the walls of the cylinder leads to gross errors, and that an attempt to base the design of an engine on the theory of a steam-engine with a non-conducting cylinder can lead only to confusion and disappointment.

The most apparent effect of the influence of the walls of the cylinder on the indicator-diagram is to change the expansion and the compression lines; the former exhibits this change most clearly. In the first place the fluid in the cylinder at cut-off consists of from twenty to fifty per cent hot water, which is found mainly adhering to the walls of the cylinder. Even if there were no action of the walls during expansion the curve would be much less steep than the adiabatic line for dry saturated steam. But the reëvaporation during expansion still further changes the curve, so that it is usually less steep than the rectangular hyperbola.

It may be mentioned that the fluctuations of temperature in the walls of a steam-engine cylinder caused by the condensation and reëvaporation of water do not extend far from the surface, but that at a very moderate depth the temperature remains constant so long as the engine runs under constant conditions.

The performance of a steam-engine is commonly stated in pounds of steam per horse-power per hour. For example, a small Corliss engine, developing 16.35 horse-power when running at 61.5 revolutions per minute under 77.4 pounds boiler-pressure, used 548 pounds of steam in an hour. The steam consumption was

$$548 \div 16.35 = 33.5$$

pounds per horse-power per hour.

This method was considered sufficient in the earlier history of the steam-engine, and may now be used for comparing simple condensing or non-condensing engines which use saturated steam and do not have a steam-jacket, for the total heat of steam, and consequently the cost of making steam from water at a given temperature increases but slowly with the pressure.

The performance of steam-engines may be more exactly stated in British thermal units per horse-power per minute. This method, or some method equivalent to it, is essential in making comparisons to discover the advantages of superheating, steam-jacketing, and compounding. For example, the engine just referred to used steam containing two per cent of moisture, so that x_1 at the steam-pressure of 77.4 pounds was 0.98. The barometer showed the pressure of the atmosphere to be 14.7 pounds, and this was also the back-pressure during exhaust. If it be assumed that the feed-water was or could be heated to the corresponding temperature of 212° F., the heat required to evaporate it against 77.4 pounds above the atmosphere or 92.1 pounds absolute was

$$x_1 r_1 + q_1 - q_2 = 0.98 \times 892.2 + 292.2 - 180.3 = 986.3 \text{ B.T.U.}$$

The thermal units per horse-power per minute were

$$\frac{986.3 \times 33.5}{60} = 551.$$

Efficiency of the Actual Engine. — When the thermal units per horse-power per minute are known or can be readily calculated, the efficiency of the actual steam-engine may be found by the following method: A horse-power corresponds to the development of 33000 foot-pounds per minute, which are equivalent to

$$33000 \div 778 = 42.42$$

thermal units. This quantity is proportional to $Q_1 - Q_2$, and the thermal units consumed per horse-power per minute are proportional to Q_1, so that the efficiency is

$$e = \frac{Q_1 - Q_2}{Q_1} = \frac{42.42}{\text{B.T.U. per H.P. per min.}}.$$

For example, the Corliss engine mentioned above had an efficiency of

$$42.42 \div 551 = 0.077.$$

This same method may evidently be applied to any heat-engine for which the consumption in thermal units per horse-power per hour can be applied.

From the tests reported in Chapter XIII it appears that the engine in the laboratory of the Massachusetts Institute of Technology on one occasion used 13.73 pounds of steam per horse-power per hour, of which 10.86 pounds were supplied to the cylinders and 2.87 pounds were condensed in steam-jackets on the cylinders. The steam in the supply-pipe had the pressure of 157.7 pounds absolute, and contained 1.2 per cent of moisture. The heat supplied to the cylinders per minute in the steam admitted was

$$\begin{aligned} &10.86\,(x_1r_1 + q_1 - q_2) \div 60 \\ &\quad = 10.86\,(0.988 \times 859.7 + 333.6 - 126.0) \div 60 \\ &\quad = 191 \text{ B.T.U.}; \end{aligned}$$

q_2 being the heat of the liquid at the temperature of the back-pressure of 4.5 pounds absolute.

The steam condensed in the steam-jackets was withdrawn at the temperature due to the pressure and could have been returned to the boiler at that temperature; consequently the heat required to vaporize it was r_1, and the heat furnished by the steam in the jackets was

$$2.87 \times 0.98 \times 859.7 \div 60 = 40.3 \text{ B.T.U.}$$

The heat consumed by the engine was

$$191 + 40.3 = 231 \text{ B.T.U.}$$

per horse-power per minute, and the efficiency was

$$e = 42.42 \div 231 = 0.183.$$

The efficiency of Carnot's cycle for the range of temperatures corresponding to 157.7 and 4.5 pounds absolute, namely, 822°.0 and 617°.4 absolute, is

$$e = \frac{T_1 - T_2}{T_1} = \frac{822.0 - 617.4}{822.0} = 0.248.$$

The efficiency for a non-conducting engine with complete expansion, calculated by equation (144), is for this case

$$e'' = 1 - \frac{x_2 r_2}{r_1 + q_1 - q_2} = 1 - \frac{0.824 \times 1002.5}{859.7 + 333.6 - 126.0} = 0.227$$

where x_2 is calculated by the equation

$$x_2 = \frac{T_2}{r_2}\left(\frac{r_1}{T_1} + \theta_1 - \theta_2\right)$$

$$= \frac{1}{1.6235}(1.0468 + 0.5192 - 0.2282) = 0.824.$$

During the test in question the terminal pressure at the end of the expansion in the low-pressure cylinder was 6 pounds absolute, which gives

$$x_c = \frac{T_c}{r_c}\left(\frac{r_1}{T_1} + \theta_1 - \theta_2\right)$$

$$= \frac{1}{1.5812}(1.0468 + 0.5192 - 0.2476) = 0.834$$

and the efficiency by equation (148) was

$$e''' = 1 - \frac{x_c r_c - q_c + q_2 - A(p_c - p_2)x_c u_c}{r_1 + q_1 - q_2}$$

$$= 1 - \frac{0.834 \times 995.5 - 138.1 + 126.0 + \frac{144}{778}(6 - 4.5)\,0.834 \times 62}{859.7 + 333.6 - 126.0}$$

$$= 0.222.$$

The real criterion of the perfection of the action of an engine is the ratio of its actual efficiency to that of a perfect engine. If for the perfect engine we choose Carnot's cycle the ratio is

$$\frac{e}{e'} = \frac{0.183}{0.2485} = 0.736.$$

But if we take for our standard an engine with a cylinder of non-conducting material the ratio for complete expansion is

$$\frac{e}{e''} = \frac{0.183}{0.227} = 0.807.$$

For incomplete expansion the ratio is

$$\frac{e}{e'''} = \frac{0.183}{0.222} = 0.824.$$

To complete the comparison it is interesting to calculate the steam-consumption for a non-conducting steam-engine by equation (149), both for complete and for incomplete expansion. For complete expansion we have

$$\frac{60 \times 33000}{778 \times 0.227\ (859.7 + 333.6 - 126.0)} = 10.5 \text{ pounds},$$

and for incomplete expansion

$$\frac{60 \times 33000}{778 \times 0.222\ (859.7 + 333.6 - 126.0)} = 10.7 \text{ pounds}.$$

per horse-power per hour.

But if these steam-consumptions are compared with the actual steam-consumption of 13.73 pounds per horse-power per hour, the ratios are

$$10.5 \div 13.73 = 0.766 \quad \text{and} \quad 10.7 \div 13.73 = 0.783,$$

which are very different from the ratios of the efficiencies. The discrepancy is due to the fact that more than a fourth of the steam used by the actual engine is condensed in the jackets and returned at full steam temperature to the boiler, while the non-conducting engine has no jacket, but is assumed to use all the steam in the cylinder.

From this discussion it appears that there is not a wide margin for improvement of a well-designed engine running under favorable conditions. Improved economy must be sought either by increasing the range of temperatures (raising the steam-pressure

or improving the vacuum), or by choosing some other form of heat-motor, such as the gas-engine.

Attention should be called to the fact that the real criterion of the economy of a heat-engine is the cost of producing power by that engine. The cost may be expressed in thermal units per horse-power per minute, in pounds of steam per horse-power per hour, in coal per horse-power per hour, or directly in money. The expression in thermal units is the most exact, and the best for comparing engines of the same class, such as steam-engines. If the same fuel can be used for different engines, such as steam- and gas-engines, then the cost in pounds of fuel per horse-power per hour may be most instructive. But in any case the money cost must be the final criterion. The reason why it is not more frequently stated in reports of tests is that it is in many cases somewhat difficult to determine, and because it is affected by market prices which are subject to change.

At the present time a pressure as high as 150 pounds above the atmosphere is used where good economy is expected. It appears from the table on page 132, showing the efficiency of Carnot's cycle for various pressures, that the gain in economy by increasing steam-pressure above 150 pounds is slow. The same thing is shown even more clearly by the following table:

EFFECT OF RAISING STEAM-PRESSURE.

Boiler-pressure by Gauge.	Efficiency, Carnot's Cycle.	Non-conducting Engine.		Probable Performance, Actual Engine.	
		Efficiency.	B.T.U. per H.P. per Minute.	B.T.U. per H.P. per Minute.	Steam per H.P. per Hour.
150	0.302	0.272	156	195	11.5
200	0.320	0.288	147	184	10.5
300	0.347	0.306	135	169	9.6

In the calculations for this table the steam is supposed to be dry as it enters the cylinder of the engine, and the back-pressure is supposed to be 1.5 pounds absolute, while the expansion for the non-conducting engine is assumed to be complete. The

heat-consumption of the non-conducting engine is obtained by dividing 42.42 by the efficiency; thus for 150 pounds

$$42.42 \div 0.272 = 156.$$

The heat-consumption of the actual engine is assumed to be one-fourth greater than that of the non-conducting engine. The steam-consumption is calculated by the reversal of the method of calculating the thermal units per horse-power per minute from the steam per horse-power per hour, and for simplicity all of the steam is assumed to be supplied to the cylinder. But an engine which shall show such an economy for a given pressure as that set down in the table must be a triple or a quadruple engine and must be thoroughly steam-jacketed. The actual steam-consumption is certain to be a little larger than that given in the table, as steam condensed in a steam-jacket yields less heat than that passed through the cylinder.

It is very doubtful if the gain in fluid efficiency due to increasing steam-pressure above 150 or 200 pounds is not offset by the greater friction and the difficulty of maintaining the engine. Higher pressures than 200 pounds are used only where great power must be developed with small weight and space, as in torpedo-boats.

Condensers. — Two forms of condensers are used to condense the steam from a steam-engine, known as jet-condensers and surface-condensers. The former are commonly used for land engines; they consist of a receptacle having a volume equal to one-fourth or one-third of that of the cylinder or cylinders that exhaust into it, into which the steam passes from the exhaust-pipe and where it meets and is condensed by a spray of cold water.

If it be assumed that the steam in the exhaust-pipe is dry and saturated and that it is condensed from the pressure p and cooled to the temperature t_k, then the heat yielded per pound of steam is

$$H - q_k,$$

where H is the total heat of steam at the pressure p, and q_k is the heat of the liquid at the temperature t_k. The heat acquired by each pound of condensing or injection water is

$$q_k - q_i,$$

where q^i is the heat of the liquid at the temperature t_i of the injection-water as it enters the condenser. Each pound of steam will require

$$G = \frac{r + q - q_k}{q_k - q_i} \quad . \quad . \quad . \quad . \quad . \quad . \quad (150)$$

pounds of injection-water.

For example, steam at 4 pounds absolute has for the total heat 1126.5. If the injection-water enters with a temperature of 60° F., and leaves with a temperature of 120° F., then each pound of steam will require

$$\frac{r + q - q_k}{q_k - q_i} = \frac{1126.5 - 88.0}{88.0 - 28.12} = 17.3$$

pounds of injection-water. This calculation is used only to aid in determining the size of the pipes and passages leading water to and from the condenser, and the dimensions of the air-pump. Anything like refinement is useless and impossible, as conditions are seldom well known and are liable to vary. From 20 to 30 times the weight of steam used by the engine is commonly taken for this purpose.

The jet-condensers cannot be used at sea when the boiler-pressure exceeds 40 pounds by the gauge; all modern steamers are consequently supplied with surface-condensers which consist of receptacles, which are commonly rectangular in shape, into which steam is exhausted, and where it is condensed on horizontal brass tubes through which cold sea-water is circulated. The condensed water is drained out through the air-pump and is returned to the boiler. Thus the feed-water is kept free from salt and other mineral matter that would be pumped into the boiler if a jet-condenser were used, and if the feed-water were drawn from the mingled water and condensed steam from such a condenser. Much trouble is, however, experienced from oil used to lubricate the cylinders of the engine, as it is likely to be pumped into the boilers with the feed-water, even though attempts are made to strain or filter it from the water.

The water withdrawn from a surface-condenser is likely to

have a different temperature from the cooling water when it leaves the condenser. If its temperature is t_1, then we have instead of equation (150)

$$G = \frac{r + q - q_1}{q_t - q_i} \quad . \quad . \quad . \quad . \quad . \quad . \quad (151)$$

for the cooling water per pound of steam. The difference is really immaterial, as it makes little difference in the actual value of G for any case.

Cooling Surface. — Experiments on the quantity of cooling surface required by a surface-condenser are few and unsatisfactory, and a comparison of condensers of marine engines shows a wide diversity of practice. Seaton says that with an initial temperature of 60°, and with 120° for the feed-water, a condensation of 13 pounds of steam per square foot per hour is considered fair work. A new condenser in good condition may condense much more steam per square foot per hour than this, but allowance must be made for fouling and clogging, especially for vessels that make long voyages.

Seaton also gives the following table of square feet of cooling surface per indicated horse-power:

Absolute Terminal Pressure, Pounds per Square Inch.	Square Feet per I. H. P.
20	1.17
15	1.57
12½	1.50
10	1.43
8	1.37
6	1.30

For ships stationed in the tropics, allow 20 per cent more; for ships which occasionally visit the tropics, allow 10 per cent more; for ships constantly in a cold climate, 10 per cent less may be allowed.

Air-Pump. — The vacuum in the condenser is maintained by the air-pump, which pumps out the air which finds its way there by leakage or otherwise; the condensing water carries

a considerable volume of air into the condenser, and the size of the air-pump can be based roughly on the average percentage of air held in solution in water; the air which finds its way into a surface-condenser enters mainly by leakage around the low-pressure piston-rod and elsewhere.

It is customary to base the size of the air-pump on the displacement of the low-pressure piston (or pistons); for example, the capacity of a single-acting vertical air-pump for a merchant steamer, with triple-expansion engines, may be about $\frac{1}{20}$ of the capacity of the low-pressure cylinder.

With the introduction of steam-turbines, the importance of a good vacuum becomes more marked, and the duty of the air-pump, which commonly removes air and also the water of condensation from the condenser, is divided between a dry-air pump, which removes air from the condenser, and a water-pump, which removes the water of condensation. Air-pumps are treated more at length on page 374, in connection with the discussion of compressed air.

Designing Engines. — The only question that is properly discussed here is the probable form of the indicator-diagram, which gives immediately the method of finding the mean effective pressure, and, consequently, the size of the cylinder of the engine.

The most reliable way of finding the expected mean effective pressure in the design of a new engine is to measure an indicator-diagram from an engine of the same or similar type and size, and working under the same conditions.

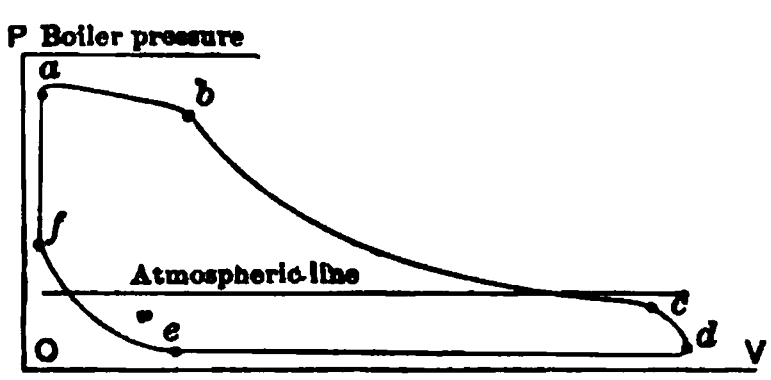

FIG. 35a.

If a new engine varies so much from the type on which the design is based that the diagram from the latter cannot be used directly, the following method may be used to allow for moderate changes of boiler pressure or expansion. The type diagram either on the original card or redrawn to a larger scale, may have added to it the axis of zero pressure and vol-

ume OV and OP (Fig. 35a). The former is laid off parallel to the atmospheric line and at a distance to represent the pressure of the atmosphere, using the scale for measuring pressure on the diagram. The latter is drawn vertical and at a distance from *af* which shall bear the same ratio to the length of the diagram as the clearance space of the cylinder has to the piston-displacement. The boiler-pressure line may be drawn as shown. The absolute pressure may now be measured from OV with the proper scale and volume from OP with any convenient scale.

Choosing points *b* and *c* at the beginning and end of expansion determine the exponent for an exponential equation by the method on page 66; do the same for the compression curve *ef*.

Draw a diagram like Fig. 35 for the new engine, making the proper allowance for change of boiler-pressure or point of cut-off, using the probable clearance for determining the position of the line *af*. Allowing for loss of pressure from the boiler to the cylinder, and for wire-drawing or loss of pressure in the valves and passages, locate the points *a* and *b*. The back-pressure line *de* can be drawn from an estimate of the probable vacuum. The volumes at *c* and *e* are determined by the action of the valve gear. By aid of the proper exponential equations locate a few points on *bc* and *ef* and sketch in those curves. Finish the diagram by hand by comparison with the type diagram. If necessary draw two such diagrams for the head and crank ends of the cylinder. The mean effective pressure can now be determined by aid of the planimeter and used in the design of the new engine.

Usually the refinements of the method just detailed are avoided, and an allowance is made for them in the lump by a practical factor. The following approximations are made: (1) the pressure in the cylinder during admission is assumed to be the boiler pressure, and during the exhaust the vacuum in the condenser; (2) the release is taken to be at the end of the stroke; (3) both expansion and compression lines are treated as hyperbolæ. The mean effective pressure is then readily computed as indicated in the following example.

Problem. — Required the dimensions of the cylinder of an engine to give 200 horse-power; revolutions 100; gauge pressure 80 pounds; vacuum 28 inches; cut-off at $\frac{1}{4}$ stroke; release at end of stroke; compression at $\frac{1}{10}$ stroke; clearance 5 per cent.

The absolute boiler-pressure is 94.7 pounds, and the absolute pressure corresponding to 28 inches of mercury is nearly one pound. It is convenient to take the piston displacement as one cubic foot and the stroke as one foot for the purpose of determining the mean effective pressure. The volume of cut-off is consequently $\frac{1}{4}$ cubic foot due to the motion of the piston plus $\frac{1}{10}$ cubic foot due to the clearance or 0.35 cubic foot; the volume at release is 1.05 cubic foot, and at compression is 0.15 cubic foot.

The work during admission (corresponding to *ab*, Fig. 35a) is

$$94.7 \times 144 \times 0.35 \text{ foot-pound,}$$

and during expansion is

$$p_1 v_1 \log_e \frac{v_2}{v_1} = 94.7 \times 144 \times 0.35 \log_e \frac{1.05}{0.35}.$$

The work during exhaust done by the piston in expelling the steam is

$$1 \times 144 \times (1 - 0.1),$$

and the work during compression is

$$1 \times 144 \times 0.15 \log_e \frac{0.15}{0.05}.$$

The mean effective pressure in pounds per square inch is obtained by adding the first two works and subtracting the last two and then dividing by 144, so that

$$\text{M.E.P.} = 94.7 \times 0.25 + 94.7 \times 0.35 \log_e \frac{1.05}{0.35}.$$

$$- 1 \times 0.9 - 1 \times 0.15 \log_e \frac{0.15}{0.05} = 59.1.$$

The probable mean effective pressure may be taken as $\frac{9}{10}$ of this computed pressure, or 53.2 pounds per square inch.

Given the diameter and stroke of an engine together with the mean effective pressure, and revolutions, we may find the horsepower by the formula

$$\text{I.H.P.} = \frac{2\,plan}{33000}$$

where p is the mean effective pressure, l is the stroke in feet, a is the area of the circle for the given diameter in square inches, and n is the number of revolutions per minute. For our case we may assume that the stroke is twice the diameter, whence

$$200 = \frac{2 \times 53.2 \times \frac{2d}{12} \times \frac{\pi d^2}{4} \times 100}{33000}.$$

$$\therefore\ d = 16.8 \text{ inches,} \qquad s = 33.6 \text{ inches.}$$

In practice the diameter would probably be made $16\frac{3}{4}$ inches and the stroke $33\frac{1}{2}$ inches.

CHAPTER IX.

COMPOUND ENGINES.

A COMPOUND engine has commonly two cylinders, one of which is three or four times as large as the other. Steam from a boiler is admitted to the small cylinder, and after doing work in that cylinder it is transferred to the large cylinder, from which it is exhausted, after doing work again, into a condenser or against the pressure of the atmosphere. If we assume that the steam from the small cylinder is exhausted into a large receiver, the back-pressure in that cylinder and the pressure during the admission to the large cylinder will be uniform. If, further, we assume that there is no clearance in either cylinder, that the back-pressure in the small cylinder and the forward pressure in the large cylinder are the same, and that the expansion in the small cylinder reduces the pressure down to the back-pressure in that cylinder, the diagram for the small cylinder will be *ABCD*,

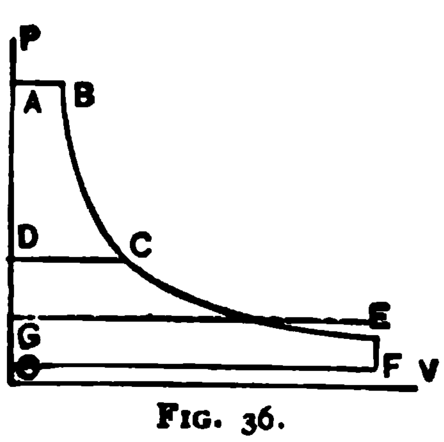

FIG. 36.

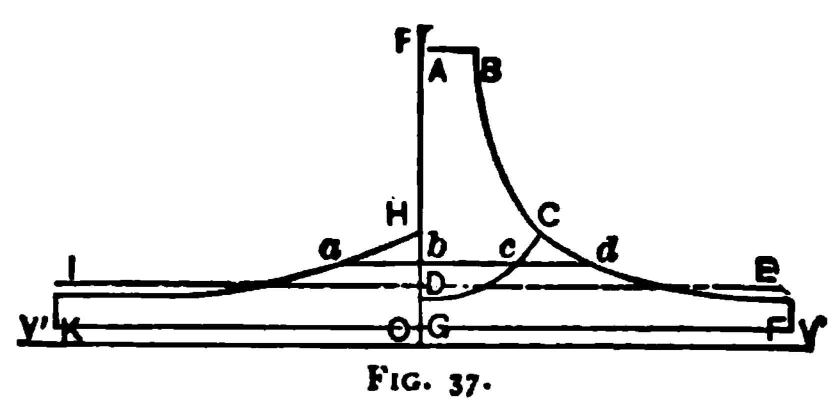

FIG. 37.

Fig. 36, and for the large cylinder *DCFG*. The volume in the large cylinder at cut-off is equal to the total volume of the small cylinder, since the large cylinder takes from the receiver the same weight of steam that is exhausted by the small cylinder, and, in this case, at the same pressure.

The case just discussed is one extreme. The other extreme occurs when the small cylinder exhausts directly into the large

cylinder without an intermediate receiver. In such engines the pistons must begin and end their strokes together. They may both act on the beam of a beam engine, or they may act on one crank or on two cranks opposite each other.

For such an engine, *ABCD*, Fig. 37, is the diagram for the small cylinder. The steam line and expansion line *AB* and *BC* are like those of a simple engine. When the piston of the small cylinder begins the return stroke, communication is opened with the large cylinder, and the steam passes from one to the other, and expands to the amount of the difference of the volume, it being assumed that the communication remains open to the end of the stroke. The back-pressure line *CD* for the small cylinder, and the admission line *HI* for the large cylinder, gradually fall on account of this expansion. The diagram for the large cylinder is *HIKG*, which is turned toward the left for convenience.

To combine the two diagrams, draw the line *abcd*, parallel to *V'OV*, and from *b* lay off *bd* equal to *ca*; then *d* is one point of the expansion curve of the combined diagram. The point *C* corresponds with *H*, and *E*, corresponding with *I*, is as far to the right as *I* is to the left.

For a non-conducting cylinder, the combined diagram for a compound engine, whether with or without a receiver, is the same as that for a simple engine which has a cylinder the same size as the large cylinder of the compound engine, and which takes at each stroke the same volume of steam as the small cylinder, and at the same pressure. The only advantage gained by the addition of the small cylinder to such an engine is a more even distribution of work during the stroke, and a smaller initial stress on the crank-pin.

Compound engines may be divided into two classes — those with a receiver and those without a receiver; the latter are called "Woolf engines" on the continent of Europe. Engines without a receiver must have the pistons begin and end their strokes at the same time; they may act on the same crank or on cranks 180° apart. The pistons of a receiver compound engine may make strokes in any order. A form of receiver compound engine with

two cylinders, commonly used in marine work, has the cranks at 90° to give handiness and certainty of action. Large marine engines have been made with one small cylinder and two large or low-pressure cylinders, both of which draw steam from the receiver and exhaust to the condenser. Such engines usually have the cranks at 120°, though other arrangements have been made.

Nearly all compound engines have a receiver, or a space between the cylinders corresponding to one, and in no case is the receiver of sufficient size to entirely prevent fluctuations of pressure. In the later marine work the receiver has been made small, and frequently the steam-chests and connecting pipes have been allowed to fulfil that function. This contraction of size involves greater fluctuations of pressure, but for other reasons it appears to be favorable to economy.

Under proper conditions there is a gain from using a compound engine instead of a simple engine, which may amount to ten per cent or more. This gain is to be attributed to the division of the change of temperature from that of the steam at admission to that of exhaust into two stages, so that there is less fluctuation of temperature in a cylinder and consequently less interchange of heat between the steam and the walls of the cylinder.

Compound Engine without Receiver. — The indicator-diagrams from a compound engine without a receiver are represented by Fig. 38. The steam line and expansion line of the small cylinder, AB and BC, do not differ from those of a simple engine. At C the exhaust opens, and the steam suddenly expands into the space between the cylinders and the clearance of the large cylinder, and the pressure falls from C to D. During the return stroke the volume in the large cylinder increases more rapidly than that of the small cylinder decreases, so that the back-pressure line DE gradually falls, as does also the admission line HI of the large cylinder, the difference between these two lines being due to the resistance to the flow of steam from one to the other.

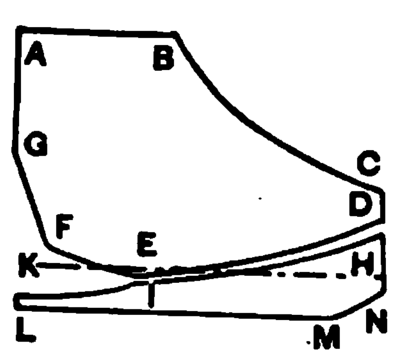

Fig. 38.

At *E* the communication between the two cylinders is closed by the cut-off of the large cylinder; the steam is then compressed in the small cylinder and the space between the two cylinders to *F*, at which the exhaust of the small cylinder closes; and the remainder of the diagram *FGA* is like that of a simple engine. From *I*, the point of cut-off of the large cylinder, the remainder of the diagram *IKLMNH* is like the same part of the diagram of a simple engine.

The difference between the lines *ED* and *HI* and the "drop" *CD* at the end of the stroke of the small piston indicate waste or losses of efficiency. The compression *EFG* and the accompanying independent expansion *IK* in the large cylinder show a loss of power when compared with a diagram like Fig. 37 for an engine which has no clearance or intermediate space; but compression is required to fill waste spaces with steam. The compression *EF* is required to reduce the drop *CD*, and the compression *FG* fills the clearance in anticipation of the next supply from the boiler. Neither compression is complete in Fig. 38.

Diagrams from a pumping engine at Lawrence, Massachusetts, are shown by Fig. 39. The rounding of corners due to the indicator makes it difficult to determine the location of points like *D*, *E*, *F*, and *I* on Fig. 38. The low-pressure diagram is taken with a weak spring, and so has an exaggerated height.

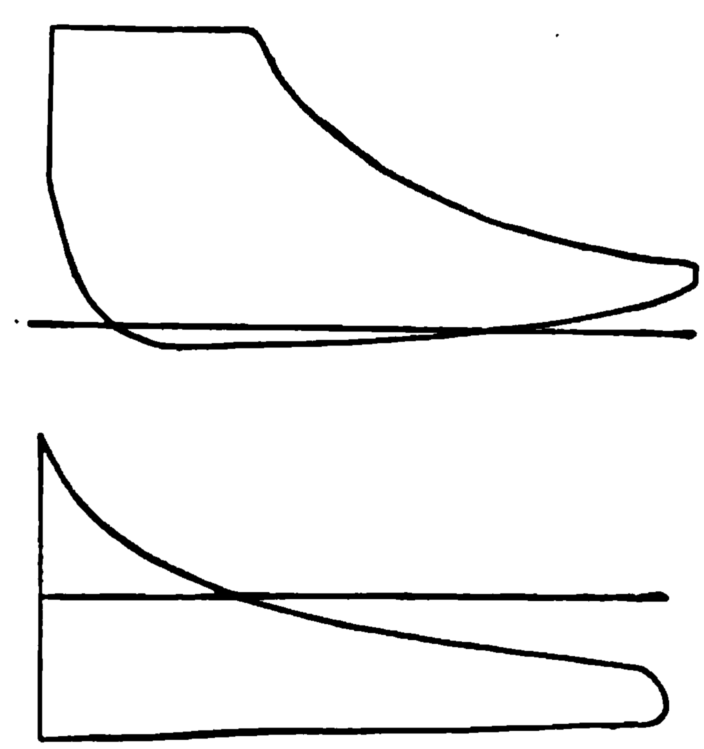

FIG. 39.

Compound Engine with Receiver. — It has already been pointed out that some receiver space is required if the cranks of a compound engine are to be placed at right angles. When the receiver space is small, as on most marine engines, the fluctuations of pressure in the receiver are very notable. This is exhibited by the diagrams in Fig. 40, which were taken from a yacht engine. An intelligent conception of the causes and meaning

of such fluctuations can be best obtained by constructing ideal diagrams for special cases, as explained on page 164.

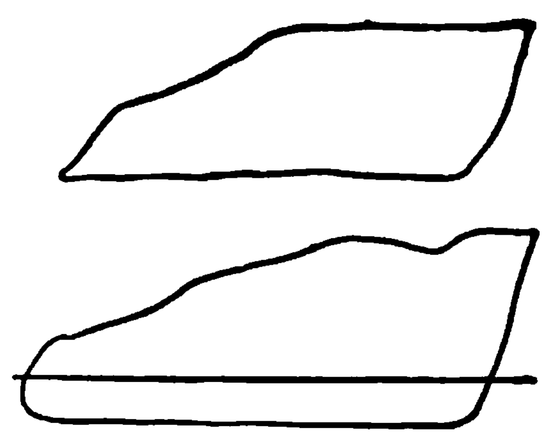

FIG. 40.

Triple and Quadruple Expansion-Engines. — The same influences which introduced the compound engines, when the common steam-pressure changed from forty to eighty pounds to the square inch, have brought in the successive expansion through three cylinders (the high-pressure, intermediate, and low-pressure cylinders) now that 150 to 200 pounds pressure are employed. Just as three or more cylinders are combined in various ways for compound engines, so four, five, or six cylinders have been arranged in various manners for triple-expansion engines; the customary arrangement has three cylinders with the cranks at 120°.

Quadruple engines with four successive expansions have been employed with high-pressure steam, but with the advisable pressures for present use the extra complication and friction make it a doubtful expedient.

Total Expansion. — In Figs. 36 and 37, representing the diagrams for compound engines without clearance and without drop between the cylinders, the total expansion is the ratio of the volumes at E and at B; that is, of the low-pressure piston displacement to the displacement developed by the high-pressure piston at cut-off. The same ratio is called the total or equivalent expansion for any compound engine, though it may have both clearance and drop. Such a conventional total expansion is commonly given for compound and multiple-expansion engines, and is a convenience because it is roughly equal to the ratio of the initial and terminal pressures; that is, of the pressure at cut-off in the high-pressure cylinder and at release in the low-pressure cylinder. It has no real significance, and is not equivalent to the expansion in the cylinder of a simple engine, by which we mean the ratio of the volume at release to that at cut-off, taking account of clearance. And further, since the clearance of

the high- and the low-pressure cylinders are different there can be no real equivalent expansion.

If the ratio of the cylinders is R and the cut-off of the high-pressure cylinder is at $\frac{1}{e}$ of the stroke, then the total expansion is represented by

$$E = eR$$

and

$$\frac{1}{e} = R \div E.$$

This last equation is useful in determining approximately the cut-off of the high-pressure cylinder.

For example, if the initial pressure is 100 pounds absolute and the terminal pressure is to be 10 pounds absolute, then the total expansions will be about 10. If the ratio of the cylinders is $3\frac{1}{2}$, then the high-pressure cut-off will be about

$$\frac{1}{e} = 3\tfrac{1}{2} \div 10 = 0.35$$

of the stroke.

Low-pressure Cut-off. — The cut-off of the low-pressure cylinders in Figs. 36 and 37 is controlled by the ratio of the cylinders, since the volumes in the low-pressure cylinder at cut-off is in each case made equal to the high-pressure piston displacement; this is done to avoid a drop. If the cut-off were lengthened there would be a loss of pressure or drop at the end of the stroke of the high-pressure piston, as is shown by Fig. 41, for an engine with a large receiver and no clearance. Such a drop will have some effect on the character of the expansion line $C''F$ of the low-pressure cylinder, both for a non-conducting and for the actual engine with or without a clearance. Its principal effect will, however, be on the distribution of work between the cylinders; for it is evident that if the cut-off of the low-pressure cylinder is shortened the

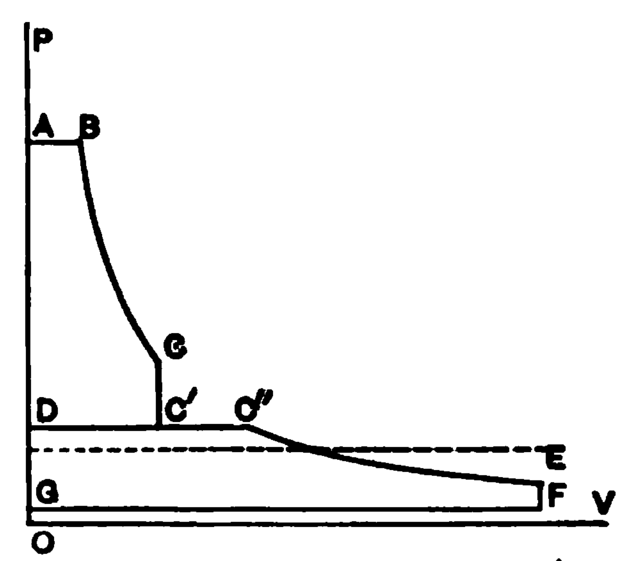

FIG. 41.

pressure at C'' will be increased because the same weight of steam is taken in a smaller volume. The back-pressure DC' of the high-pressure cylinder will be raised and the work will be diminished; while the forward pressure DC'' of the low-pressure cylinder will be raised, increasing the work in that cylinder.

Ratio of Cylinders. — In designing compound engines, more especially for marine work, it is deemed important for the smooth action of the engine that the total work shall be evenly distributed upon the several cranks of the engines, and that the maximum pressure on each of the cranks shall be the same, and shall not be excessive. In case two or more pistons act on one crank, the total work and the resultant pressure on those pistons are to be considered; but more commonly each piston acts on a separate crank, and then the work and pressure on the several pistons are to be considered.

In practice both the ratio of the cylinders and the total expansions are assumed, and then the distribution of work and the maximum loads on the crank-pins are calculated, allowing for clearance and compression. Designers of engines usually have a sufficient number of good examples at hand to enable them to assume these data. In default of such data it may be necessary to assume proportions, to make preliminary calculations, and to revise the proportions till satisfactory results are obtained. For compound engines using 80 pounds steam-pressure the ratio is 1 : 3 or 1 : 4. For triple-expansion engines the cylinders may be made to increase in the ratio 1 : 2 or 1 : 2½.

Approximate Indicator-Diagrams. — The indicator-diagrams from some compound and multiple-expansion engines are irregular and apparently erratic, depending on the sequence of the cranks, the action of the valves, and the relative volumes of the cylinders and the receiver spaces. A good idea of the effects and relations of these several influences can be obtained by making approximate calculations of pressures at the proper parts of the diagrams by a method which will now be illustrated.

For such a calculation it will be assumed that all expansion

lines are rectangular hyperbolæ, and in general that any change of volume will cause the pressure to change inversely as the volume. Further, it will be assumed that when communication is opened between two volumes where the pressures are different, the resultant pressure may be calculated by adding together the products of each volume by its pressure, and dividing by the sum of the volumes. Thus if the pressure in a cylinder having the volume v_c is p_c, and if the pressure is p_r in a receiver where the volume is v_r, then after the valve opens communication from the cylinder to the receiver the pressure will be

$$p = \frac{p_c v_c + p_r v_r}{v_c + v_r}.$$

The same method will be used when three volumes are put into communication.

It will be assumed that there are no losses of pressure due to throttling or wire-drawing; thus the steam line for the high-pressure cylinder will be drawn at the full boiler-pressure, and the back-pressure line in the low-pressure cylinder will be drawn to correspond with the vacuum in the condenser. Again, cylinders and receiver spaces in communication will be assumed to have the same pressure.

For sake of simplicity the motions of pistons will be assumed to be harmonic.

Diagrams constructed in this way will never be realized in any engine; the degree of discrepancy will depend on the type of engine and the speed of rotation. For slow-speed pumping-engines the discrepancy is small and all irregularities are easily accounted for. On the other hand the discrepancies are large for high-speed marine-engines, and for compound locomotives they almost prevent the recognition of the events of the stroke.

Direct-expansion Engine. — If the two pistons of a compound engine move together or in opposite directions the diagrams are like those shown by Fig. 42. Steam is admitted to the high-pressure cylinder from a to b; cut-off occurs at b, and bc represents expansion to the end of the stroke; bc being a rectangular

hyperbola referred to the axes OV and OP, from which a, b, and c are laid off to represent absolute pressures and volumes, including clearance.

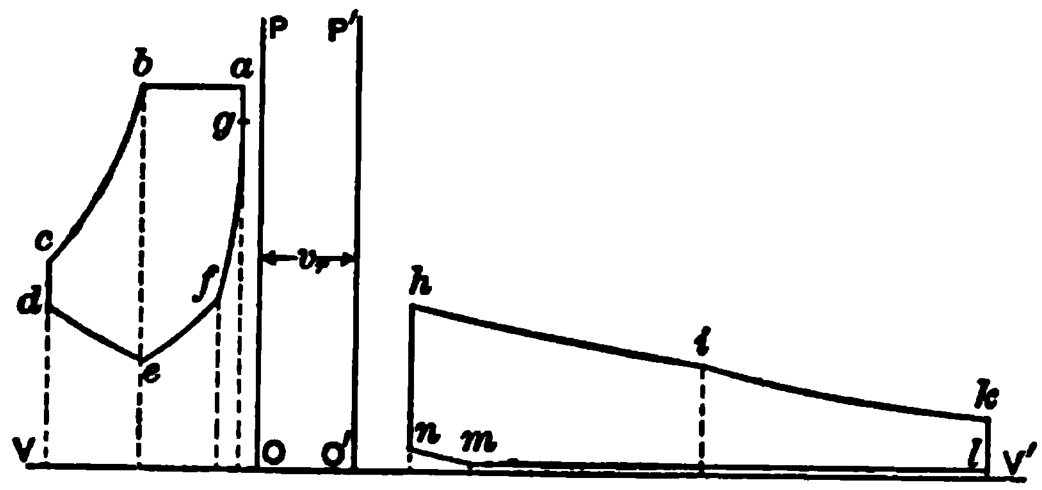

FIG. 42.

At the end of the stroke release from the high-pressure cylinder and admission to the low-pressure cylinder are assumed to take place, so that communication is opened from the high-pressure cylinder through the receiver space into the low-pressure cylinder. As a consequence the pressure falls from c to d, and rises from n to h in the low-pressure cylinder. The line $O'P'$ is drawn at a distance from OP, which corresponds to the volume of the receiver space, and the low-pressure diagram is referred to $O'P'$ and $O'V'$ as axes.

The communication between the cylinders is maintained until cut-off occurs at i for the low-pressure cylinder. The lines de and hi represent the transfer of steam from the high-pressure to the low-pressure cylinder, together with the expansion due to the increased size of the large cylinder. After the cut-off at i, the large cylinder is shut off from the receiver, and the steam in it expands to the end of the stroke. The back-pressure and compression lines for that cylinder are not affected by compounding, and are like those of a simple engine. Meanwhile the small piston compresses steam into the receiver, as represented by ef, till compression occurs, after which compression into the clearance space is represented by fg. The expansion and compression lines ik and mn are drawn as hyperbolæ referred to the axes $O'P'$ and $O'V'$. The compression line ef is drawn as an hyperbola referred to $O'V$ and $O'P'$, while fg is referred to OV and OP.

In Fig. 42 the two diagrams are drawn with the same scale for volume and pressure, and are placed so as to show to the eye the relations of the diagrams to each other. Diagrams taken from such an engine resemble those of Fig. 39, which have the same length, and different vertical scales depending on the springs used in the indicators.

Some engines have only one valve to give release and compression for the high-pressure cylinder and admission and cut-off for the low-pressure cylinder. In such case there is no receiver space, and the points e and f coincide.

When the receiver is closed by the compression of the high-pressure cylinder it is filled with steam with the pressure represented by f. It is assumed that the pressure in the receiver remains unchanged till the receiver is opened at the end of the stroke. It is evident that the drop cd may be reduced by shortening the cut-off of the low-pressure cylinder so as to give more compression from e to f and consequently a higher pressure at f when the receiver is closed.

Representing the pressure and volume at the several points by p and v with appropriate subscript letters, and representing the volume of the receiver by v_r, we have the following equations:

$$p_a = p_b = \text{initial pressure};$$
$$p_l = p_m = \text{back-pressure};$$
$$p_c = p_b v_b \div v_c;$$
$$p_n = p_m v_m \div v_n;$$
$$p_d = p_h = (p_c v_c + p_n v_n + p_f v_r) \div (v_c + v_n + v_r);$$
$$p_e = p_i = p_d\,(v_c + v_n + v_r) \div (v_e + v_i + v_r);$$
$$p_f = p_e\,(v_e + v_r) \div (v_f + v_r);$$
$$p_g = p_f v_f \div v_g;$$
$$p_k = p_i v_i \div v_k.$$

The pressures p_c and p_n can be calculated directly. Then the equations for p_d, p_e, and p_f form a set of three simultaneous equations with three unknown quantities, which can be easily solved. Finally, p_g and p_k may be calculated directly.

For example, let us find the approximate diagram for a direct-expansion engine which has the low-pressure piston displacement equal to three times the high-pressure piston displacement. Assume that the receiver space is half the smaller piston displacement, and that the clearance for each cylinder is one-tenth of the corresponding piston displacement. Let the cut-off for each cylinder be at half-stroke, and the compression at nine-tenths of the stroke; let the admission and release be at the end of the stroke. Let the initial pressure be 65.3 pounds by the gauge or 80 pounds absolute, and let the back-pressure be two pounds absolute.

If the volume of the high-pressure piston displacement be taken as unity, then the several required volumes are:

$$\begin{array}{ll}
v_b = 0.5 + 0.1 = 0.6 & v_h = v_n = 3 \times 0.1 = 0.3 \\
v_c = v_d = 1.0 + 0.1 = 1.1 & v_i = 3\,(0.5 + 0.1) = 1.8 \\
v_e = 0.5 + 0.1 = 0.6 & v_k = v_l = 3\,(1.0 + 0.1) = 3.3 \\
v_f = 0.1 + 0.1 = 0.2 & v_m = 3\,(0.1 + 0.1) = 0.6 \\
v_g = 0.1 & v_r = 0.5
\end{array}$$

The pressures may be calculated as follows:

$$p_a = p_b = 80; \quad p_l = p_m = 2;$$

$$p_c = p_b v_b \div v_c = 80 \times 0.6 \div 1.1 = 43.6;$$

$$p_n = p_m v_m \div v_n = 2 \times 0.6 \div 0.3 = 4;$$

$$p_e = p_d\,(v_c + v_n + v_r) \div (v_e + v_i + v_r) = p_d\,(1.1 + 0.3 + 0.5) \div (0.6 + 1.8 + 0.5) = 0.655\,p_d;$$

$$p_f = p_e\,(v_e + v_r) \div (v_f + v_r) = p_e\,(0.6 + 0.5) \div (0.2 + 0.5) = 1.57\,p_e = 1.57 \times 0.655\,p_d = 1.03\,p_d;$$

$$\begin{aligned}
p_d &= (p_c v_c + p_n v_n + p_f v_r) \div (v_c + v_n + v_r) \\
&= (80 \times 0.6 + 4 \times 0.3 + 0.5\,p_f) \div (0.6 + 0.3 + 0.5) \\
&= 25.89 + 0.26\,p_f;
\end{aligned}$$

$$p_d = 25.89 + 0.26 \times 1.03\,p_d; \quad p_d = 35.36;$$

$$p_e = p_i = 0.655\,p_d = 0.655 \times 35.36 = 23.2;$$

$$p_f = 1.03\,p_d = 1.03 \times 35.36 = 36.5;$$

$$p_g = p_f v_f \div v_g = 36.5 \times 0.2 \div 0.1 = 73;$$

$$p_k = p_i v_i \div v_k = 23.2 \times 1.8 \div 3.3 = 12.6.$$

As the pressures and volumes are now known the diagrams of Fig. 42 may be drawn to scale. Or, if preferred, diagrams like Fig. 39 may be drawn, making them of the same length and using convenient vertical scales of pressure. If the engine runs slowly and has abundant valves and passages the diagrams thus obtained will be very nearly like those taken from the engine by indicators. If losses of pressure in valves and passages are allowed for, a closer approximation can be made.

The mean effective pressures of the diagrams may be readily obtained by the aid of a planimeter, and may be used for estimating the power of the engine. For this purpose we should either use the modified diagrams allowing for losses of pressure, or we should affect the mean effective pressures by a multiplier obtained by comparison of the approximate with the actual diagrams from engines of the same type. For a slow-speed pumping-engine the multiplier may be as large as 0.9 or even more; for high-speed engines it may be as small as 0.6.

The mean effective pressures of the diagrams may be calculated from the volumes and pressures if desired, assuming, of course, that the several expansion and compression curves are hyperbolæ. The process can be best explained by applying it to the example already considered. Begin by finding the mean pressure during the transfer of steam from the high-pressure cylinder to the low-pressure cylinder as represented by *de* and *hi*. The net effective work during the transfer is

$$\int p dv = p_1 v_1 \log_e \frac{v_2}{v_1} = 144\, p_d\, (v_d + v_h + v_r) \log_e \frac{v_e + v_i + v_r}{v_d + v_h + v_r}$$

$$= 144 \times 35.4\, (1.1 + 0.3 + 0.5) \log_e \frac{0.6 + 1.8 + 0.5}{1.1 + 0.3 + 0.5}$$

$$= 4120 \text{ foot-pounds}$$

for each cubic foot of displacement of the high-pressure piston. This corresponds with our previous assumption of unity for the displacement of that piston. The increase of volume is

$$v_e + v_i + v_r - (v_d + v_h + v_r) = 0.6 + 1.8 + 0.5 - (1.1 + 0.3 + 0.5) = 1;$$

so that the mean pressure during the transfer is

$$4120 \div 1 \times 144 = 28.6 = p_x$$

pounds per square inch, which acts on both the high- and the low-pressure pistons.

The effective work for the small cylinder is obtained by adding the works under *ab* and *bc* and subtracting the works under *de*, *ef*, and *fg*. So that

$$W_H = 144 \left\{ p_a (v - v_a) + p_b v_b \log_e \frac{v_c}{v_b} - p_x (v_d - v_e) \right.$$
$$\left. - p_e (v_e + v_r) \log_e \frac{v_e + v_r}{v_f + v_r} - p_f v_f \log_e \frac{v_f}{v_g} \right\}$$
$$= 144 \left\{ 80 (0.6 - 0.1) + 80 \times 0.6 \log_e \frac{1.1}{0.6} - 28.6 (1.1 - 0.6) \right.$$
$$\left. - 23.2 (0.6 + 0.5) \log_e \frac{0.6 + 0.5}{0.2 + 0.5} - 36.5 \times 0.2 \log_e \frac{0.2}{0.1} \right\}$$
$$= 144 \times 33.26 = 4789 \text{ foot-pounds.}$$

This is the work for each cubic foot of the high-pressure piston displacement, and the mean effective pressure on the small piston is evidently 33.26 pounds per square inch.

In a like manner the work of the large piston is

$$W_L = 144 \left\{ p_x (v_i - v_h) + p_i v_i \log_e \frac{v_k}{v_i} - p_l (v_l - v_m) - p_m v_m \log_e \frac{v_m}{v_n} \right\}$$
$$= 144 \left\{ 28.6 (1.8 - 0.3) + 23.2 \times 1.8 \log_e \frac{3.3}{1.8} \right.$$
$$\left. - 2 (3.3 - 0.6) - 2 \times 0.6 \log_e \frac{0.6}{0.3} \right\} = 144 \times 61.92 = 8916 \text{ foot-pounds.}$$

Since the ratio of the piston displacements is 3, the work for each cubic foot of the low-pressure piston displacement is one-third of the work just calculated; and the mean effective pressure on the large piston is

$$61.92 \div 3 = 20.64$$

pounds per square inch.

The proportions given in the example lead to a very uneven distribution of work; that of the large cylinder being nearly twice as much as is developed in the small cylinder. The dis-

tribution can be improved by lengthening the cut-off of the large cylinder, or by changing the proportions of the engine.

As has already been pointed out, the works just calculated and the corresponding mean effective pressures are in excess of those that will be actually developed, and must be affected by multipliers which may vary from 0.6 to 0.9, depending on the type and speed of the engine.

Cross-compound Engine. — A two-cylinder compound engine with pistons connected to cranks at right angles with each other is frequently called a cross-compound engine. Unless a large receiver is placed between the cylinders the pressure in the space between the cylinders will vary widely.

Two cases arise in the discussion of this engine according as

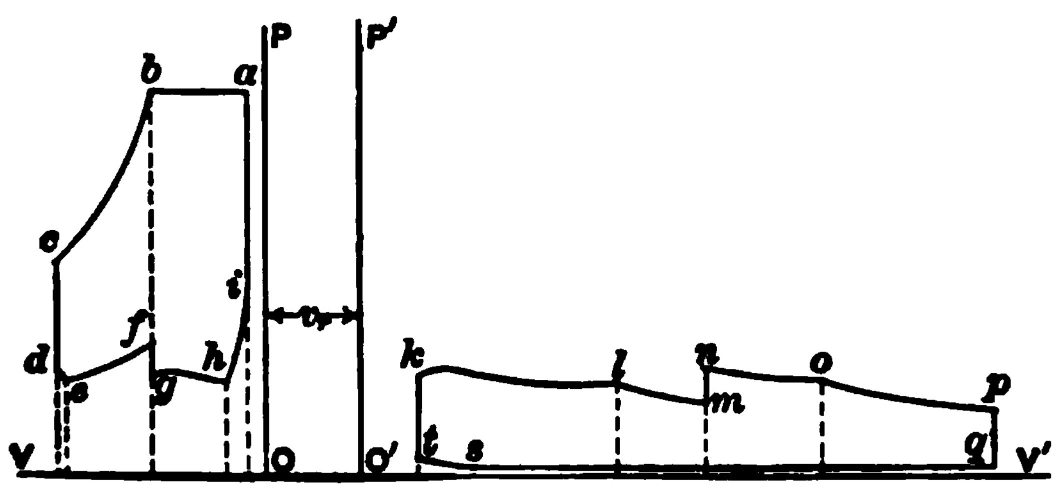

FIG. 43.

the cut-off of the large cylinder is earlier or later than half-stroke; in the latter case there is a species of double admission to the low-pressure cylinder, as is shown in Fig. 43. For sake of simplicity the release, and also the admission for each cylinder, is assumed to be at the end of the stroke. If the release is early the double admission occurs before half-stroke.

The admission and expansion of steam for the high-pressure cylinder are represented by *ab* and *bc*. At *c* release occurs, putting the small cylinder in communication with the intermediate receiver, which is then open to the large cylinder. There is a drop at *cd* and a corresponding rise of pressure *mn* on the large piston, which is then at half-stroke; it will be assumed that the pressures at *d* and at *n* are identical. From *d* to *e* the

steam is transferred from the small to the large cylinder, and the pressure falls because the volume increases; *no* is the corresponding line on the low-pressure diagram. The cut-off at *o* for the large cylinder interrupts this transfer, and steam is then compressed by the small piston into the intermediate receiver with a rise of pressure as represented by *ef*. The admission to the large cylinder, *tk*, occurs when the small piston is at the middle of its stroke, and causes a drop, *fg*, in the small cylinder. From *g* to *h* steam is transferred through the receiver from the small to the large cylinder. The pressure rises at first because the small piston moves rapidly while the large one moves slowly until its crank gets away from the dead-point; afterwards the pressure falls. The line *kl* represents this action on the low-pressure diagram. At *h* compression occurs for the small cylinder, and *hi* shows the rise of pressure due to compression. For the large cylinder the line *lm* represents the supply of steam from the receiver, with falling pressure which lasts till the double admission at *mn* occurs.

The expansion, release, exhaust, and compression in the large cylinder are not affected by compounding.

Strictly, the two parts of the diagram for the low-pressure cylinder, *mnopq* and *stklm*, belong to opposite ends of the cylinder, one belonging to the head end and one to the crank end. With harmonic motion the diagrams from the two ends are identical, and no confusion need arise from our neglect to determine which end of the large cylinder we are dealing with at any time. Such an analysis for the two ends of the cylinder, taking account of the irregularity due to the action of the connecting-rod, would lead to a complexity that would be unprofitable.

A ready way of finding corresponding positions of two pistons connected to cranks at right angles with each other is by aid of the diagram of Fig. 44. Let O be the centre of the crank-shaft and pR_yR_xq be the path of the crank-pin. When one piston has the displacement py and its crank is at OR_y, the other crank may be 90° ahead at OR_x and the corresponding piston displacement will be px. The same construction may be used if the

crank is 90° behind or if the angle R_yOR_x is other than a right angle. The actual piston position and crank angles when affected by the irregularity due to the connecting-rod will differ from those found by this method, but the positions found by such a diagram will represent the average positions very nearly.

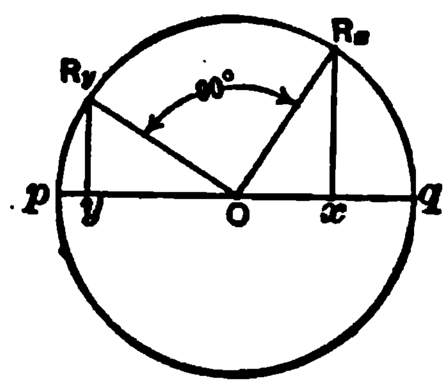

FIG. 44.

The several pressures may be found as follows:

$$p_b = p_a = \text{initial pressure;}$$
$$p_s = p_q = \text{back-pressure;}$$
$$p_c = p_b v_b \div v_c;$$
$$p_t = p_s v_s \div v_t;$$
$$p_d = p_n = \{p_c v_c + p_m(v_m + v_r)\} \div (v_c + v_n + v_r);$$
$$p_e = p_o = p_d\ (v_c + v_m + v_r) \div (v_e + v_o + v_r);$$
$$p_f = p_e\ (v_e + v_r) \div (v_f + v_r);$$
$$p_g = p_k = \{p_f\ (v_f + v_r) + p_t v_t\} \div (v_f + v_t + v_r);$$
$$p_h = p_l = p_g\ (v_f + v_t + v_r) \div (v_h + v_l + v_r);$$
$$p_m = p_l\ (v_l + v_r) \div (v_m + v_r);$$
$$p_i = p_h v_h \div v_i;$$
$$p_p = p_o v_o \div v_p.$$

The pressures p_c and p_t can be found directly from the initial pressure and the back-pressure, and finally the last two equations give direct calculations for p_i and p_p so soon as p_h and p_o are found. There remain six equations containing six unknown quantities, which can be readily solved after numerical values are assigned to the known pressures and to all the volumes.

The expansion and compression lines, *bc* and *hi*, for the high-pressure diagrams are hyperbolæ referred to the axes OV and OP; and in like manner the expansion and compression lines *op* and *st*, for the low-pressure diagram, are hyperbolæ referred to $O'V'$ and $O'P'$. The curve *ef* is an hyperbola referred to $O'V$ and $O'P'$, and the curve *lm* is an hyperbola referred to OV' and OP. The transfer lines *de* and *no*, *gh* and *kl*, are not hyperbolæ. They may be plotted point by point by finding corre-

sponding intermediate piston positions, p_x and p_y, by aid of Fig. 44, and then calculating the pressure for these positions by the equation

$$p_x = p_y = p_d \ (v_d + v_m + v_r) \div (v_x + v_y + v_r).$$

The work and mean effective pressure may be calculated in a manner similar to that given for the direct-expansion engine; but the calculation is tedious, and involves two transfers, *de* and *no*, and *gh* and *kl*. The first involves only an expansion, and presents no special difficulty; the second consists of a compression and an expansion, which can be dealt with most conveniently by a graphical construction. All things considered, it is better to plot the diagrams to scale and determine the areas and mean effective pressures by aid of a planimeter.

If the cut-off of the low-pressure is earlier than half-stroke so as to precede the release of the high-pressure cylinder the transfer represented by *de* and *no*, Fig. 43, does not occur. Instead there is a compression from *d* to *f* and an expansion from *l* to *m*. The number of unknown quantities and the number of equations are reduced. On the other hand, a release before the end of the stroke of the high-pressure piston requires an additional unknown quantity and one more equation.

Triple Engines. — The diagrams from triple and other multiple-expansion engines are likely to show much irregularity, the form depending on the number and arrangement of the cylinders and the sequence of the cranks. A common arrangement for a triple engine is to have three pistons acting on cranks set equidistant around the circle, as shown by Fig. 45. Two cases arise depending on the sequence of the cranks, which may be in the order, from one end of the engine, of high-pressure, low-pressure, and intermediate, as shown by Fig. 45; or in the order of high-pressure, intermediate, and low-pressure.

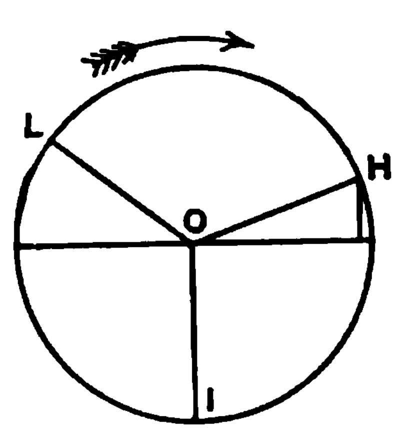

FIG. 45.

With the cranks in the order, high-pressure, low-pressure, and

intermediate, as shown by Fig. 45, the diagrams are like those given by Fig. 46. The admission and expansion for the high-pressure cylinder are represented by *abc*. When the high-pressure piston is at release, its crank is at *H*, Fig. 45, and the intermediate crank is at *I*, so that the intermediate piston is near half-stroke. If the cut-off for that cylinder is later than

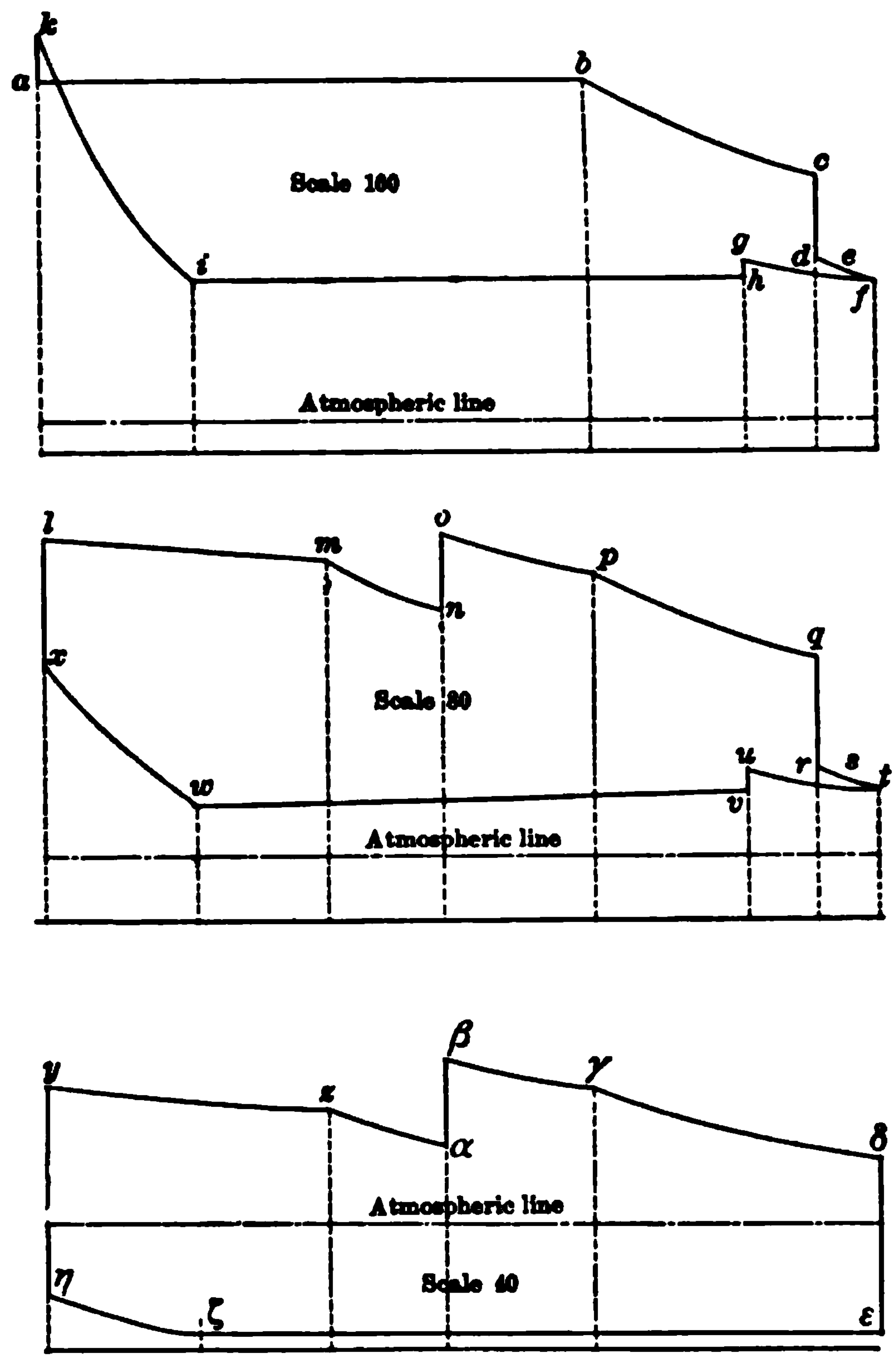

FIG. 46.

half-stroke, it is in communication with the first receiver when its crank is at *I*, and steam may pass through the first receiver from the high-pressure to the intermediate cylinder, and there is a drop *cd*, and a corresponding rise of pressure *no* in the intermediate cylinder. The transfer continues till cut-off for the

intermediate cylinder occurs at p, corresponding to the piston position e for the high-pressure cylinder. From the position e the high-pressure piston moves to the end of the stroke at f, causing an expansion, and then starts to return, causing the compression fg. When the high-pressure piston is at g the intermediate cylinder takes steam at the other end, causing the drop gh and the rise of pressure xl. Then follows a transfer of steam from the high-pressure to the intermediate cylinder represented by hi and lm. At i the high-pressure compression ik begins, and is carried so far as to produce a loop at k. After compression for the high-pressure cylinder shuts it from the first receiver, the steam in that receiver and in the intermediate cylinder expands as shown by mn. The expansion in the intermediate cylinder is represented by pq and the release by qr, corresponding to a rise of pressure $\alpha\beta$ in the low-pressure cylinder. rs and $\beta\gamma$ represent a transfer of steam from the intermediate cylinder to the low-pressure cylinder. The remainder of the back-pressure line of the intermediate cylinder and the upper part of the low-pressure diagram for the low-pressure cylinder correspond to the same parts of the high-pressure and the intermediate cylinders, so that a statement of the actions in detail does not appear necessary.

The equations for calculating the pressure are numerous, but they are not difficult to state, and the solution for a given example presents no special difficulty. Thus we have

$p_a = p_b =$ initial pressure; $\qquad v_\rho =$ vol. first receiver;

$p_c = p_b v_b \div v_c$; $\qquad v_R =$ vol. second receiver;

$$\text{I.} \quad p_d = p_o = \{p_c v_c + p_n (v_o + v_\rho)\} \div (v_d + v_o + v_\rho);$$
$$p_e = p_p = p_d (v_d + v_o + v_\rho) \div (v_e + v_p + v_\rho);$$
$$p_f = p_e (v_e + v_\rho) \div (v_f + v_\rho);$$
$$p_g = p_f (v_f + v_\rho) \div (v_g + v_\rho);$$

$$\text{II.} \quad p_h = p_l = \{p_g (v_g + v_\rho) + p_x v_x\} \div (v_h + v_l + v_\rho);$$
$$p_i = p_m = p_h (v_h + v_l + v_\rho) \div (v_i + v_m + v_\rho);$$
$$p_k = p_i v_i \div v_k;$$
$$p_n = p_m (v_m + v_\rho) \div (v_n + v_\rho);$$
$$p_q = p_p v_p \div v_q;$$

$$
\begin{aligned}
III.\quad & p_r = p_\beta = \{p_q v_q + p_s (v_s + v_R)\} \div (v_r + v_s + v_R); \\
& p_s = p_\gamma = p_r (v_r + v_s + v_R) \div (v_s + v_s + v_R); \\
& p_t = p_s (v_s + v_R) \div (v_t + v_R); \\
& p_u = p_t (v_t + v_R) \div (v_u + v_R);
\end{aligned}
$$

$$
\begin{aligned}
IV.\quad & p_v = \{p_u (v_u + v_R) + p_\eta v_\eta\} \div (v_v + v_\eta + v_R); \\
& p_w = p_v (v_v + v_\eta + v_R) \div (v_w + v_x + v_R); \\
& p_x = p_w v_w \div v_x; \\
& p_e = (v_x + v_R) \div (v_e + v_R); \\
& p_\delta = p_\gamma v_\gamma \div v_\delta; \\
& p_\epsilon = p_\zeta = \text{back-pressure}; \\
& p_\eta = p_\zeta v_\zeta \div v_\eta.
\end{aligned}
$$

The pressures at c and at η can be calculated immediately from the initial pressure and from the back-pressure. Then it will be recognized that there are four individual equations for finding p_f, p_k, p_t, and p_δ. The fourteen remaining equations, solved as simultaneous equations, give the corresponding fourteen required pressures, some of which are used in calculating the four pressures which are determined by the four individual equations. The most ready solution may be made by continuous substitution in the four equations which are numbered at the left hand. Thus for p_g in equation II, we may substitute,

$$
p_g = p_f \frac{v_f + v_p}{v_g + v_p} = p_e \frac{v_e + v_p}{v_f + v_p} \frac{v_f + v_p}{v_g + v_p} = p_d \frac{v_d + v_o + v_p}{v_e + v_p + v_p} \frac{v_e + v_p}{v_g + v_p}.
$$

In the actual computation the several volumes and the proper sums of volumes are to be first determined; consequently the factors following p_d will be numerical factors which may be conveniently reduced to the lowest terms before introduction in the equation. This system of substitution will give almost immediately four equations with four unknown quantities which may readily be solved; after which the determination of individual pressures will be easy. In handling these equations the letters representing smaller pressures should be eliminated first, thus giving values of higher pressure like p_d to tenths of a pound; afterward the lower pressure can be determined to a like degree

of accuracy. A skilled computer may make a complete solution of such a problem in two or three hours, which is not excessive for an engineering method.

If the cut-off for the intermediate cylinder occurs before the release of the high-pressure cylinder, then the transfer represented by *de* and *op* does not occur. In like manner, if the cut-off for the low-pressure cylinder occurs before the release for the intermediate cylinder, the transfer represented by *rs* and $\beta\gamma$ does not occur. The omission of a transfer of course simplifies the work of deducing and of solving equations.

In much the same way, equations may be deduced for calculating pressures when the cranks have the sequence high-pressure, intermediate, and low-pressure. The diagrams take forms which are distinctly unlike those for the other sequence of cranks. In general, the case solved, i.e., high-pressure, low-pressure, and intermediate, gives a smoother action for the engine.

For example, the engines of the U. S. S. *Machias* have the following dimensions and proportions:

•	High-pressure.	Inter-mediate.	Low-pressure.
Diameter of piston, inches	$15\frac{3}{4}$	$22\frac{1}{2}$	35
Piston displacement, cubic feet	2.71	5.53	13.39
Clearance, per cent	13	14	7
Cut-off, per cent stroke	66	66	66
Release, per cent stroke	93	93	93
Compression, per cent stroke	18	18	18
Ratio of piston displacements	1	2.04	4.94
Volume first receiver, cubic feet		2.22	
Volume second receiver, cubic feet		6.26	
Ratio of receiver volumes to high-pressure piston displacement	0.82		2.31
Stroke, inches		24	
Boiler-pressure, absolute, pounds per sq. in.		180	
Pressure in condenser, pounds per sq. in.		2	

If the volume of the high-pressure piston displacement is taken to be unity, then the volumes required in the equations for

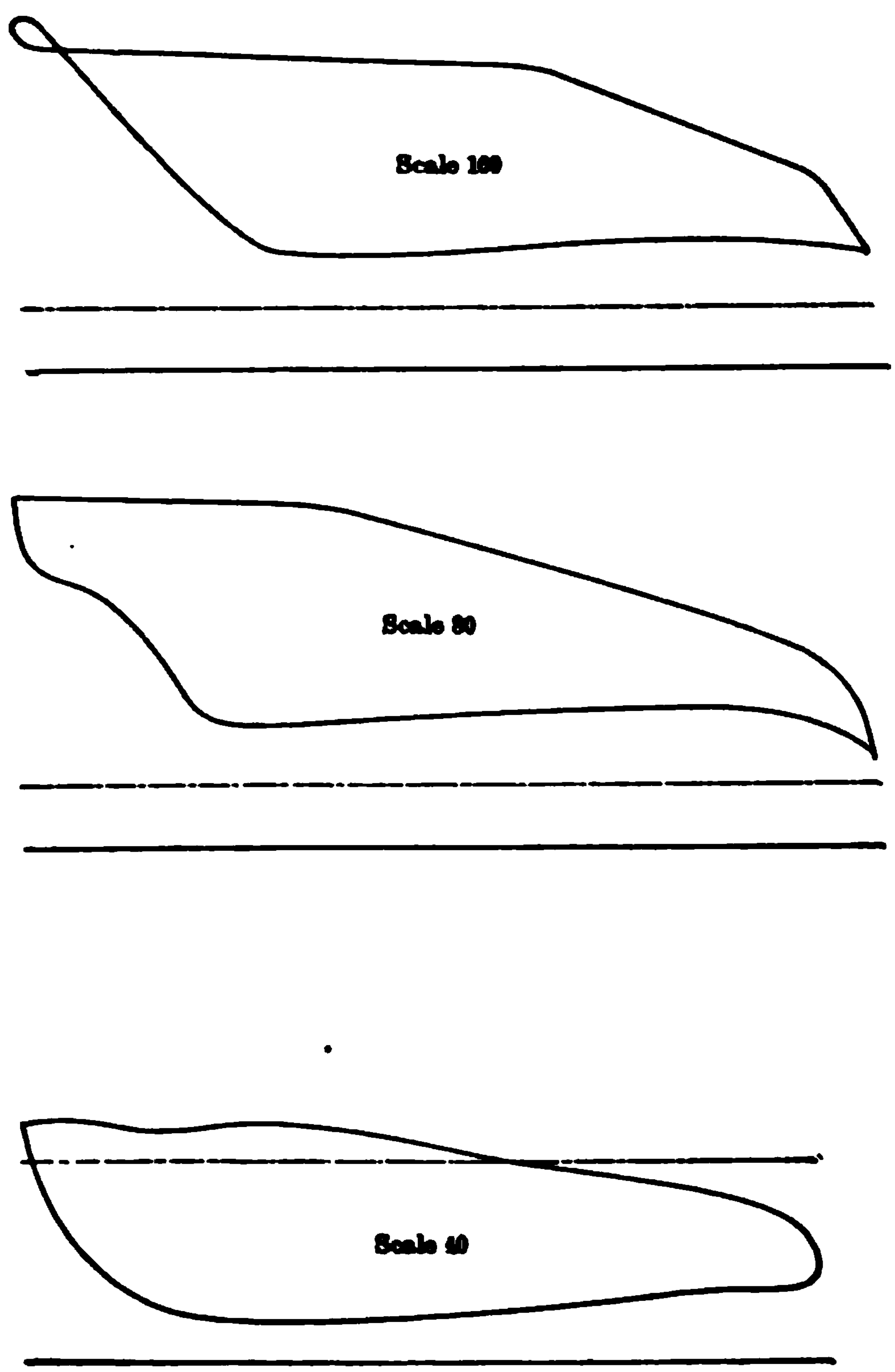

Fig. 47.

calculating pressures, when the cranks have the order high-pressure, low-pressure, and intermediate, are as follows:

$v_b = 0.79$	$v_l = v_x = 0.29$	$v_y = v_\eta = 0.35$
$v_c = v_d = 1.06$	$v_m = 0.98$	$v_z = 2.02$
$v_e = 1.10$	$v_n = v_o = 1.26$	$v_\alpha = v_\beta = 2.72$
$v_f = 1.13$	$v_p = 1.63$	$v_\gamma = 3.60$
$v_g = v_h = 0.88$	$v_q = v_r = 2.18$	$v_\delta = v_\epsilon = 4.94$
$v_i = 0.31$	$v_s = 2.28$	$v_\zeta = 1.23$
$v_k = v_a = 0.13$	$v_t = 2.33$	
	$v_u = v_v = 1.85$	
	$v_w = 0.63$	

The required pressures are:

$p_a = p_b = 150$	$p_k = 165$	$p_w = p_z = 25.6$
$p_c = 112$	$p_n = 60.0$	$p_x = 52.3$
$p_d = p_o = 76.5$	$p_q = 50.5$	$p_\alpha = 22.1$
$p_e = p_p = 67.5$	$p_r = p_\beta = 28.3$	$p_\delta = 18.5$
$p_f = 67.5$	$p_s = p_\gamma = 25.3$	$p_\epsilon = p_\zeta = 5$
$p_g = 76.9$	$p_t = 25.1$	$p_\eta = 17.6$
$p_h = p_l = 73.5$	$p_u = 29.0$	
$p_i = p_m = 69.3$	$p_v = p_y = 28.2$	

Diagrams with the volumes and pressures corresponding to this example are plotted in Fig. 46 with convenient vertical scales. Actual indicator-diagrams taken from the engine are given by Fig. 47. The events of the stroke come at slightly different piston positions on account of the irregularity due to the connecting-rod, and the drops and other fluctuations of pressure are gradual instead of sudden; moreover, there is considerable loss of pressure from the boiler to the engine, from one cylinder to another, and from the low-pressure cylinder to the condenser. Nevertheless the general forms of the diagrams are easily recognized, and all apparent erratic variations are accounted for.

Designing Compound Engines. — The designer of compound and multiple-expansion engines should have at hand a well-systematized fund of information concerning the sizes, proportions, and powers of such engines, together with actual indicator-diagrams. He may then, by a more or less direct method of interpolation or exterpolation, assign the dimensions and proportions to a new design, and can, if deemed advisable, draw or determine a set of probable indicator-diagrams for the new engines. If the new design differs much from the engines for which he has information, he may determine approximate diagrams both for an actual engine from which indicator-diagrams are at hand, and for the new design. He may then sketch diagrams for the new engine, using the approximate

diagrams as a basis and taking a comparison of the approximate and actual diagrams from the engine already built, as a guide. It is convenient to prepare and use a table showing the ratios of actual mean effective pressures and approximate mean effective pressures. Such a table enables the designer to assign mean effective pressures to a cylinder of the new engine and to infer very closely what its horse-power will be. It is also very useful as a check in sketching probable diagrams for a new engine, which diagrams are not only useful in estimating the power of the new engine, but in calculating stresses on the members of that engine.

A rough approximation of the power of an engine may be made by calculating the power as though the total or equivalent expansion took place in the low-pressure cylinder, neglecting clearance and compression. The power thus found is to be affected by a factor which according to the size and type of the engine may vary from 0.6 to 0.9 for compound engines and from 0.5 to 0.8 for triple engines. Seaton and Rounthwaite * give the following table of multipliers for compound marine engines:

MULTIPLIERS FOR FINDING PROBABLE M.E.P. COMPOUND AND TRIPLE MARINE ENGINES.

Description of Engine.	Jacketed.	Unjacketed.
Receiver-compound, screw-engines	0.67 to 0.73	0.58 to 0.68
Receiver-compound, paddle-engines	.....	0.55 to 0.65
Direct expansion	.....	0.71 to 0.75
Three-cylinder triple, merchant ships	0.64 to 0.68	0.60 to 0.66
Three-cylinder triple, naval vessels	0.55 to 0.65	.....
Three-cylinder triple, gunboats and torpedo-boats	.....	0.60 to 0.67

For example, let the boiler-pressure be 80 pounds by the gauge, or 94.7 pounds absolute; let the back-pressure be 4 pounds absolute; and let the total number of expansions be six, so that the volume of steam exhausted to the condenser is six times the

* *Pocket Book of Marine Engineering.*

volume admitted from the boiler. Neglecting the effect of clearance and compression, the mean effective pressure is

$$94.7 \times \tfrac{1}{6} + 94.7 \times \tfrac{1}{6} \log_e \tfrac{6}{1} - 4 \times 1 = 40.06 = \text{M.E.P.}$$

If the large cylinder is 30 inches in diameter, and the stroke is 4 feet, the horse-power at 60 revolutions per minute is

$$\frac{\pi\, 30^2}{4} \times 40.06 \times 2 \times 4 \times 60 \div 33000 = 412 \text{ H.P.}$$

If the factor to be used in this case is 0.75, then the actual horse-power will be about

$$0.75 \times 400 = 300 \text{ H.P.}$$

Binary Engines. — Another form of compound engines using two fluids like steam and ether, was proposed by du Trembly* in 1850, to extend the lower range of temperature of vapor-engines. At that time the common steam-pressure was not far from thirty pounds by the gauge, corresponding to a temperature of 250° F. If the back-pressure of the engine be assumed to be 1.5 pounds absolute (115° F.), the efficiency for Carnot's cycle would be approximately

$$\frac{250 - 115}{250 + 460} = 0.19.$$

If, however, by the use of a more volatile fluid the result at lower temperature could be reduced to 65° F., the efficiency could be increased to

$$\frac{250 - 65}{250 + 460} = 0.26.$$

At the present time when higher steam-pressures are common, the comparison is less favorable. Thus the temperature of steam at 150 pounds by the gauge is 365° F., so that with 1.5

**Manuel du Conducteur des Machines à Vaporous combinées au Machines Binaires*, also *Rankine Steam Engine*, p. 444.

pounds absolute (or 115° F.) for the back-pressure the efficiency for Carnot's cycle is

$$\frac{365 - 115}{365 + 460} = 0.30,$$

and for a resultant temperature of 65° F., the efficiency would be

$$\frac{365 - 65}{365 + 460} = 0.36.$$

If a like gain of economy could be obtained in practice, it would represent a saving of 17 per cent, which would be well worth while. Further discussion of this matter of economy will be given in Chapter XI, in connection with experiments on binary engines using steam and sulphur-dioxide.

The practical arrangement of a binary engine substitutes for the condenser an appliance having somewhat the same form as a tubular surface-condenser, the steam being condensed on the outside of the tubes and withdrawn in the form of water of condensation at the bottom. Through the tubes is forced the more volatile fluid, which is vaporized much as it would be in a "water-tube" boiler. The vapor is then used in a cylinder differing in no essential from that for a steam-engine, and in turn is condensed in a surface-condenser which is cooled with water at the lowest possible temperature.

An ideal arrangement for a binary engine avoiding the use of air-pumps would appear to be the combination of a compound non-condensing steam-engine with a third cylinder on the binary system which should have for its working substance a fluid that would give a convenient pressure at 212° F., and a pressure somewhat above the atmosphere at 60° F. There is no known fluid that gives both these conditions; thus ether at 212° F. gives a pressure of about 96 pounds absolute, but its boiling-point at atmospheric pressure is 94° F., consequently it would from necessity require a vacuum and an air-pump unless the ether could be entirely freed from air, and leakage into the vacuum space entirely prevented. Sulphur-dioxide gives a pressure of 41

pounds absolute at 60° F., so that it can always be worked at a pressure above the atmosphere; but 212° F. would give an inconvenient pressure, and in practice it has been found convenient to run the steam-engine with a vacuum of 22 inches of mercury or about 4 pounds absolute, which gives a temperature of 155° F., at which sulphur-dioxide has a pressure of 180 pounds per square inch by the gauge.

The attempt of du Trembly to use ether for the second fluid in a binary engine did not result favorably, as his fuel-consumption was not less than that of good engines of that time, which appears to indicate that he could not secure favorable conditions.

CHAPTER X.

TESTING STEAM-ENGINES.

The principal object of tests of steam-engines is to determine the cost of power. Series of engine tests are made to determine the effect of certain conditions, such as superheating and steam-jackets, on the economy of the engine. Again, tests may be made to investigate the interchanges of heat between the steam and the walls of the cylinder by the aid of Hirn's analysis, and thus find how certain conditions produce effects that are favorable or unfavorable to economy.

The two main elements of an engine test are, then, the measurement of the power developed and the determination of the cost of the power in terms of thermal units, pounds of steam, or pounds of coal. Power is most commonly measured by aid of the steam-engine indicator, but the power delivered by the engine is sometimes determined by a dynamometer or a friction brake; sometimes, when an indicator cannot be used conveniently, the dynamic or brake power only is determined. When the engine drives a dynamo-electric generator the power applied to the generator may be determined electrically, and thus the power delivered by the engine may be known.

When the cost of power is given in terms of coal per horse-power per hour, it is sufficient to weigh the coal for a definite period of time. In such case a combined boiler and engine test is made, and all the methods and precautions for a careful boiler test must be observed. The time required for such a test depends on the depth of the fire on the grate and the rate of combustion. For factory boilers the test should be twenty-four hours long if exact results are desired.

When the cost of power is stated in terms of steam per horse-power per hour, one of two methods may be followed. When

the engine has a surface-condenser the steam exhausted from the engine is condensed, collected, and weighed. One hour is usually sufficient for tests under favorable conditions; shorter intervals, ten or fifteen minutes, give fairly uniform results. The chief objection to this method is that the condenser is liable to leak water at the tube packings. Under favorable conditions the results of tests by this method and by determining the feed-water supplied to the boiler are substantially the same. In tests on non-condensing and on jet-condensing engines the steam-consumption is determined by weighing or measuring the feed-water supplied to the boiler or boilers that furnish steam to the engine. Steam used for any other purpose than running the engine, for example, for pumping, heating, or making determinations of the quality of the steam, must be determined and allowed for. The most satisfactory way is to condense and weigh such steam; but small quantities, as for determining quality by a steam calorimeter, may be gauged by allowing it to flow through an orifice. Tests which depend on measuring the feed-water should be long enough to minimize the effect of the uncertainty of the amount of water in a boiler corresponding to an apparent height of water in a water-gauge; for the apparent height of the water-level depends largely on the rate of vaporization and the activity of convection currents.

When the cost of power is expressed in thermal units it is necessary to measure the steam-pressure, the amount of moisture in the steam supplied to the cylinder, and the temperature of the condensed steam when it leaves the condenser. If steam is used in jackets or reheaters it must be accounted for separately. But it is customary in all engine tests to take pressures and temperatures, so that the cost may usually be calculated in thermal units, even when the experimenter is content to state it in pounds of steam.

For a Hirn's analysis it is necessary to weigh or measure the condensing water, and to determine the temperatures at admission to and exit from the condenser.

Important engines, with their boilers and other appurtenances,

are commonly built under contract to give a certain economy, and the fulfilment of the terms of a contract is determined by a test of the engine or of the whole plant. The test of the entire plant has the advantage that it gives, as one result, the cost of power directly in coal; but as the engine is often watched with more care during a test than in regular service, and as auxiliary duties, such as heating and banking fires, are frequently omitted from such an economy test, the actual cost of power can be more justly obtained from a record of the engine in regular service, extending for weeks or months. The tests of engine and boilers, though made at the same time, need not start and stop at the same time, though there is a satisfaction in making them strictly simultaneous. This requires that the tests shall be long enough to avoid drawing the fires at beginning and end of the boiler test.

In anticipation of a test on an engine it must be run for some time under the conditions of the test, to eliminate the effects of starting or of changes. Thus engines with steam-jackets are commonly started with steam in the jackets, even if they are to run with steam excluded from the jackets. An hour or more must then be allowed before the effect of using steam in the jackets will entirely pass away. An engine without steam-jackets, or with steam in the jackets, may come to constant conditions in a fraction of that time, but it is usually well to allow at least an hour before starting the test.

It is of the first importance that all the conditions of a test shall remain constant throughout the test. Changes of steam-pressure are particularly bad, for when the steam-pressure rises the temperature of the engine and all parts affected by the steam must be increased, and the heat required for this purpose is charged against the performance of the engine; if the steam-pressure falls a contrary effect will be felt. In a series of tests one element at a time should be changed, so that the effect of that element may not be confused by other changes, even though such changes have a relatively small effect. It is, however, of more importance that steam-pressure should remain constant

than that all tests at a given pressure should have identically the same steam-pressure, because the total heat of steam varies more slowly than the temperature.

All the instruments and apparatus used for an engine test should be tested and standardized either just before or just after the test; preferably before, to avoid annoyance when any instrument fails during the test and is replaced by another.

Thermometers. — Temperatures are commonly measured by aid of mercurial thermometers, of which three grades may be distinguished. For work resembling that done by the physicist the highest grade should be used, and these must ordinarily be calibrated, and have their boiling- and freezing-points determined by the experimenter or some qualified person; since the freezing-point is liable to change, it should be redetermined when necessary. For important data good thermometers must be used, such as are sold by reliable dealers, but it is preferable that they should be calibrated or else compared with a thermometer that is known to be reliable. For secondary data or for those requiring little accuracy common thermometers with the graduation on the stem may be used, but these also should have their errors determined and allowed for. Thermometers with detachable scales should be used only for crude work.

Gauges. — Pressures are commonly measured by Bourdon gauges, and if recently compared with a correct mercury column these are sufficient for engineering work. The columns used by gauge-makers are sometimes subject to minor errors, and are not usually corrected for temperature. It is important that such gauges should be frequently retested.

Dynamometers. — The standard for measurement of power is the friction-brake. For smooth continuous running it is essential that the brake and its band shall be cooled by a stream of water that does not come in contact with the rubbing surfaces. Sometimes the wheel is cooled by a stream of water circulating through it, sometimes the band is so cooled, or both may be. A rubbing surface which is not cooled should be of non-conducting material. If both rubbing surfaces are metallic they

must be freely lubricated with oil. An iron wheel running in a band furnished with blocks of wood requires little or no lubrication.

To avoid the increase of friction on the brake-bearings due to the load applied at a single brake-arm, two equal arms may be used with two equal and opposite forces applied at the ends to form a statical couple.

With care and good workmanship a friction-brake may be made an instrument of precision sufficient for physical investigations, but with ordinary care and workmanship it will give results of sufficient accuracy for engineering work.

An engine which drives an electric-generator may readily have the dynamic or brake-power determined from the electric output, provided that the efficiency of the generator is properly determined.

The only power that can be measured for a steam-turbine is the dynamic or brake-power; when connected with an electric-generator this involves no difficulty. For marine propulsion it is customary to determine the power of steam-turbines by some form of torsion-metre applied to the shaft that connects the turbine to the propeller. This instrument measures the angle of torsion of the shaft while running, and consequently, if the modulus of elasticity has been determined, gives a positive determination of the power delivered to the propeller. Under favorable conditions a torsion-metre may have an error of not more than one per cent.

Indicators. — The most important and at the same time the least satisfactory instrument used in engine-testing is the indicator. Even when well made and in good condition it is liable to have an error which may amount to two per cent when used at moderate speeds. At high speeds, three hundred revolutions per minute and over, it is likely to have two or three times as much error. As a rule, an indicator cannot be used at more than four hundred revolutions per minute.

The mechanism for reducing the motion of the crosshead of the engine and transferring it to the paper drum of an indicator

should be correct in design and free from undue looseness. It should require only a very short cord leading to the paper drum, because all the errors due to the paper drum are proportional to the length of the cord and may be practically eliminated by making the cord short.

The weighing and recording of the steam-pressure by the indicator-piston, pencil-motion, and pencil are affected by errors which may be classified as follows:

1. Scale of the spring.
2. Design of the pencil-motion.
3. Inertia of moving parts.
4. Friction and backlash.

Good indicator-springs, when tested by direct loads out of the indicator, usually have correct and uniform scales; that is, they collapse under pressure the proper amount for each load applied. When enclosed in the cylinder of an indicator the spring is heated by conduction and radiation to the temperature of the cylinder, and that temperature is sensibly equal to the mean temperature in the engine-cylinder. But a spring is appreciably weaker at high temperatures, so that when thus enclosed in the indicator-cylinder, it gives results that are too large; the error may be two per cent or more.

Outside spring-indicators avoid this difficulty and are to be preferred for all important work. They have only one disadvantage, in that the moving parts are heavier, but this may be overcome by increasing the area of the piston from half a square inch to one square inch.

The motion of the piston of the indicator is multiplied five or six times by the pencil-motion, which is usually an approximate parallel motion. Within the proper range of motion (about two inches) the pencil draws a line which is nearly straight when the paper drum is at rest, and it gives a nearly uniform scale provided that the spring is uniform. The errors due to the geometric design of this part of the indicator are always small.

When steam is suddenly let into the indicator, as at admission to the engine-cylinder, the indicator-piston and attached parts forming the pencil-motion are set into vibration, with a natural time of vibration depending on the stiffness of the spring. A weak spring used for indicating a high-speed engine may throw the diagram into confusion, because the pencil will give a few deep undulations which make it impossible to recognize the events of the stroke of the engine, such as cut-off and release. A stiffer spring will give more rapid and less extensive undulations, which will be much less troublesome. As a rule, when the undulations do not confuse the diagram the area of the diagram is but little affected by the undulations due to the inertia of the piston and pencil-motion.

The most troublesome errors of the indicator are due to friction and backlash. The various joints at the piston and in the pencil-motion are made as tight as can be without undue friction, but there is always some looseness and some friction at those joints. There is also some friction of the piston in the cylinder and of the pencil on the paper. Errors from this source may be one or two per cent, and are liable be excessive unless the instrument is used with care and skill. A blunt pencil pressed up hard on the paper will reduce the area of the diagram five per cent or more; on the other hand, a medium pencil drawing a faint but legible line will affect the area very little. Any considerable friction of the piston of the indicator will destroy the value of the diagram.

Errors of the scale of the spring can be readily determined and investigated by loading the spring with known weights, when properly supported, out of the indicator. This method is probably sufficient for outside spring indicators. Those that have the spring inside the cylinder are tested under steam pressure, measuring the pressure either by a gauge or a mercury column. Considerable care and skill are required to get good results, especially to avoid excessive friction of the piston as it remains at rest or moves slowly in the cylinder. It must be borne in mind that the indicator cylinder heats readily when subjected to

progressively higher steam pressures, but that it parts with heat slowly, and that consequently tests made with falling steam pressures are not valuable.

Scales. — Weighing should be done on scales adapted to the load; overloading leads to excessive friction at the knife-edges and to lack of delicacy. Good commercial platform scales, when tested with standard weights, are sufficient for engineering work.

Coal and ashes are readily weighed in barrows, for which the tare is determined by weighing empty. Water is weighed in barrels or tanks. The water can usually be pumped in or allowed to run in under a head, so that the barrel or tank can be filled promptly. Large quick-opening valves must be used to allow the water to run out quickly. As the receptacle will seldom drain properly, it is not well to wait for it to drain, but to close the exit-valve and weigh empty with whatever small amount of water may be caught in it. Neither is it well to try to fill the receptacle to a given weight, as the jet of water running in may affect the weighing. With large enough scales and tanks the largest amounts of water used for engine tests may be readily handled.

Measuring Water. — When it is not convenient to weigh water directly, it may be measured in tanks or other receptacles of known volume. Commonly two are used, so that one may fill while the other is emptied. The volume of a receptacle may be calculated from its dimensions, or may be determined by weighing in water enough to fill it. When desired a receptacle may be provided with a scale showing the depth of the water, and so partial volumes can be determined. A closed receptacle may be used to measure hot water or other fluids.

Water-Meters of good make may be used for measuring water when other methods are not applicable, provided they are tested and rated under the conditions for which they are used, taking account of the amount and temperature of the water measured. Metres are most convenient for testing marine engines because water cannot be weighed at sea on account of the motion of the ship, and arrangements for measuring water in tanks are expensive and inconvenient. For such tests the metre may be placed

on a by-pass through which the feed-water from the surface-condenser can be made to pass by closing a valve on the direct line of feed-pipe. If necessary the metre can be quickly shut off and the direct line can be opened. The chief uncertainty in the use of a metre is due to air in the water; to avoid error from this source, the metre should be frequently vented to allow air to escape without being recorded by the metre.

Weirs and Orifices. — So far as possible the use of weirs and orifices for water during engine tests should be avoided, for, in addition to the uncertainties unavoidably connected with such hydraulic measurements, difficulties are liable to arise from the temperature of the water and from the oil in it. A very little oil is enough to sensibly affect the coefficient for a weir or orifice. The water flowing from the hot-well of a jet-condensing engine is so large in amount that it is usually deemed advisable to measure it on a weir; the effect of temperature and oil is less than when the same method is used for measuring condensed steam from a surface-condenser.

Priming-Gauges. — When superheated steam is supplied to an engine it is sufficient to take the temperature of the steam in the steam-pipe near the engine. When moist steam is used the amount of moisture must be determined by a separated test. Originally such tests were made by some form of calorimeter, and that name is now commonly attached to certain devices which are not properly heat-measurers. Three of these will be mentioned: (1) the throttling-calorimeter, which can usually be applied to all engine tests; (2) the separating-calorimeter, which can be applied when the steam is wet; and (3) the Thomas electric calorimeter, intended for use with steam-turbines to determine the moisture in steam at any stage of the turbine whatever may be the pressure or quality of the steam.

Throttling-Calorimeter. — A simple form of calorimeter, devised by the author, is shown by Fig. 48, where *A* is a reservoir about 4 inches in diameter and about 12 inches long to which steam is admitted through a half-inch pipe *b*, with a throttle-valve near the reservoir. Steam flows away through an

inch pipe *d*. At *f* is a gauge for measuring the pressure, and at *e* there is a deep cup for a thermometer to measure the temperature. The boiler-pressure may be taken from a gauge on the main steam-pipe near the calorimeter. It should not be taken from a pipe in which there is a rapid flow of steam as in the pipe *b*, since the velocity of the steam will affect the gauge-reading, making it less than the real pressure. The reservoir is wrapped with hair-felt and lagged with wood to reduce radiation of heat.

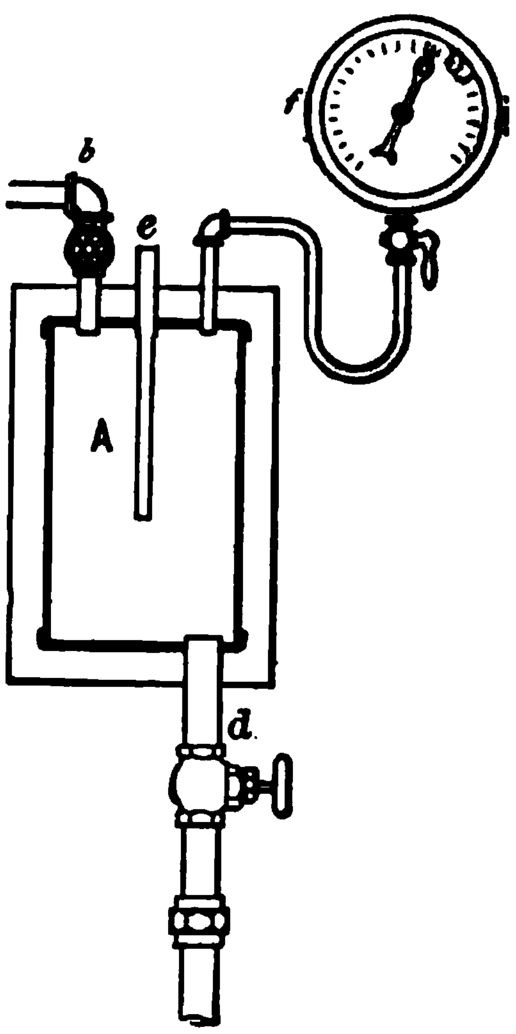

FIG. 48.

When a test is to be made, the valve on the pipe *d* is opened wide (this valve is frequently omitted), and the valve at *b* is opened wide enough to give a pressure of five to fifteen pounds in the reservoir. Readings are then taken of the boiler-gauge, of the gauge at *f*, and of the thermometer at *e*. It is well to wait about ten minutes after the instrument is started before taking readings so that it may be well heated. Let the boiler-pressure be p, and let r and q be the latent heat and heat of the liquid corresponding. Let p_1 be the pressure in the calorimeter, r_1 the heat of vaporization, q_1 the heat of the liquid, and t_1 the temperature of saturated steam at that pressure, while t_s is the temperature of the superheated steam in the calorimeter. Then

$$xr + q = r_1 + q_1 + c_p\,(t_s - t_1);$$

$$\therefore \quad x = \frac{r_1 + q_1 + c_p\,(t_s - t_1) - q}{r} \quad . \quad . \quad . \quad (152)$$

Example. — The following are the data of a test made with this calorimeter:

Pressure of the atmosphere	14.8 pounds;
Steam-pressure by gauge	69.8 "
Pressure in the calorimeter, gauge .	12.0 "
Temperature in the calorimeter . .	268°.2 F.

Specific heat of superheated steam for the condition of the test 0.46

$$x = \frac{948.8 + 212.6 + 0.46\,(268.2) - 243.9) - 286.2}{896.8} = 0.988;$$

Per cent of priming, 1.2.

A little consideration shows that this type of calorimeter can be used only when the priming is not excessive; otherwise the throttling will fail to superheat the steam, and in such case nothing can be told about the condition of the steam either before or after throttling. To find this limit for any pressure t_2 may be made equal to t_1 in equation (152); that is, we may assume that the steam is just dry and saturated at that limit in the calorimeter. Ordinarily the lowest convenient pressure in the calorimeter is the pressure of the atmosphere, or 14.7 pounds to the square inch. The table following has been calculated for several pressures in the manner indicated. It shows that the limit is higher for higher pressures, but that the calorimeter can be applied only where the priming is moderate.

When this calorimeter is used to test steam supplied to a condensing-engine the limit may be extended by connecting the exhaust to the condenser. For example, the limit at 100 pounds absolute, with 3 pounds absolute in the calorimeter, is 0.064 instead of 0.040 with atmospheric pressure in the calorimeter.

LIMITS OF THE THROTTLING-CALORIMETER.

Pressure.		Priming.
Absolute.	Gauge.	
300	285.3	0.077
250	235.3	0.070
200	185.3	0.061
175	160.3	0.058
150	135.3	0.052
125	110.3	0.046
100	85.3	0.040
75	60.3	0.032
50	35.3	0.023

In case the calorimeter is used near its limit — that is, when the superheating is a few degrees only — it is essential that the thermometer should be entirely reliable; otherwise it might happen that the thermometer should show superheating when the steam in the calorimeter was saturated or moist. In any other case a considerable error in the temperature will produce an inconsiderable effect on the result. Thus at 100 pounds absolute with atmospheric pressure in the calorimeter, 10° F. of superheating indicates 0.035 priming, and 15° F. indicates 0.032 priming. So also a slight error in the gauge-reading has little effect. Suppose the reading to be apparently 100.5 pounds absolute instead of 100, then with 10° of superheating the priming appears to be 0.033 instead of 0.032.

It has been found by experiment that no allowance need be made for radiation from this calorimeter if made as described, provided that 200 pounds of steam are run through it per hour. Now this quantity will flow through an orifice one-fourth of an inch in diameter under the pressure of 70 pounds by the gauge, so that if the throttle-valve be replaced by such an orifice the question of radiation need not be considered. In such case a stop-valve will be placed on the pipe to shut off the calorimeter when not in use; it is opened wide when a test is made. If an orifice is not provided the throttle-valve may be opened at first a small amount, and the temperature in the calorimeter noted; after a few minutes the valve may be opened a trifle more, whereupon the temperature may rise, if too little steam was used at first. If the valve is opened little by little till the temperature stops rising, it will then be certain that enough steam is used to reduce the error from radiation to a very small amount.

Separating-Calorimeter. — If steam contains more than three per cent of moisture the priming may be determined by a good separator which will remove nearly all the moisture. It remains to measure the steam and water separately. The water may be best measured in a calibrated vessel or receiver, while the steam may be condensed and weighed, or may be gauged by allowing it to flow through an orifice of known size.

A form of separating-calorimeter devised by Professor Carpenter * is shown by Fig. 49.

Steam enters a space at the top which has sides of wire gauze and a convex cup at the bottom. The water is thrown against the cup and finds its way through the gauze into an inside chamber or receiver and rises in a water-glass outside. The receiver is calibrated by trial, so that the amount of water may be read directly from a graduated scale. The steam meanwhile passes into the outer chamber which surrounds the inner receiver and escapes from an orifice at the bottom. The steam may be determined by condensing, collecting, and weighing it; or it may be calculated from the pressure and the size of the orifice. When the steam is weighed there is no radiation error, since the inner chamber is protected by the steam in the outer chamber. This instrument may be guarded against radiation by wrapping and lagging, and then if steam enough is used the radiation will be insignificant, just as was found to be the case for the throttling-calorimeter.

Fig. 49.

The Thomas Electric Calorimeter.—The essential feature of this instrument (Fig. 50) is the drying and superheating of the steam by a measured amount of electric energy. Steam is admitted at *B* and passes through numerous holes in a block of soapstone which occupies the middle of the instrument; these holes are partially filled with coils of German silver wire which are heated by an electric current that enters and leaves at the binding-screws. The steam emerges dry or superheated at the upper part of the chamber and passes down through wire gauze, which surrounds the central escape pipe; this central pipe surrounds

* *Trans. Am. Soc. Mech. Engs.*, vol. xvii, p. 608.

the thermometer cup, and leads to the exit at the top, which has two orifices, either of which may be piped to a condenser or elsewhere.

To use the instrument it is properly connected by a sampling-tube to the space from which steam is drawn, and valves are adjusted to supply a convenient amount of steam which is assumed to be uniform for steady pressure; this last is a matter of some importance.

Fig. 30.

The current of electricity is then adjusted to dry the steam; this may be determined by noting the temperature by the thermometer in the central thermometer cup, because that thermometer will show a slight rise corresponding to a very small degree of superheating which is sufficient to indicate the disappearance of moisture, but not enough to affect the determination of quality by the instrument. The wire gauze surrounding the thermometer is an essential feature of this operation, as it insures the homogeneity of the steam, which, without the gauze, would be likely to be a mixture of superheated steam and moist steam. Readings are taken of the proper electrical instruments from which the electrical energy imparted can be determined in watts; let this energy required to dry the steam be denoted by E_1. Now let the electric current be increased till the steam is superheated 30°, and let E_2 be the increase of electric input which is required to superheat the steam.

If W is the weight of steam flowing per hour through the

instrument under the first conditions, the weight when superheated will be CW, where C is a factor less than unity which has been determined by exhaustive tests on the instrument. The amount of electric energy required to superheat one pound of steam 30° from saturation at various pressures has also been determined and may be represented by S; this constant has been so determined as to include an allowance for radiation, and is more convenient than the specific heat of superheated steam, in this place. Making use of the factors C and S, we may write

$$E_2 = CSW, \text{ or } W = \frac{E_2}{CS},$$

which affords a means of eliminating the weight of steam used; this is an important feature in the use of the instrument.

Returning now to the first condition of the instrument when steam is dried by the application of E_1 watts of electric energy, we have for the equivalent heat

$$3.42\ E_1;$$

and dividing by the expression for the weight of steam flowing per hour, we have for the heat required to dry one pound of steam

$$\frac{3.42\ E_1}{W} = 3.42\ CS\ \frac{E_1}{E_2} = (1 - x)r,$$

where r is the heat of vaporization and $1 - x$ is the amount of water in one pound of moist steam.

Solving the above equation for x, we have

$$x = 1 - \frac{3.42\ CS}{r}\frac{E_1}{E_2}.$$

If desired, the constant factors may be united into one term, and the equation may be written

$$x = 1 - \frac{K}{r}\frac{E_1}{E_2}.$$

With each instrument is furnished a diagram giving values of K for all pressures, so that the use of the instrument involves

only two readings of a wattmeter and the application of the above simple equation.

For example, suppose that the use of the instrument in a particular case gave the values $E_1 = 240$, and $E_2 = 93.0$ for the absolute pressure 100 pounds per square inch. The value of K from the diagram is 54.2, and r from the steam-tables is 884, consequently

$$x = 1 - \frac{54.2}{884} \frac{240}{93.0} = 0.84.$$

Method of Sampling Steam. — It is customary to take a sample of steam for a calorimeter or priming-gauge through a small pipe leading from the main steam-pipe. The best method of securing a sample is an open question; indeed, it is a question whether we ever get a fair sample. There is no question but that the composition of the sample is correctly shown by any of the calorimeters described, when the observer makes tests with proper care and skill. It is probable that the best way is to take steam through a pipe which reaches at least halfway across the main steam-pipe, and which is closed at the end and drilled full of small holes. It is better to have the sampling-pipe at the side or top of the main, and it is better to take a sample from a pipe through which steam flows vertically upward. The sampling-pipe should be short and well wrapped to avoid radiation.

CHAPTER XI.

INFLUENCE OF THE CYLINDER WALLS.

IN this chapter a discussion will be given of the discrepancy between the theory of the steam-engine as detailed in the previous chapter, and the actual performance as determined by tests on engines. It was early evident that this discrepancy was due to the interference of the metal of the cylinder walls which abstracted heat from the steam at high pressure and gave it out at low pressure. In consequence there followed a long struggle to determine precisely what action the walls exerted and how to allow for that action in the design of new engines. The first part has been accomplished; we can determine to a nicety the influence of the cylinder walls for any engine already built and tested; but as yet all attempts to systematize the information derived from such tests in such a manner that it can be used in the design of new engines has been utterly futile. Consequently the discussion in this chapter is important mainly in that it allows us to understand the real action of certain devices that are intended to improve the economy of engines, and to form a just opinion on the probability of future improvements.

As soon as the investigations by Clausius and Rankine and the experiments by Regnault made a precise theory of the steam engine possible, it became evident that engines used from quarter to half again as much steam as the adiabatic theory indicated, and in particular that expansion down to the back-pressure was inadvisable. An early and a satisfactory exposition of these points was made by Isherwood after his tests on the U. S. S. *Michigan*, which are given in Table III.

TABLE III.

TESTS ON THE ENGINE OF THE U. S. S. *MICHIGAN*.

CYLINDER DIAMETER, 36 INCHES; STROKE, 8 FEET.

By Chief-Engineer ISHERWOOD, *Researches in Experimental Steam Engineering*.

	I.	II.	III.	IV.	V.	VI.	VII.
Duration, hours	72	72	72	72	72	72	72
Cut-off	11/12	7/10	4/9	3/10	1/4	1/6	4/45
Revolutions per minute	20.6	15.6	17.3	13.7	13.9	11.2	14.1
Boiler-pressure, pounds per sq in. above atmosphere	21.0	19.5	21.0	21.0	21.0	21.0	22.0
Barometer, inches of mercury	30.1	29.8	29.7	30.1	29.9	29.9	29.9
Vacuum, inches of mercury	26.5	26.1	26.3	25.8	25.8	25.6	24.1
Steam per horse-power per hour, pounds	38.0	33.8	32.7	34.7	34.5	36.8	41.4
Per cent of water in cylinder at release	10.7	15.3	27.2	41.7	39.6	42.1	45.1

In the first place the best economy for this engine was 32.7 pounds instead of 26.5 pounds as calculated by the expression

$$\frac{60 \times 33000}{778\,(r_1 + q_1 - x_2 r_2 - q_2)}$$

deduced on page 141 for the steam-consumption for a non-conducting engine with complete expansion. This result was obtained with cut-off at four-ninths of the stroke which gave a terminal pressure of one pound above the atmosphere.

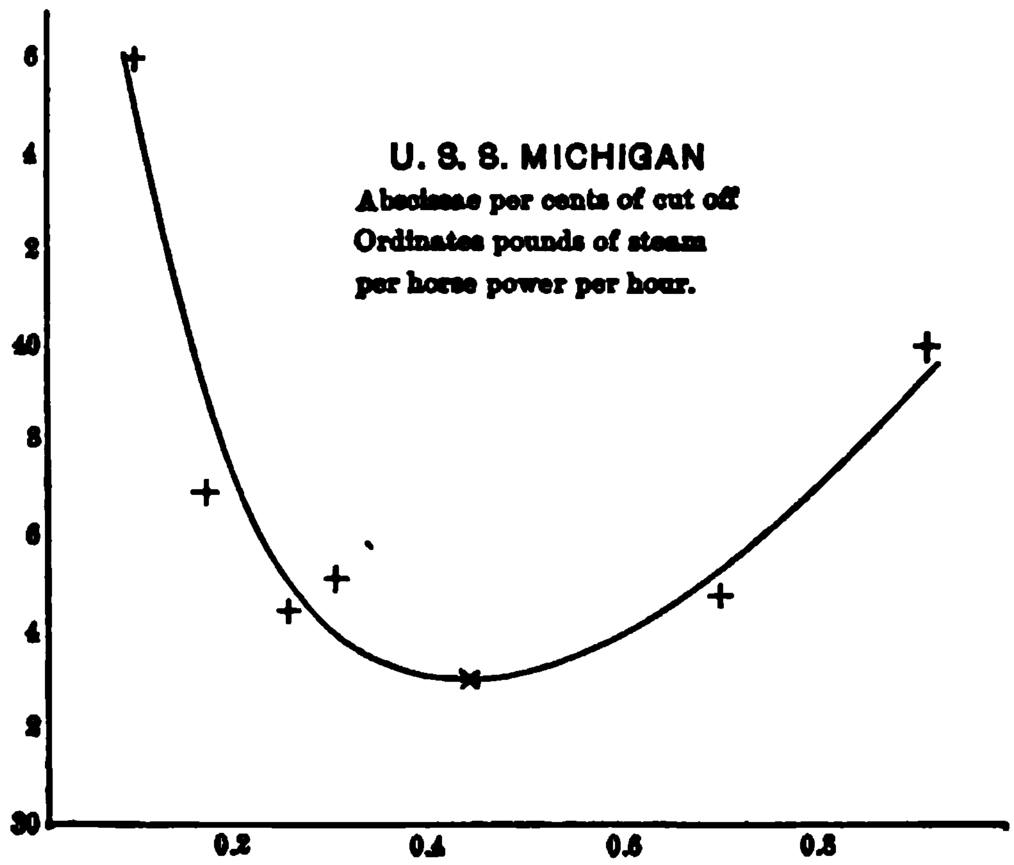

FIG. 51.

The manner of the variation of the steam consumption with the cut-off is clearly shown by Fig. 51, in which the fraction of stroke at cut-off is taken for abscissæ and the steam-consumptions as ordinates.

To make the diagram clear and compact, the axis of abscissæ is taken at 30 pounds of steam per horse-power per hour. An inspection of this diagram and of the figures in the table shows a regularity in the results which can be attained only when tests are made with care and skill. The only condition purposely varied is the cut-off; the only condition showing important accidental variation is the vacuum, and consequently the back-pressure in the cylinder. To allow for the small variations in the back-pressure Isherwood changed the mean effective pressure for each test by adding or subtracting, as the case might require, the difference between the actual back-pressure and the mean back-pressure of 2.7 pounds per square inch, as deduced from all the tests.

An inspection of any such a series of tests having a wide range of expansions will show that the steam-consumption decreases as the cut-off is shortened till a minimum is reached, usually at $\frac{1}{3}$ to $\frac{1}{6}$ stroke; any further shortening of the cut-off will be accompanied by an increased steam-consumption, which may become excessive if the cut-off is made very short. Some insight into the reason for this may be had from the per cent of water in the cylinder, calculated from the dimensions of the cylinder and the pressures in the cylinder taken from the indicator-diagram. The method of the calculation will be given in detail a little later in connection with Hirn's analysis. It will be sufficient now to notice that the amount of water in the cylinder of the engine of the *Michigan* at release increased from 10.7 per cent for a cut-off at $\frac{11}{12}$ of the stroke to 45.1 per cent for a cut-off at $\frac{4}{45}$ of the stroke. Now all the water in the cylinder at release is vaporized during the exhaust, the heat for this purpose being abstracted from the cylinder walls, and the heat thus abstracted is wasted, without any compensation. The walls may be warmed to some extent in consequence of the rise of pressure and temperature during compression, but by far the greater part of the heat abstracted during exhaust must be supplied by the incoming steam at admission. There is, therefore, a large condensation of steam during admission and up to cut-off, and the greater part

of the steam thus condensed remains in the form of water and does little if anything toward producing work. This may be seen by inspection of the table of results of Dixwell's tests on page 270. With saturated steam and with cut-off at 0.217 of the stroke, 52.2 per cent of the working substance in the cylinder was water. Of this 19.8 per cent was reëvaporated during expansion, and 32.4 per cent remained at release to be reëvaporated during exhaust. When the cut-off was lengthened to 0.689 of the stroke, there was 27.9 per cent of water at cut-off and 23.9 per cent at release. The statement in percentages gives a correct idea of the preponderating influence of the cylinder walls when the cut-off is unduly shortened; it is, however, not true that there is more condensation with a short than with a long cut-off. On the contrary, there is more water condensed in the cylinder when the cut-off is long, only the condensation does not increase as fast as do the weight of steam supplied to the cylinder and the work done, and consequently the condensation has a less effect.

Graphical Representation. — The divergence of the actual expansion line from the adiabatic line can be shown in a striking manner by plotting the former on the temperature-entropy diagram as shown in Fig. 53 which is constructed from the indicator-diagram in Fig. 52, shown with the axes of zero pressure and zero volume drawn in the usual manner, allowing for clearance and for the pressure of the atmosphere.

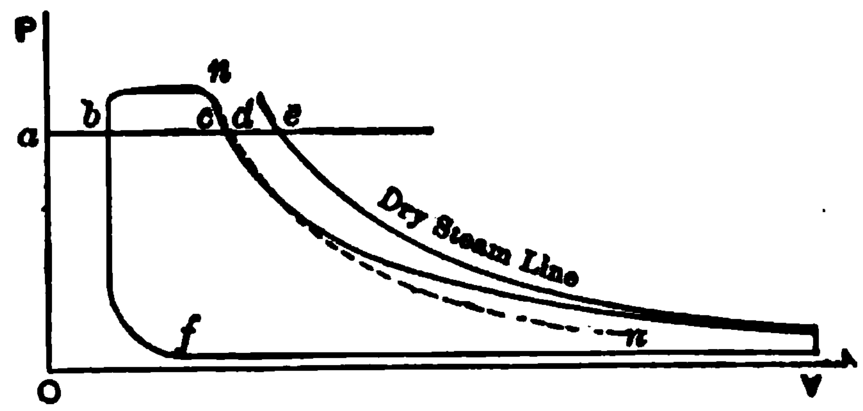

FIG. 52.

In order to undertake this construction the weight of *steam per stroke* W as determined from the test of the engine during which the diagrams were taken, must be determined, and the weight of steam W_0 caught in the clearance must be computed from the pressure and volume f, the beginning of compression.

The dry steam line (Fig. 52) is drawn by the following process:

a line *ae* is drawn at a convenient pressure, and on it is laid off the volume of $W + W_0$ pounds of dry steam as determined from the steam-table to the proper scale of the drawing. Thus if s_e is the specific volume of the steam at the pressure p_e the volume of steam present if dry and saturated would be

$$(W + W_0)\, s_e.$$

But the length of the diagram L, in inches is proportional to the piston displacement D in cubic feet. The latter is obtained by multiplying the area of the piston in square feet by its stroke in feet. For the crank end the net area of the piston is to be used, allowing for the piston-rod. Consequently the proper abscissa, representing the volume is obtained by multiplying by $\frac{L}{D}$, giving

$$s\,\frac{(W + W_0)\, L}{D};$$

and of this all except s is a constant for which a numerical result can be found.

The diagram shown by Fig. 52 was taken from the head end of the high-pressure cylinder of an experimental engine in the laboratory of the Massachusetts Institute of Technology. The value of $W + W_0$ was found to be 0.075 of a pound; the piston displacement was 1.102 cubic feet, and the length of the diagram was 3.69 inches; consequently

$$\frac{(W + W_0)\, L}{D} = 0.251.$$

The line *ae* was drawn at 90 pounds absolute at which $s = 4.86$ cubic feet; the length of the line *ae* was consequently

$$0.251 \times 4.86 = 1.22 \text{ inch.}$$

Neglecting the volume of the water present, the volume of steam actually present bore the same ratio to the volume of the steam when saturated, that *ac* had to *ae*. This gave in the figure at *c*

$$x_e = \frac{ac}{ae} = \frac{0.94}{1.219} = 0.771.$$

To plot the point e on the temperature-entropy diagram, Fig. 53, we may find the temperature at 90 pounds absolute, namely, 320° F., and on a line with that temperature as an ordinate we may interpolate between the lines for constant values of x. Other points can be drawn in a like manner, and the curve cg can be sketched in; showing that the steam continues to yield heat to the cylinder walls from cut-off till c is reached on Fig. 52, and perhaps a trifle longer. Beyond c the steam receives heat from the walls until exhaust opens.

Fig. 53.

The same feature is exhibited in Fig. 52, by drawing the adiabatic line xdn from the point of cut-off. The point d can be located by multiplying the length ae, which represents the volume of steam in the cylinder when dry by the value of x after adiabatic expansion from the point of cut-off n. This point n is readily included in the preceding investigation, so that x_n can be determined. Locating n on the temperature-entropy diagram, Fig. 53, we may draw through it a vertical constant entropy line and note where it cuts the lines corresponding to the pressure lines like ae in Fig. 52, and interpolate for the values of x. For example, the entropy at n in Fig. 53 appears to be 1.36, and at 320° F., which corresponds to 90 pounds, this entropy line gives by interpolation 0.78, so that the length of ad is

$$0.78 \times 1.22 = 0.95.$$

In this discussion no attempt is made to distinguish the moisture which may be in contact with the wall from the remainder of steam and water in the cylinder. In reality that moisture has furnished the heat which the cylinder walls acquire during admission, and it abstracts heat from the walls during the expan-

sion. The mixture, moreover, is not homogeneous, because the moisture on the cylinder walls is likely to be colder than the steam, though naturally it cannot be warmer.

Finally, the indicator-pencil is subject to a friction lag that operates to produce the effect shown by Figs. 52 and 53 and is liable to exaggerate them. That is to say, the pencil draws a horizontal line and tends to remain at the same height after the steam-pressure falls. It then lets go and falls sharply some little time after the valve has closed at cut-off. Afterwards it lags behind and shows a higher pressure than it should.

Hirn's Analysis. — Though the methods just illustrated give a correct idea of the influence of the walls of the cylinder of a steam-engine, our first clear insight into the action of the walls is due to Hirn,* who accompanied his exposition by quantitative results from certain engine tests. The statement of his method which will be given here is derived from a memoir by Dwelshauvers-Dery.†

Let Fig. 54 represent the cylinder of a steam-engine and the diagram of the actual cycle. For sake of simplicity the diagram is represented without lead of admission or release, but the equations to be deduced apply to engines having either or both. The points 1, 2, 3, and 0 are the points of cut-off, release, compression, and admission. The part of the cycle from 0 to 1, that is, from admission to cut-off, is represented by *a*; in like manner, *b*, *c*, and *d* represent the parts of the cycle during expansion, exhaust, and compression. The numbers will be used as subscripts to designate the properties of the working fluid under the conditions represented by the points indicated, and the letters will be used in connection with the operations taking place during the several parts of the cycle. Thus at cut-off the

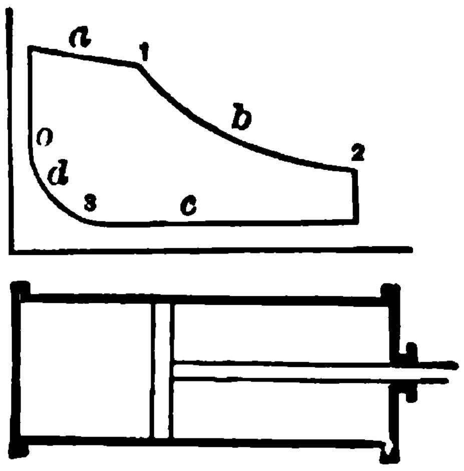

FIG. 54.

* *Bulletin de la Soc. Ind. de Mulhouse*, 1873; *Théorie Méchanique de la Chaleur*, vol. ii, 1876.

† *Revue universelle des Mines*, vol. viii, p. 362, 1880.

pressure is p_1, and the temperature, heat of the liquid, heat of vaporization, quality, etc., are represented by t_1, q_1, r_1, x_1, etc. The external work from cut-off to release is W_b, and the heat yielded by the walls of the cylinder due to reëvaporation is Q_b.

Suppose that M pounds of steam are admitted to the cylinder per stroke, having in the supply-pipe the pressure p and the condition x; that is, each pound is x part steam mingled with $1 - x$ of water. The heat brought into the cylinder per stroke, reckoned from freezing-point, is

$$Q = M\ (q + xr) \quad . \quad . \quad . \quad . \quad . \quad . \quad (153)$$

Should the steam be superheated in the supply-pipe to the temperature t_s, then

$$Q = M\ [r + q + \int c dt] \quad . \quad . \quad . \quad . \quad . \quad . \quad (154)$$

for which a numerical value can be found in the temperature-entropy table.

Let the heat-equivalent of the intrinsic energy of the entire weight of water and steam in the cylinder at any point of the cycle be represented by I; then at admission, cut-off, release, and compression we have

$$I_0 = M_0\ (q_0 + x_0\rho_0); \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (155)$$
$$I_1 = (M + M_0)\ (q_1 + x_1\rho_1); \quad . \quad . \quad . \quad . \quad . \quad . \quad (156)$$
$$I_2 = (M + M_0\ )\ (q_2 + x_2\rho_2); \quad . \quad . \quad . \quad . \quad . \quad . \quad (157)$$
$$I_3 = M_0\ (q + x_3\rho_3); \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (158)$$

in which ρ is the heat-equivalent of the internal work due to vaporization of one pound of steam, and M_0 is the weight of water and steam caught in the cylinder at compression, calculated in a manner to be described hereafter.

At admission the heat-equivalent of the fluid in the cylinder is I_0, and the heat supplied by the entering steam up to the point of cut-off is Q. Of the sum of these quantities a part, AW_a, is used in doing external work, and a part remains as intrinsic energy at cut-off. The remainder must have been absorbed by

the walls of the cylinder, and will be represented by Q_a. Hence

$$Q_a = Q + I_0 - I_1 - AW_a.$$

From cut-off to release the external work W_b is done, and at release the heat-equivalent of the intrinsic energy is I_2. Usually the walls of the cylinder, during expansion, supply heat to the steam and water in the cylinder. To be more explicit, some of the water condensed on the cylinder walls during admission and up to cut-off is evaporated during expansion. This action is so energetic that I_2 is commonly larger than I_1. Since heat absorbed by the walls is given a positive sign, the contrary sign should be given to heat yielded by them; it is, however, convenient to give a positive sign to all the interchanges of heat in the equations, and then in numerical problems a negative sign will indicate that heat is yielded during the operation under consideration. For expansion, then,

$$Q_b = I_1 - I_2 - AW_b.$$

During the exhaust the external work W_c is done by the engine on the steam, the water resulting from the condensation of the steam in the condenser carries away the heat Mq_4, the cooling water carries away the heat $G\ (q_k - q_i)$, and there remains at compression the heat-equivalent of intrinsic energy I_3. So that

$$Q_c = I_2 - I_3 - Mq_4 - G\ (q_k - q_i) + AW_c,$$

in which q_4 is the heat of the liquid of the condensed steam, and G is the weight of cooling water per stroke which has on entering the heat of the liquid q_i, and on leaving the heat of the liquid q_k.

During compression the external work W_d is done by the engine on the fluid in the cylinder, and at the end of compression, i.e., at admission, the heat-equivalent of the intrinsic energy is I_0. Hence

$$Q_d = I_3 - I_0 + AW_d.$$

It should be noted (Fig. 54) that the work W_a is represented

by the area which is bounded by the steam line, the ordinates through 0 and 1 and by the base line. And in like manner the works W_b, W_c, and W_d are represented by areas which extend to the base line. In working up the analysis from a test the line of absolute zero of pressure may be drawn under the atmospheric line as in Fig. 55, or proper allowance may be made after the calculation has been made with reference to the atmospheric line.

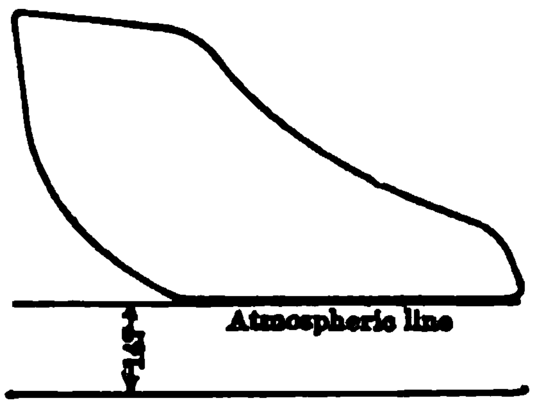

FIG. 55.

For convenience these four equations will be assembled as follows:

$$Q_a = Q + I_0 - I_1 - AW_a \quad \ldots \ldots \quad (159)$$

$$Q_b = I_1 - I_2 - AW \quad \ldots \ldots \quad (160)$$

$$Q_c = I_2 - I_3 - Mq_4 - G\,(q_k - q_i) + AW_c \quad . \quad (161)$$

$$Q_d = I_3 - I_0 + AW_d \quad \ldots \ldots \quad (162)$$

A consideration of these equations shows that all the quantities of the right-hand members can be obtained directly from the proper observations of an engine test except the several values of I, the heat-equivalents of the intrinsic energies in the cylinder. These quantities are represented by equations (155) to (158), in which there are five unknown quantities, namely, x_0, x_1, x_2, x_3, and M_0.

Let the volume of the clearance-space between the valve and the piston when it is at the end of its stroke be V_0; and let the volumes developed by the piston up to cut-off and release be V_1 and V_2; finally, let V_3 represent the corresponding volume at compression. The specific volume of one pound of mixed water and steam is

$$v = xu + \sigma,$$

and the volume of M pounds is

$$V = Mv = M\,(xu + \sigma).$$

At the points of admission, cut-off, release, and compression,

$$V_0 = M_0 (x_0 u_0 + \sigma) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (163)$$

$$V_0 + V_1 = (M + M_0)(x_1 u_1 + \sigma) \quad . \quad . \quad . \quad . \quad . \quad (164)$$

$$V_0 + V_2 = (M + M_0)(x_2 u_2 + \sigma) . \quad . \quad . \quad . \quad . \quad . \quad (165)$$

$$V_0 + V_3 = M_0 (x_3 u_3 + \sigma) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (166)$$

There is sufficient evidence that the steam in the cylinder at compression is nearly if not quite dry, and as there is comparatively little steam present at that time, there cannot be much error in assuming

$$x_3 = 1.$$

This assumption gives, by equation (166),

$$M_0 = \frac{V_0 + V_3}{u_3 + \sigma} = \frac{V_0 + V_3}{s_3} = (V_0 + V_3)\,\gamma_3 \quad . \quad . \quad (167)$$

in which γ_3 is the density or weight of one cubic foot of dry steam at compression.

Applying this result to equations (263) to (265) gives

$$x_0 = \frac{V_0}{M_0 u_0} - \frac{\sigma}{u_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (168)$$

$$x_1 = \frac{V_0 + V_1}{(M + M_0)\, u_1} - \frac{\sigma}{u_1} \quad . \quad . \quad . \quad . \quad (169)$$

$$x_2 = \frac{V_0 + V_2}{(M + M_0)\, u_2} - \frac{\sigma}{u_2} \quad . \quad . \quad . \quad . \quad (170)$$

We are now in condition to find the values of I_0, I_1, I_2, and I_3, and consequently can calculate all the interchanges of heat by equations (159) to (162).

Should the value of x in any case appear to be greater than unity it indicates that the steam is superheated; this may happen for x_0, and then as the weight of steam M_0 is relatively small, and as the superheating is usually slight, it will be sufficient to make x_0 equal to unity. It is unlikely to be the case for x_1 or x_2, even though the steam is strongly superheated in the steam-pipe;

should the computation give a value slightly larger than unity the steam may be assumed to be dry without appreciable error, and the work may proceed as indicated. If in the use of very strongly superheated steam a computed value of x_2 is appreciably larger than unity, we may replace the equation (166) by

$$V_0 + V_2 = (M + M_0) v_2,$$

where v_2 is the specific volume of superheated steam; consequently

$$v_2 = \frac{V_0 + V_2}{M + M_0}.$$

By aid of the temperature-entropy table we may find (by interpolation if necessary) the corresponding temperature t_2 and the value of the heat-contents or total heat. The heat-equivalent of the intrinsic energy is then equal to this quantity minus Ap_2v_2.

In the diagram, Fig. 54, the external work during exhaust is all work done by the piston on the fluid, since the release is assumed to be at the end of the stroke. If the release occurs before the end of the stroke, some of the work, namely, from release to the end of the stroke, will be done by the steam on the piston, and the remainder, from the end of the stroke back to compression, will be done by the piston on the fluid. In such case W_c will be the difference between the second and the first quantities. If an engine has lead of admission, a similar method may be employed; but at that part of the diagram the curves of compression and admission can be distinguished with difficulty, if at all, and little error can arise from neglecting the lead.

The several pressures at admission, cut-off, release, and compression are determined by the aid of the indicator-diagram, and the pressures in the steam-pipe and exhaust-pipe or condenser are determined by gauges. The weight M of steam supplied to the cylinder per stroke is best determined by condensing the exhaust-steam in a surface-condenser and collecting and weighing it in a tank. If the engine is non-condensing, or if it has a jet-condenser, or if for any reason this method cannot

be used, then the feed-water delivered to the boiler may be determined instead. The cooling or condensing water, either on the way to the condenser or when flowing from it, may be weighed, or for engines of large size may be measured by a metre or gauged by causing it to flow over a weir or through an orifice. The several temperatures t_4, t_i, and t_k must be taken by proper thermometers. When a jet-condenser is used, and the condensing water mingles with the steam, t_4 is identical with t_k. The quality x of the steam in the supply-pipe must be determined by a steam-calorimeter. A boiler with sufficient steam-space will usually deliver nearly dry steam; that is, x will be nearly unity. If the steam is superheated, its temperature t_s may be taken by a thermometer.

Let the heat lost by radiation, conduction, etc., be Q_e; this is commonly called the radiation. Let the heat supplied by the jacket be Q_j. Of the heat supplied to the cylinder per stroke, a portion is changed into work, a part is carried away by the condensed steam and the cooling or condensing water, and the remainder is lost by radiation; therefore

$$Q_e = Q + Q_j - Mq_4 - G\,(q_k - q_i) - A\,(W_a + W_b - W_c - W_d) \quad . \quad (171)$$

The heat Q_j supplied by a steam-jacket may be calculated by the equation

$$Q_j = m\,(x'r' + q' - q'') \quad . \quad . \quad . \quad . \quad (172)$$

in which m is the weight of water collected per stroke from the jacket; x', r', and q' are the quality, the heat of vaporization, and the heat of the liquid of the steam supplied; and q'' is the heat of the liquid when the water is withdrawn. When the jacket is supplied from the main steam-pipe, x' is the same as the quality in that pipe. When supplied direct from the boiler, x' may be assumed to be unity. If the jacket is supplied through a reducing-valve, the pressure and quality may be determined either before or after passing the valve, since throttling does not change the amount of heat in the steam. Should

the steam applied to the jacket be superheated from any cause, we may use the equation

$$Q_j = m\,[r' + q' + c_p\,(t_x' - t') - q''] \quad . \quad . \quad . \quad (173)$$

in which r' and q' are the heat of vaporization and heat of the liquid of saturated steam at the temperature t', and t_x' is the temperature of the superheated steam.

Equation (171) furnishes a method of calculating the heat lost by radiation and conduction; but since Q_e is obtained by subtraction and is small compared with the quantities on the right-hand side of the equation, the error of this determination may be large compared with Q_e itself. The usual way of determining Q_e for an engine with a jacket is to collect the water condensed in the jacket for a known time, an hour for example, when the engine is at rest, and then the radiation of heat per hour may be calculated. If it be assumed that the rate of radiation at rest is the same as when the engine is running, the radiation for any test may be inferred from the time of the test and the determined rate. But the engine always loses heat more rapidly when running than when at rest, so that this method of determining radiation always gives a result which is too small.

If a steam-engine has no jacket it is difficult or impossible to determine the rate of radiation. The only available way appears to infer the rate from that of some similar engine with a jacket. Probably the best way is to get an average value of Q_e from the application of equation (171) to a series of carefully made tests.

It is well to apply equation (171) to any test before beginning the calculation for Hirn's analysis, as any serious error is likely to be revealed, and so time may be saved.

When the radiation Q_e is known from a direct determination of the rate of radiation, we may apply Hirn's analysis to a test on an engine even though the quantities depending on the condenser have not been obtained. For from equation (171)

$$-Mq_4 - G\,(q_k - q_i) = Q_e - Q - Q_j + A(W_a + W_b - W_e - W_d),$$

and consequently

$$Q_c = I_2 - I_3 - Q - Q_j + Q_e + A\ (W_a + W_b - W_d)\ .\ .\ (174)$$

Thus it is possible to apply the analysis to a non-condensing engine or to the high-pressure cylinder of a compound engine.

It is apparent that the heat Q_c, thrown out from the walls of the cylinder during exhaust, passes without compensation to the condenser, and is a direct loss. Frequently it is the largest source of loss, and for this reason Hirn proposed to make it a test of the performance and perfection of the engine; but such a use of this quantity is not justifiable, and is likely to lead to confusion.

The heat Q_b that is restored during expansion is supplied at a varying and lower temperature than that of the source of heat, namely, the boiler, and, though not absolutely wasted, is used at a disadvantage. It has been suggested that an early compression, as found in engines with high rotative speed, warms up the cylinder and so checks initial condensation, thereby reducing Q_a and finally Q_c also. Such a storing of heat during compression and restoring during expansion is considered to act like the regenerator of a hot-air engine, and to make the efficiency of the actual cycle approach the efficiency of the ideal cycle more nearly than would be the case without compression. It does not, however, appear that engines of that type have exceeded, if they have equalled, the performance of slow-speed engines with small clearance and little compression.

Application. — In order to show the details of the method of applying Hirn's analysis the complete calculation for a test made on a small Corliss engine in the laboratory of the Massachusetts Institute of Technology will be given. Its usefulness is mainly as a guide to any one who may wish to apply the method for the first time.

Diameter of the cylinder . . . . 8 inches.
Stroke of the piston . . . . 2 feet.
Piston displacement: crank end . . . . 0.6791 cu. ft.
head end . . . . 0.7016 " "

Clearance, per cent of piston displacement:

crank end 3.75
head end 5.42
Boiler-pressure by gauge . . . . 77.4 pounds.
Barometer . . . . 14.8 "

Condition of steam, two per cent of moisture.

Events of the stroke:

Cut-off: crank end . . . . 0.306 of stroke.
head end . . . . 0.320 "

Release at end of stroke.

Compression: crank end . . . . 0.013 of stroke.
head end . . . . 0.0391 "

Duration of the test, one hour.

Total number of revolutions 3692
Weight of steam used . . . . 548 pounds.
Weight of condensing water used . . . . 14,568 "

Temperatures:

Condensed steam . . . . $t_4 = 141°.1$ F.
Condensing water: cold . . . . $t_i = 52°.9$ F.
warm . . . . $t_h = 88°.3$ F.

* ABSOLUTE PRESSURES, FROM INDICATOR-DIAGRAMS, AND CORRESPONDING PROPERTIES OF SATURATED STEAM.

	Crank End.				Head End.			
	p	q	ρ	u	p	q	ρ	u
Cut-off . . .	83.6	284.6	813.0	5.190	83.3	284.4	813.2	5.207
Release . . .	29.2	217.8	864.8	13.924	31.9	222.9	860.8	12.804
Compression.	14.8	181.1	893.2	26.464	14.8	181.1	893.2	26.464
Admission .	21.8	201.5	877.4	18.344	29.8	219.0	863.9	13.664

* These values are taken from the first edition of the Tables of Properties of Saturated Steam.

MEAN PRESSURES, AND HEAT-EQUIVALENTS OF EXTERNAL WORKS.

	Crank End.		Head End.	
	Mean Pressures.	Equivalents of Work.	Mean Pressures.	Equivalents of Work.
Admission	87.7	3.369	89.3	3.711
Expansion	44.5	3.877	47.1	4.159
Exhaust	14.8	1.836	14.8	1.847
Compression	18.3	0.0290	21.8	0.1104

VOLUMES, CUBIC FEET.

	Crank End.	Head End.
At cut-off, $V_0 + V_1$	0.2333	0.2626
At release, $V_0 + V_2$	0.7046	0.7396
At compression, $V_0 + V_3$	0.0343	0.0655
At admission, V_0	0.02550	0.03806

At the boiler-pressure, 92.1 pounds absolute, we have

$$r = 888.4, \qquad q = 291.7.$$

The steam used per stroke is

$$M = \frac{548}{2 \times 3692} = 0.0742 \text{ pound.}$$

The steam caught in the clearance space at compression, on the assumption that the steam is then dry and saturated, is obtained by multiplying the mean volume at that point by the weight of one cubic foot of steam at the pressure at compression, which is 0.03781 of a pound.

$$\therefore M_0 = \frac{0.0343 + 0.0655}{2} \times 0.03781 = 0.0019 \text{ of a pound;}$$

$$M + M_0 = 0.0742 + 0.0019 = 0.0761 \text{ pound.}$$

The condensing water used per stroke is

$$G = \frac{14568}{2 \times 3692} = 1.973.$$

$$Q = M\,(xr + q) = 0.0742(0.98 \times 888.3 + 291.8) = 86.243;$$

$$x_0 = \frac{V_0}{M_0 u_0} - \frac{\sigma}{u_0};$$

$$\therefore x_0 = \frac{\frac{1}{2}\,(0.02550 + 0.03806)}{0.0019 \times \frac{1}{2}(18.344 + 13.664)} - \frac{1}{62.4 \times \frac{1}{2}(18.344 + 13.664)}$$
$$= 1.043.$$

This indicates that the steam is superheated at admission. Such may be the case, or the appearance may be due to an error in the assumption of dry steam at compression, or to errors of observation. It is convenient to assume $x_0 = 1$.

$$x_1 = \frac{V_0 + V_1}{(M + M_0)\,u_1} - \frac{\sigma}{u_1};$$

$$\therefore\; x_1 = \frac{\frac{1}{2}\,(0.2333 + 0.2626)}{0.0761 \times \frac{1}{2}\,(5.190 + 5.207)} - \frac{1}{62.4 \times \frac{1}{2}(5.190 + 5.207)}$$
$$= 0.6236.$$

$$x_2 = \frac{V_0 + V_2}{(M + M_0)\,u_1} - \frac{\sigma}{u_2};$$

$$\therefore\; x_2 = \frac{\frac{1}{2}\,(0.7046 + 0.7396)}{0.0761 \times \frac{1}{2}(13.924 + 12.804)} - \frac{1}{62.4 \times \frac{1}{2}(13.924 + 12.804)}$$
$$= 0.7088.$$

$$I_0 = M_0\,(q_0 + x_0\rho_0);$$

$$\therefore\; I_0 = \tfrac{1}{2} \times 0.0019\,[201.5 + 219.0 + 1.00\,(877.4 + 863.9)]$$
$$= 2.054.$$

$$I_1 = (M + M_0)\,(q_1 + x_1\rho_1);$$

$$\therefore\; I_1 = \tfrac{1}{2} \times 0.0761\,[284.6 + 284.4 + 0.6236\,(813.0 + 813.2)]$$
$$= 60.238.$$

$$I_2 = (M + M_0)\,(q_2 + x_2\rho_2);$$

$$\therefore\; I_2 = \tfrac{1}{2} \times 0.0761\,[217.8 + 222.0 + 0.7088\,(864.8 + 861.8)]$$
$$= 63.311.$$

$$I_3 = M_0(q_3 + x_3\rho_3);$$

$$\therefore\; I_3 = 0.0019\,(181.1 + 893.2) = 2.041.$$

$$Q_a = Q + I_0 - I_1 - AW_a;$$

$$\therefore Q_a = 86.243 + 2.054 - 60.238 - \tfrac{1}{2}(3.369 + 3.711) = 24.519.$$

$$Q_b = I_1 - I_2 - AW_b;$$

$$\therefore Q_b = 60.238 - 63.311 - \tfrac{1}{2}(3.877 + 4.159) = -7.091.$$

$$Q_c = I_2 - I_3 - Mq_4 - G(q_k - q_i) + AW_c;$$

$$\therefore Q_c = 63.311 - 2.041 - 0.0742 \times 109.3$$
$$- 1.973(56.35 - 21.01) + \tfrac{1}{2}(1.836 + 1.847)$$
$$= -14.721.$$

$$Q_d = I_3 - I_0 + AW_d;$$

$$\therefore Q_d = 2.041 - 2.054 + \tfrac{1}{2}(0.0299 + 0.1104) = 0.157.$$

$$Q_e = Q_a + Q_b + Q_c + Q_d = 2.764.$$

Also, equation (171) for this case gives

$$Q_e = Q - Mq_4 - G(q_k - q_i) - AW$$
$$= 86.243 - 8.110 - 69.723 - (3.540 + 4.018 - 1.841 - 0.070)$$
$$= 86.243 - 8.110 - 69.723 - 5.647 = 2.764.$$

It is to be remembered that the heat lost by radiation and conduction per stroke, when estimated in this manner, is affected by the accumulated errors of observation and computation, which may be a large part of the total value of Q_e.

Dropping superfluous significant figures, we have in B.T.U.

$$Q = 86.2, \qquad Q_a = 24.5, \qquad Q_b = -7.1,$$
$$Q_c = -14.7, \qquad Q_d = .06, \qquad Q_e = 2.8.$$

Noting that 5.647 are the B.T.U. changed into work per stroke and 3692 the total revolutions the horse-power of the engine is

$$\frac{778 \times 5.647 \times 3692 \times 2}{60 \times 33000} = 16.35 \text{ H.P.}$$

and the steam per horse-power per hour is

$$\frac{548}{16.35} = 33.5 \text{ pounds.}$$

For data and results of this test and others see Table IV.

TABLE IV.

APPLICATION OF HIRN'S ANALYSIS TO A SMALL CORLISS ENGINE AT THE MASSACHUSETTS INSTITUTE OF TECHNOLOGY.

DIAMETER, 8 INCHES; STROKE, 24 INCHES.

Number.	Weight of condensing-water, pounds.	Per cent of priming in steam. $1-x$	Temperatures, degrees Fahrenheit: Condensed steam. t_s	Condensing-water cold. t_i	Condensing-water, warm. t_o	Barometer, pounds on the square inch.	Boiler-pressure by gauge, pounds. p	Absolute pressures, pounds. Cut-off. p_1 C. E.	Cut-off. p_1 H. E.	Release. p_2 C. E.	Release. p_2 H. E.	Compression. p_3 C. E.	Compression. p_3 H. E.	Admission. p_4 C. E.	Admission. p_4 H. E.
1	3853	2.	118.6	39.2	74.3	14.8	68.7	77.1	76.3	16.3	16.8	14.8	14.8	24.8	30.8
2	10077	2.	130.5	54.8	86.0	14.7	73.0	83.7	82.7	17.7	19.7	14.7	14.7	21.7	25.7
3	4294	2.	118.0	39.2	77.0	14.9	68.5	74.2	77.5	18.6	20.3	14.9	14.9	20.9	24.9
4	8881	2.	132.8	53.6	132.8	14.8	72.9	77.1	80.5	21.8	23.8	14.8	14.8	23.8	34.8
5	14568	2.	141.1	52.0	88.3	14.8	77.4	83.6	83.3	29.2	31.9	14.8	14.8	21.8	29.8

Number.	Events of the stroke — per cent of stroke from beginning. Cut-off. C. E.	Cut-off. H. E.	Release. C. E.	Release. H. E.	Compression. C. E.	Compression. H. E.	Heat-equivalents of work. B.T.U. Admission. AW_a C. E.	Admission. AW_a H. E.	Expansion. AW_b C. E.	Expansion. AW_b I. E.	Exhaust. AW_c C. E.	Exhaust. AW_c H. E.	Compression. AW_d C. E.	Compression. AW_d H. E.	Interchanges of heat. B.T.U. Heat in M pounds of moist steam. Q	Absorbed by walls during admission. Q_a	Yielded by walls during expansion. Q_b	Yielded by walls during exhaust. Q_c	Absorbed by walls during compression. Q_d	Radiation and error. Q_e	Indicated horse-power.	Steam per horse-power per hour, pounds.
1	10.0	9.0	100	100	2.2	7.7	1.027	0.970	3.11	3.284	1.817	1.773	0.052	0.248	45.10	20.91	12.27	6.43	0.43	2.65	6.53	44.3
2	12.2	12.2	100	100	1.3	4.6	1.305	1.386	3.285	3.783	1.822	1.820	0.035	0.114	52.61	22.26	11.60	9.28	0.15	1.53	8.08	38.6
3	15.0	15.0	100	100	0.9	3.4	1.471	1.633	3.202	3.564	1.854	1.868	0.022	0.099	53.60	21.28	11.15	7.77	0.05	2.42	8.72	39.0
4	19.5	19.5	100	100	1.5	4.0	2.108	2.217	3.527	3.929	1.830	1.844	0.039	0.116	64.10	23.39	10.64	9.43	0.06	3.58	11.13	35.3
5	30.6	32.0	100	100	1.3	3.9	3.369	3.711	3.877	4.159	1.836	1.847	0.0299	0.1104	86.24	24.52	7.09	14.72	0.06	2.76	16.35	35.5

Effect of Varying Cut-off. — An inspection of the interchanges of heat shows that the values of Q_a, the heat absorbed by the walls during admission, increase regularly as the cut-off is lengthened, and that the heat returned during expansion decreases at the same time, so that there is a considerable increase in the value of the heat Q_c which is rejected during exhaust. Nevertheless there is a large gain in economy from restricting the cut-off so that it shall not come earlier than one-third stroke. Unfortunately tests on this engine with longer cut-off than one-third stroke have not been made, and consequently the poorer economy for long cut-off cannot be shown for this engine as for the engine of the *Michigan*.

Hallauer's Tests. — In Table V are given the results of a number of tests made by Hallauer on two engines, one built by Hirn having four flat gridiron valves, and the other a Corliss engine having a steam-jacket. Two tests were made on the former with saturated steam and six with superheated steam. Three tests were made on the latter with saturated steam and with steam supplied to the jackets. These tests have a historic interest, for though not the first to which Hirn's analysis was applied, they are the most widely known, and brought about the acceptance of his method. They have also a great intrinsic value, as they exhibit the action of two different methods of ameliorating the effect of the action of the cylinder walls, namely, by the use of superheated steam and of the steam-jacket. In all these tests there was little compression, and Q_d, the interchange of heat during compression, is ignored.

Superheated Steam. — Steam from a boiler is usually slightly moist, x, the quality, being commonly 0.98 or 0.99. Some boilers, such as vertical boilers with tubes through the steam space, give steam which is somewhat superheated, that is, the steam has a temperature higher than that of saturated steam at the boiler-pressure. Strongly superheated steam is commonly obtained by passing moist steam from a boiler through a coil of pipe, or a system of piping, which is exposed to hot gases beyond the boiler.

TABLE V.

TESTS ON A HIRN AND A CORLISS ENGINE.

By HALLAUER, *Bulletin de la Soc. Ind. de Mulhouse*, vol. XLVII, 1877.

	Condition.	Temperature, superheated steam, Fahr.	Revolutions per minute.	Expansions.	Boiler-pressure, absolute, pounds per square inch.	Back-pressure, absolute, pounds per square inch.	Horse-power.	Steam per H. P. per hour, pounds.	B.T.U. per H. P. per minute.	Exchanges of heat in per cent of total heat furnished per stroke. Q_a	Q_b	Q_c	Q_d	Q_f	B.T.U. rejected per stroke. Q_e
1			30.4	6.1	70.7	3.0	106.4	19.9	353	28.3	5.0	21.6	1.5		147
2			30.6	3.9	66.0	5.2	134.6	22.5	390	23.9	7.3	15.4	1.0		149
3	Superheated	383	30.0	6.1	71.1	2.7	111.6	16.0	296	22.4	8.4	12.5	1.6		75
4	Superheated	419	30.0	4.7	71.0	2.7	134.0	15.7	295	....	...	9.7	...		70
5	Superheated	448	30.2	3.9	69.6	5.2	142.4	17.2	320	11.0	2.0	7.8	1.2		66
6	Superheated	433	30.3	2.2	68.4	2.7	124.5	18.2	341	....	...	10.5	1.3		81
7	Superheated	428	30.1	2.2	71.6	2.5	89.6	20.2	380	....	...	14.2	1.6		87
8*	Superheated	428	30.0	3.5	62.2	...	77.3	27.9	...	....	...	†−1.2	...	...	−7
9	Jacketed		50.4	12		2.1	103.6	17.8	...	26.4	17.5	12.3	1.65	5.1	44
10	Jacketed		51.1	8		2.4	129.1	17.7	...	21.1	15.0	9.8	1.3	4.9	44
11	Jacketed		49.3	6		2.6	155.9	17.8	...	15.4	10.4	8.0	1.1	4.1	44

* Non-condensing. † The true value is probably zero.

The throttle-value was partly closed for 6, 7, and 8.

Superheated steam may yield a considerable amount of heat before it begins to condense; consequently where superheated steam is used in an engine a portion of the heat absorbed by the walls during admission is supplied by the superheat of the steam and less condensation of steam occurs. This is very evident in Dixwell's tests given by Table XXV, on page 271, where the water in the cylinder at cut-off is reduced from 52.2 per cent to 27.4 per cent, when the cut-off is two-tenths of the stroke, by the use of superheated steam; with longer cut-off the effect is even greater. This reduction of condensation is accompanied by a very marked gain in economy.

The way in which superheated steam diminishes the action of the cylinder walls and improves the economy of the engine is made clear by Hallauer's tests in Table V. A comparison of tests 1 and 3, having six expansions, shows that the heat Q_a absorbed during admission is reduced from 28.3 to 22.4 per cent of the total heat supplied, and that the exhaust waste is correspondingly reduced from 21.6 to 12.5 per cent. A similar comparison of tests 2 and 5, having nearly four expansions, shows even more reduction of the action of the cylinder walls. The effect on the restoration of heat Q_b during expansion appears to be contradictory: in one case there is more and in the other case less. It does not appear profitable to speculate on the meaning of this discrepancy, as it may be in part due to errors and is certainly affected by the unequal degree of superheating in tests 3 and 5. It may be noted that the actual value of Q_c in calories is nearly the same for tests 1 and 2, there being a small apparent increase with the increase of cut-off, which is, however, less than the probable error of the tests. The exhaust waste Q_c is much more irregular for tests 3 to 7 for superheated steam. The increase from 81 to 87 B.T.U. from test 6 to test 7 may properly be attributed to a less degree of superheating; the increase from 66 to 81 B.T.U. for tests 5 and 6 is due to longer cut-off and less superheating; finally, the steady reduction from 75 to 66 B.T.U. for the three tests 3, 4, and 5 is probably due to the rise of temperature of the superheated steam, which more

than compensates for the effect of lengthening the cut-off. Finally in test 8 the exhaust waste is practically reduced to zero by the use of strongly superheated steam in a non-condensing engine; this shows clearly that the exhaust waste Q_e by itself is no criterion of the value of a certain method of using steam.

Steam-Jackets. — If the walls of the cylinder of a steam-engine are made double, and if the space between the walls is filled with steam, the cylinder is said to be steam-jacketed. Both barrel and heads may be jacketed, or the barrel only may have a jacket; less frequently the heads only are jacketed. The principal effect of a steam-jacket is to supply heat during the vaporization of any water which may be condensed on the cylinder walls. The consequence is that more heat is returned to the steam during expansion and the walls are hotter at the end of exhaust than would be the case for an unjacketed engine. This is evident from a comparison of tests 1 and 11 in Table V. In test 1 only a small part of the heat absorbed during admission is returned during expansion, and by far the larger part is wasted during exhaust. In test 11 the heat returned during expansion is equal to two-thirds that absorbed during admission, though a part of this heat of course comes from the jacket. About half as much is wasted during exhaust as is absorbed during admission. The condensation of steam is thus reduced indirectly; that is, the chilling of the cylinder during expansion, and especially during exhaust, is in part prevented by the jacket, and consequently there is less initial condensation and less exhaust waste, and in general a gain in economy. The heat supplied during expansion, though it does some work, is first subjected to a loss of temperature in passing from the steam in the jacket to the cooler water on the walls of the cylinder, and such a non-reversible process is necessarily accompanied by a loss of efficiency. On the other hand, the heat supplied by a jacket during exhaust passes with the steam directly into the exhaust-pipe. It appears, then, that the direct effect of a steam-jacket is to waste heat; the indirect effect (drying and warming the cylinder)

reduces the initial condensation and the exhaust waste and often gives a notable gain in economy.

Application to Multiple-expansion Engines. — The application of Hirn's analysis to the high-pressure cylinder of a compound or multiple-expansion engine may be made by using equations (159), (160), and (162) for calculating Q_a, Q_b, and Q_d, while equation (174) may be used to find Q_c.

A similar set of equations may be written for the next cylinder, whether it be the low-pressure cylinder of a compound engine or the intermediate cylinder of a triple engine, provided we can determine the value of Q', the heat supplied to that cylinder. But of the heat supplied to the high-pressure cylinder a part is changed into work, a part is radiated, and a part is rejected in the exhaust waste. The heat rejected is represented by

$$Q + Q_j - AW - Q_e \quad . \quad . \quad . \quad . \quad . \quad . \quad (175)$$

where Q is the heat supplied by the steam entering the cylinder, Q_j is the heat supplied by the jacket, AW is the heat-equivalent of the work done in the cylinder, and Q_e is the heat radiated. Suppose the steam from the high-pressure cylinder passes to an intermediate receiver, which by means of a tubular reheater or by other means supplies the heat Q_r, while there is an external radiation Q_{re}. The heat supplied to the next cylinder is consequently

$$Q' = Q + Q_j - AW - Q_e + Q_r - Q_{re} \quad . \quad . \quad (176)$$

In a like manner we may find the heat Q'' supplied to the next cylinder; for example, to the low-pressure cylinder of a triple engine.

It is clear that such an application of Hirn's analysis can be made only when the several steam-jackets on the high- and the low-pressure cylinders, and the reheater of the receiver, etc., can be drained separately, so that the heat supplied to each may be determined individually.

Table VI gives applications of Hirn's analysis to four tests on the experimental triple-expansion engine in the laboratory of the Massachusetts Institute of Technology.

It will be noted that the steam in the cylinders becomes drier in its course through the engine, under the influence of thorough steam-jacketing with steam at boiler-pressure, and is practically dry at release in the low-pressure cylinder. All of the tests show superheating in the low-pressure cylinder, which is of course possible, for the steam in the jackets is at full boiler-pressure while the steam in the cylinder is below atmospheric pressure. The superheating was small in all cases — not more than would be accounted for by the errors of the tests. The exhaust waste Q_e'' from the low-pressure cylinder in the triple-expansion tests is very small in all cases — less than two per cent of the heat supplied to the cylinders. The apparent absurdity of a positive value for Q_e'' in two of the tests (indicating an absorption of heat by the cylinder walls during exhaust) may properly be attributed to the unavoidable errors of the test.

In the fourth test, when the engine was developing 120.3 horse-power, there were 1305 pounds of steam supplied to the cylinders in an hour, and 345 pounds to the steam-jackets; so that the steam per horse-power per hour passing through the cylinders was

$$1305 \div 120.3 = 10.86 \text{ pounds,}$$

while the condensation in the jackets was

$$345 \div 120.3 = 2.87 \text{ pounds.}$$

So that, as shown on page 145, the B.T.U. per horse-power per minute supplied to the cylinders by the entering steam was 191.1, while the jackets supplied 40.6 B.T.U., making in all 231.7 B.T.U. per horse-power per minute for the heat-consumption of the engine. In the same connection it was shown that the thermal efficiency of the engine for this test was 0.183, while the efficiency for incomplete expansion in a non-conducting cylinder corresponding to the conditions of the test was 0.222; so that the engine was running with 0.824 of the possible efficiency. In light of this satisfactory conclusion some facts with regard to the test are interesting.

TABLE VI.

APPLICATION OF HIRN'S ANALYSIS TO THE EXPERIMENTAL ENGINE IN THE LABORATORY OF THE MASSACHUSETTS INSTITUTE OF TECHNOLOGY.

TRIPLE-EXPANSION; CYLINDER DIAMETERS, 9, 16, AND 24 INCHES; STROKE, 30 INCHES.

Trans. Am. Soc. Mech. Engrs., vol. xii, p. 740.

	I.	II.	III.	IV.
Duration of test, minutes	60	60	60	60
Total number of revolutions	5299	5228	5173	5148
Revolutions per minute	88.3	87.1	86.2	85.8
Steam-consumption during test, lbs.:				
Passing through cylinders	1193	1157	1234	1305
Condensation in h.p. jacket	57	50	29	30
in first receiver-jacket	61	64	69	72
in inter. jacket	85	92	97	105
in second receiver-jacket	53	50	52	51
in l.p. jacket	89	76	90	87
Total	1538	1489	1571	1650
Condensing water for test, lbs.	22847	22186	20244	20252
Priming, by calorimeter	0.013	0.012	0.011	0.012
Temperatures, Fahrenheit:				
Condensed steam	95.4	92.1	102.4	105.3
Condensing-water, cold	41.9	42.1	43.0	42.8
Condensing-water, hot	96.1	96.6	106.3	109.6
Pressure of the atmosphere, by the barometer, lbs. per sq. in.	14.8	14.8	14.7	14.7
Boiler pressure, lbs. per sq. in. absolute	155.3	155.5	156.9	157.7
Vacuum in condenser, inches of mercury	25.0	25.1	24.1	23.9
Events of the stroke:				
High-pressure cylinder —				
Cut-off, crank end	0.192	0.194	0.245	0.183
head end	0.215	0.205	0.271	0.305
Release, both ends	1.00	1.00	1.00	1.00
Compression, crank end	0.05	0.05	0.04	0.04
head end	0.05	0.05	0.05	0.05
Intermediate cylinder —				
Cut-off, both ends	0.29	0.29	0.29	0.29
Release, both ends	1.00	1.00	1.00	1.00
Compression, crank end	0.03	0.03	0.03	0.03
head end	0.04	0.04	0.04	0.04
Low-pressure cylinder —				
Cut-off, crank end	0.38	0.38	0.38	0.38
head end	0.39	0.39	0.39	0.39
Release, both ends	1.00	1.00	1.00	1.00

TABLE VI — *Continued.*

	I.	II.	III.	IV.
Absolute pressures in the cylinder, pounds per sq. in.:				
High-pressure cylinder —				
Cut-off, crank end	145.9	145.9	138.8	138.3
head end	143.2	143.1	140.3	140.6
Release, crank end	41.3	41.5	44.7	48.4
head end	41.5	40.5	45.7	49.8
Compression, crank end	43.7	45.3	48.5	53.2
head end	48.7	47.9	54.5	62.0
Admission, crank end	64.5	68.8	72.2	81.2
head end	75.3	74.8	86.7	97.8
Intermediate cylinder —				
Cut-off, crank end	37.2	37.6	38.6	40.9
head end	35.0	35.3	39.6	42.6
Release, crank end	13.6	14.2	14.7	16.0
head end	13.4	13.8	14.9	16.0
Compression, crank end	16.3	17.3	18.2	19.0
head end	17.9	18.8	20.3	22.4
Admission, crank end	20.4	20.8	22.2	23.1
head end	21.1	22.8	24.2	26.7
Low-pressure cylinder —				
Cut-off, crank end	12.1	12.6	12.4	13.2
head end	12.0	12.4	13.1	14.0
Release, crank end	5.6	5.3	5.1	5.7
head end	5.4	5.8	5.9	6.4
Compression and admission —				
crank end	3.7	3.8	4.1	4.2
head end	4.3	4.5	4 6	4.7
Heat-equivalents of external work, B.T.U., from a reason indicator-diagram to line of absolute vacuum:				
High-pressure cylinder —				
During admission,				
AW_a, crank end	5.71	5.78	7.00	8.19
head end	6.61	6.37	8.42	9.50
During expansion,				
AW_b, crank end	10.65	10.76	10.40	10.25
head end	10.81	11.04	11.22	11.09
During exhaust,				
AW_e, crank end	7.73	7.89	8.44	9.02
head end	8.08	8.15	9.04	9.66
During compression,				
AW_d, crank end	0.48	0.60	0.49	0.50
head end	0.62	0.64	0.73	0.81
Intermediate cylinder —				
During admission,				
AW_a', crank end	7.58	7.57	7.98	8.64
head end	7.43	7.55	8.46	9.10
During expansion,				
AW_b', crank end	9.54	9.54	9.91	10.64
head end	9.22	9.31	10.37	11.14

TABLE VI — *Continued.*

	I.	II.	III.	IV.
Intermediate cylinder —				
During exhaust,				
AW_c', crank end	9.27	9.47	9.64	10.54
head end	9.27	9.47	10.18	10.84
During compression,				
AW_d', crank end	0.39	0.43	0.57	0.46
head end	0.60	0.70	0.78	0.84
Low-pressure cylinder —				
During admission,				
AW_a'', crank end	7.75	7.95	8.33	8.97
head end	7.99	8.19	8.66	9.39
During expansion,				
AW_b'', crank end	6.83	7.10	6.86	7.45
head end	6.87	7.12	7.34	7.87
During exhaust,				
AW_c'', crank end	5.08	5.08	4.62	5.09
head end	5.08	5.16	4.81	5.00
During compression,				
AW_d'', crank end	0.00	0.00	0.00	0.00
head end	0.00	0.00	0.00	0.00
Quality of the steam in the cylinder. At admission and at compression the steam was assumed to be dry and saturated:				
High-pressure cylinder —				
At cut-off . . . . . . x_1 .	0.785	0.784	0.848	0.875
At release . . . . . . x_2 .	0.899	0.903	0.920	0.931
Intermediate cylinder —				
At cut-off . . . . . . x_1' .	0.899	0.912	0.906	0.908
At release . . . . . . x_2' .	0.994	* * *	* * *	* * *
Low-pressure cylinder —				
At cut-off . . . . . . x_1'' .	0.978	* * *	0.970	0.974
At release . . . . . . x_2'' .	* * *	* * *	* * *	* * *
Interchanges of heat between the steam and the walls of the cylinders, in B. T. U. Quantities affected by the positive sign are absorbed by the cylinder walls; quantities affected by the negative sign are yielded by the walls: . .				
High-pressure cylinder —				
Brought in by steam . Q . . .	132.93	130.77	141.11	149.84
During admission . . . Q_a . .	23.54	23.43	17.49	14.93
During expansion . . . Q_b . .	−18.69	−19.28	−15.33	−14.03
During exhaust . . . . Q_c . .	− 8.36	− 7.22	− 3.50	− 2.38
During compression . . Q_d . .	0.45	0.51	0.49	0.52
Supplied by jacket . . Q_j . .	4.56	4.08	2.39	2.50
Lost by radiation . . . Q_e . .	1.50	1.52	1.54	1.54
First intermediate receiver —				
Supplied by jacket . . Q_r . .	4.92	5.20	5.67	5.95
Lost by radiation . . . Q_{re} . .	0.58	0.58	0.59	0.59

* Superheated.

TABLE VI — *Continued.*

	I.	II.	III.	IV.
Intermediate cylinder —				
Brought in by steam . Q' . .	131.89	129.61	137.87	146.64
During admission . . . Q_a' . .	13.62	11.74	11.33	11.75
During expansion . . . Q_b' . .	−18.65	−18.84	−20.30	−21.88
During exhaust . . . . Q_c' . .	0.22	1.57	2.88	3.41
During compression . . Q_d' . .	0.44	0.51	0.62	0.59
Supplied by jacket . . Q_j' . .	6.82	7.50	7.97	8.64
Lost by radiation . . . Q_e' . .	2.45	2.48	2.50	2.51
Second intermediate receiver—				
Supplied by jacket . . Q_r' . .	4.20	4.04	4.27	4.22
Lost by radiation . . . Q_n' . .	1.20	1.22	1.23	1.24
Low-pressure cylinder —				
Brought in by steam . Q'' . .	132.14	130.50	138.61	147.33
During admission . . . Q_a'' . .	5.85	3.05	5.57	5.29
During expansion . . . Q_b'' . .	− 9.51	− 7.09	− 8.65	−10.13
During exhaust . . . . Q_c'' . .	2.53	2.23	− 1.44	− 0.11
During compression . . Q_d'' . .	0.00	0.00	0.00	0.00
Supplied by jacket . . Q_j'' . .	7.08	6.20	7.41	7.14
Lost by radiation . . . Q_e'' . .	4.34	4.40	4.45	4.47
Total loss by radiation —				
By preliminary tests . . ΣQ_e . .	10.07	10.20	10.31	10.35
By equation (171)	11.68	10.19	8.75	8.07
Power and economy:				
Heat-equivalents of works per stroke —				
H. P. cylinder . . . . AW . .	8.44	8.34	9.17	9.52
Interm. cylinder. . . . AW' .	7.12	6.95	7.77	8.42
L. P. cylinder . . . . AW'' .	9.64	10.06	10.87	11.79
Totals	25.20	25.35	27.81	29.73
Total heat furnished by jackets . .	27.58	27.02	27.71	28.45
Distribution of work —				
High-pressure cylinder	1.00	1.00	1.00	1.00
Intermediate cylinder	0.84	0.83	0.85	0.88
Low-pressure cylinder	1.14	1.21	1.19	1.24
Horse-power	104.9	104.2	113.1	120.3
Steam per H.P. per hour	14.65	14.31	13.90	13.73
B.T.U. per H.P. per minute . . .	247	241	236	232

It will be noted that for test IV 149.84 B.T.U. per stroke are brought in by the steam supplied to the high-pressure cylinder and that 28.45 B.T.U. per stroke are supplied by the steam-jackets; and that, further, 29.73 B.T.U. are changed into work while 10.35 are radiated. Thus it appears that the jackets furnished almost as much heat as was required to do all the work developed. Of the heat furnished by the jackets something more than a third

was radiated; the other two-thirds may fairly be considered to have been changed into work, since the exhaust waste of the low-pressure cylinder was practically zero.

Quality of Steam at Compression. — In all the work of this chapter the steam in the cylinder at compression has been considered to be dry and saturated, and it has been asserted that little if any error can arise from this assumption. It is clear that some justification for such an assumption is needed, for a relatively large weight of water in the cylinder would occupy a small volume and might well be found adhering to the cylinder walls in the form of a film or in drops; such a weight of water would entirely change our calculations of the interchanges of heat. The only valid objection to Hirn's analysis is directed against the assumption of dry steam at compression. Indeed, when the analysis was first presented some critics asserted that the assumption of a proper amount of water in the cylinder is all that is required to reduce the calculated interchanges of heat to zero. It is not difficult to refute such an assertion from almost any set of analyses, but unfortunately such a refutation cannot be made to show conclusively that there is little or no water in the cylinder at compression; in every case it will show only that there must be a considerable interchange of heat.

For the several tests on the Hirn engine given in Table V, Hallauer determined the amount of moisture in the steam in the exhaust-pipe, and found it to vary from 3 to 10 per cent. Professor Carpenter * says that the steam exhausted from the high-pressure cylinder of a compound engine showed 12 to 14 per cent of moisture. Numerous tests made in the laboratory of the Massachusetts Institute of Technology show there is never a large percentage of water in exhaust-steam. Finally, such a conclusion is evident from ordinary observation. Starting from this fact and assuming that the steam in the cylinder at compression is at least as dry as the steam in the exhaust-pipe, we are easily led to the conclusion that our assumption of dry steam is proper. Professor Carpenter reports also that a calorimeter

* *Trans. Am. Soc. Mech. Engrs.*, vol. xii, p. 811.

test of steam drawn from the cylinder during compression showed little or no moisture. Nevertheless, there would still remain some doubt whether the assumption of dry steam at compression is really justified, were we not so fortunate as to have direct experimental knowledge of the fluctuations of temperature in the cylinder walls.

Dr. Hall's Investigations. — For the purpose of studying the temperatures of the cylinder walls Dr. E. H. Hall used a thermo-electric couple, represented by Fig. 56. *I* is a cast-iron plug about three-quarters of an inch in diameter, which could be screwed into the hole provided for attaching an indicator-cock to the cylinder of a steam-engine. The inner end of the plug carried a thin cast-iron disk, which was assumed to act as a part of the cylinder wall when the plug was in place. To study the temperature of the outside surface of the disk a nickel rod *N* was soldered to it, making a thermo-electric couple. Wires from *I* and *N* led to another couple made by soldering together cast-iron and nickel, and this second couple was placed in a bath of paraffine which could be maintained at any desired temperature. In the electric circuit formed by the wires joining the two thermo-electric couples there was placed a galvanometer and a circuit-breaker. The circuit-breaker was closed by a cam on the crank-shaft, which could be set to act at any point of the revolution. If the temperature of the outside of the disk *S* differed from the temperature of the paraffine bath at the instant when contact was made by the cam, a current passed through the wires and was indicated by the galvanometer. By properly regulating the temperature of the bath, the current could be reduced and made to cease, and then a thermometer in the bath gave the temperature at the surface of the disk for the instant when the cam closed the electric circuit. Two points in the steam-cycle were chosen for investigation, one immediately after cut-off and the other immediately after compression, since

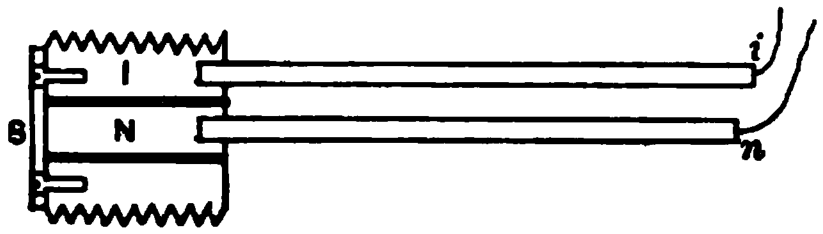

FIG. 56.

they gave the means of investigating the heat absorbed during compression and admission of steam, and the heat given up during expansion and exhaust.

Three different disks were used: the first one half a millimetre thick, the second one millimetre thick, and a third two millimetres thick. From the fluctuations of temperature at these distances from the inside surface of the wall some idea could be obtained concerning the variations of temperature at the inner surface of the cylinder, and also how far the heating and cooling of the walls extended.

The account given here is intended only to show the general idea of the method, and does not adequately indicate the labor difficulties of the investigation which involved many secondary investigations, such as the determination of the conductivity of nickel. Having shown conclusively that there is an energetic action of the walls of the cylinder, Dr. Hall was unable to continue his investigations.

Callendar and Nicolson's Investigations. — A very refined and complete investigation of the temperature of the cylinder walls and also of the steam in the cylinder was made by Callendar and Nicolson * in 1895 at the McGill University, by the thermo-electric method.

The wall temperatures were determined by a thermo-electric couple of which the cylinder itself was one element and a wrought-iron wire was the other element. To make such a couple, the cylinder wall was drilled nearly through, and the wire was soldered to the bottom of the hole. Eight such couples were established in the cylinder-head, the thickness of the unbroken wall varying from 0.01 of an inch to 0.64 of an inch. Four pairs of couples were established along the cylinder-barrel, one near the head, and the others at 4 inches, 6 inches, and 12 inches from the head. One of each pair of wall couples was bored to within 0.04 of an inch, and the other to 0.5 of an inch of the inside surface of the cylinder. Other couples were established along the side of the cylinder to study the flow of heat from the

* *Proceedings of the Inst. Civ. Engrs.*, vol. cxxxii.

head toward the crank end. The temperature of the steam near the cylinder-head was measured by a platinum thermometer capable of indicating correctly rapid fluctuations of temperature.

The engine used for the investigations was a high-speed engine, with a balanced slide-valve controlled by a fly-wheel governor. During the investigations the cut-off was set at a fixed point (about one-fifth stroke), and the speed was controlled externally. By the addition of a sufficient amount of lap to prevent the valve from taking steam at the crank end the engine was made single-acting. The normal speed of the engine was 250 revolutions per minute, but during the investigations the speed was from 40 to 90 revolutions per minute. The diameter of the cylinder was 10.5 inches and the stroke of the piston was 12 inches. The clearance was ten per cent of the piston displacement.

From the indicator-diagrams an analysis, nearly equivalent to Hirn's analysis, showed the heat yielded to or taken from the walls by the steam; on the other hand the thermal measurements gave an indication of the heat gained by or yielded by the walls. The results are given in the following table; and considering the difficulty of the investigation and the large allowance for leakage, the concordance must be admitted to be very satisfactory.

TABLE VII.

INFLUENCE OF THE WALLS OF THE CYLINDER.

CALLENDAR AND NICOLSON, *Proc. Inst. Civ. Engrs.*, 1897.

	I.	II.	III.	IV.	V.	VI.	VII.
Duration, minutes	37	68	55	79	76	35	25
Revolutions per minute	43.8	45.7	47.7	70.4	73.4	81.7	97.0
Mean gauge-pressure	87.9	89.2	94.4	98.1	92.0	94.2	96.0
Gross steam per revolution	0.1422	0.1437	0.1483	0.1094	0.1036	0.1000	0.0856
Leakage correction	0.1004	0.0976	0.0990	0.0697	0.0627	0.0576	0.0494
Net steam per revolution	0.0418	0.0461	0.0493	0.0397	0.0409	0.0424	0.0362
Steam caught at compression	0.0107	0.0104	0.0103	0.0099	0.0098	0.0100	0.0105
Weight of mixture in cylinder	0.0525	0.0565	0.0596	0.0496	0.0507	0.0524	0.0467
Indicated steam at quarter stroke	0.0407	0.0414	0.0437	0.0418	0.0394	0.0408	0.0393
Indicated steam at release	0.0466	0.0456	0.0488	0.0460	0.0436	0.0454	0.0426
Increase of indicated weight	0.0059	0.0042	0.0051	0.0042	0.0042	0.0046	0.0033
Adiabatic condensation	0.0019	0.0020	0.0021	0.0020	0.0019	0.0020	0.0019
Indicated evaporation	0.0078	0.0062	0.0072	0.0062	0.0061	0.0066	0.0052
Calculated evaporation	0.0076	0.0073	0.0072	0.0048	0.0046	0.0041	0.0035
Indicated condensation	0.0118	0.0151	0.0159	0.0078	0.0113	0.0116	0.0074
Calculated condensation	0.0148	0.0142	0.0136	0.0092	0.0089	0.0080	0.0067
Indicated horse-power	4.10	4.34	4.78	7.02	6.67	7.71	8.81
Steam per H.P. per hour, pounds	26.8	29.1	29.5	23.8	27.1	26.9	23.8

The platinum thermometer near the cylinder-head showed superheating throughout compression, thus confirming our idea that steam can be treated as dry and saturated at the beginning of compression. This same thermometer fell rapidly during admission and showed saturation practically up to cut-off, as of course it should; after cut-off it began again to show a temperature higher than that due to the indicated pressure, which shows that the cylinder-head probably evaporated all the moisture from its surface soon after cut-off. If this conclusion is correct, there would appear to be little advantage from steam-jacketing a cylinder-head, a conclusion which is borne out by tests on the experimental engine at the Massachusetts Institute of Technology.

The following table gives the areas, temperatures, and the heat absorbed during a given test by the various surfaces exposed to steam at the end of the stroke, i.e., the clearance surface.

TABLE VIII.

CYCLICAL HEAT-ABSORPTION FOR CLEARANCE SURFACES.

Portions of surface considered.	Area of surface, square feet.	Mean temperature, F.	Heat absorbed B.T.U. per minute.
Cover face, 10.5 inches diameter . .	0.60	305	68
Cover side, 3.0 inches	0.70	305	79
Piston face, 10.5 inches diameter. . .	0.60	295	110
Piston side, 0.5 inch	0.11	295	20
Barrel side, 3.0 inches	0.71	297	123
Counterbore, 0.5 inch	0.12	291	28
Ports and valves	0.90	305	102
Sums and means	3.74	301	530

The heat absorbed by the side of the cylinder wall uncovered by the piston up to 0.25 of the stroke was estimated to be 55 B.T.U. per minute, which added to the above sum gives 585 B.T.U.; from which it appears that 90 per cent of the condensation is chargeable to the clearance surfaces, which were exceptionally large for this type of engine. Further inspection shows that the condensation on the piston and the barrel is much more

energetic than on the cover or head. For example, the face of the piston absorbs 110 B.T.U., while the face of the cover absorbs only 68 B.T.U., and the sides of the cover and of the barrel, each 3 inches long, absorb 79 and 123 B.T.U. respectively. This relatively small action of the surface of the head indicates in another form that less gain is to be anticipated from the application of a steam-jacket to the head than to the barrel of a steam-engine.

The exposed surfaces at the side of the cylinder-head and the corresponding side of the barrel are due to the use of a deeply cored head which protrudes three inches into the counter-bore of the cylinder, and which has the steam-tight joint at the flange of the head. It would appear from this that a notable reduction of condensation could be obtained by the simple expedient of making a thin cylinder-head.

Leakage of Valves.—Preliminary tests when the engine was at rest showed that the valve and piston were tight. The valve was further tested by running it by an electric motor when the piston was blocked, the stroke of the valve being regulated so that it did not quite open the port, whereupon it appeared that there was a perceptible but not an important leak past the valve into the cylinder. There was also found to be a small leakage past the piston from the head to the crank end.

But the most unexpected result was the large amount of leakage past the valve from the steam-chest into the exhaust. This was determined by blocking up the ports with lead and running the valve in the normal manner by an electric motor. This leakage appeared to be proportional to the difference of pressure causing the leak, and to be independent of the number of reciprocations of the valve per minute. From the tests thus made on the leakage to the exhaust, the leakage correction in Table VII was estimated. Although the investigators concluded that their experimental rate of leakage was quite definite, it would appear that much of the discrepancy between the indicated and calculated condensation and vaporization can be attributed to this correction, which was two or three times as large as the

weight of steam passing through the cylinder. Under the most favorable condition (for the seventh test) the leakage was 0.0494 of a pound per stroke, and since there were 97 strokes per minute, it amounted to

$$0.0494 \times 97 \times 60 = 287.5$$

pounds per hour, or 32.6 pounds per horse-power per hour, so that the steam supplied per horse-power per hour amounted to 56.4 pounds. If it be assumed that the horse-power is proportional to the number of revolutions, then the engine running double-acting will develop about 44 horse-power, and the leakage then would be reduced to 6.5 pounds per horse-power per hour. Such a leakage would have the effect of increasing the steam-consumption from 23.5 to 30 pounds of steam per horse-power per hour.

To substantiate the conclusions just given concerning the leakage to the exhaust, the investigators made similar tests on the leakage of the valves of a quadruple-expansion engine, which had plain unbalanced slide-valves. The valves chosen were the largest and smallest; both were in good condition, the largest being absolutely tight when at rest. Allowing for the size and form of the valve and for the pressure, substantially identical results were obtained.

The following provisional equation is proposed for calculating the leakage to the exhaust for slide-valves:

$$\text{leakage} = \frac{kep}{l},$$

where l is the lap and e is the perimeter of the valve, both in inches, and p is the pressure in pounds in the steam-chest in excess of the exhaust-pressure. The value of the constant in the above equation is 0.021 for the high-speed engine used by Callendar and Nicolson, and is 0.019 for one test each of the valves for the quadruple engine, while another test on the large valve gave 0.021.

This matter of the leakage to the exhaust is worthy of further investigation. Should it be found to apply in general to slide-valve and piston-valve engines it would go far towards explaining the superior economy of engines with separate admission- and exhaust-valves, and especially of engines with automatic drop-cut-off valves which are practically at rest when closed. It may be remarked that the excessive leakage for the engine tested appears to be due to the size and form of valves. The valve was large so as to give a good port-opening when the cut-off was shortened by the fly-wheel governor, and was faced off on both sides so that it could slide between the valve-seat and a massive cover-plate. The cover-plate was recessed opposite the steam-ports, and the valve was constructed so as to admit steam at both faces; from one the steam passed directly into the cylinder, and from the other it passed into the cover-plate and thence into the steam-port. This type of valve has long been used on the Porter-Allen and the Straight-line engines; the former, however, has separate steam- and exhaust-valves. Such a valve has a very long perimeter which accounts for the very large effect of the leakage.

Callendar and Nicolson consider that the leakage is probably in the form of water which is formed by condensation of steam on the surface of the valve-seat uncovered by the valve, and say further, that it is modified by the condition of lubrication of the valve-seat, as oil hinders the leakage.

CHAPTER XII.

ECONOMY OF STEAM-ENGINES.

In this chapter an attempt is made to give an idea of the economy to be expected from various types of steam-engines and the effects of the various means that are employed when the best performance is desired.

Table X gives the economy of various types of engines, and represents the present state of the art of steam-engine construction. It must be considered that in general the various engines for which results are given in the table were carefully worked up to their best performance when these tests were made. In ordinary service these engines under favorable conditions may consume five or ten per cent more steam or heat; under unfavorable conditions the consumption may be half again or twice as much.

All the examples in the table are taken from reliable tests; a few of these tests are stated at length in the chapter on the influence of the cylinder walls; others are taken from various series of tests which will be quoted in connection with the discussion of the effects of such conditions as steam-jacketing and compounding; the remaining tests will be given here, together with some description of the engines on which the tests were made. These tables of details are to be consulted in case fuller information concerning particular tests is desired.

The first engine named in the table is at the Chestnut Hill pumping-station for the city of Boston. Its performance is the best known to the writer for engines using saturated steam. Some engines using superheated steam have a notably less steam-consumption; but the heat-consumption, which is a better criterion of engine performance for such tests, is little if any better. The first compound engine for which results are given, used 9.6

TABLE X.

EXAMPLES OF STEAM-ENGINE ECONOMY.

Type of Engine.	Revolutions per minute.	Steam-pressure. Pounds per square inch.	Horse-power.	Steam per horse-power per hour. Pounds.	B.T.U. per horse-power per min.	Coal per horse-power per hour. Pounds.
Triple-expansion engines:						
Leavitt pumping-engine at Chestnut Hill	50.6	176	576	11.2	204	1.15
Sulzer mill-engine at Augsburg	56	149	1823	11.3	...	1.19
Experimental engine at the Massachusetts Institute of Technology	92	147	125	13.7	231	...
Marine engine *Iona*	61	165	645	13.4	...	1.46
Marine engine *Meteor*	72	145	1994	15.0	...	2.01
Marine engine *Brookline*	94	154	1136	15.5	263	2
Compound engines:						
Horizontal mill-engine:						
superheated	128	135	115	9.6	199	...
saturated	127	135	127	11.8	213	...
Leavitt pumping-engine at Louisville	18.6	137	643	12.2	222	...
Marine engine *Rush*	71	69	266	18.4	...	2.45
Marine engine *Fusi Yama*	56	57	371	21.2	...	2.66
Simple engines, condensing:						
Corliss engine at Creusot	60	84	176	16.9	...	...
Corliss engine without jacket	59	61	150	18.1	...	...
Harris-Corliss engine at Cincinnati	76	96	145	19.4	...	...
Marine engine *Gallatin*	51	65	260	22	...	...
Simple engines, non-condensing:						
Corliss engine at Creusot	63	104	237	21.5	...	...
Corliss engine without jacket	61	78	209	24.2	...	...
Harris-Corliss engine at Cincinnati	76	96	120	23.9	...	...
Harris-Corliss engine at the Massachusetts Institute of Technology	61	77	16	33.5	548	...
Direct-acting steam-pumps:						
Fire-pump at the Massachusetts Institute of Technology	*90	47	41	67	1110	...
at reduced power	*50	59	6.8	125	2070	...
Steam- and feed-pump on the *Minneapolis*	*11	...	8.8	91	...	...
at reduced power	*2.6	...	1.6	243	...	...

pounds of steam and 199 B.T.U. per minute, the gain being hardly more than the variation that might be attributed to difference in apparatus, etc. The Chestnut Hill engine, which was de-

* Strokes per minute.

signed by Mr. E. D. Leavitt, has three vertical cylinders with their pistons connected to cranks at 120°. Each cylinder has four gridiron valves, each valve being actuated by its own cam on a common cam-shaft; the cut-off for the high-pressure cylinder is controlled by a governor. Steam-jackets are applied to the heads and barrels of each cylinder, and tubular reheaters are placed between the cylinders. Steam at boiler-pressure is supplied to all the jackets and to the tubular reheaters.

TABLE XI.

TRIPLE-EXPANSION LEAVITT PUMPING-ENGINE AT THE CHESTNUT HILL STATION, BOSTON, MASSACHUSETTS.

CYLINDER DIAMETERS 13.7, 24.375, AND 39 INCHES; STROKE 6 FEET.

By Professor E. F. MILLER, *Technology Quarterly*, vol. ix, p. 72.

Duration, hours	24
Total expansion	21
Revolutions per minute	50.6
Steam-pressure above atmosphere, pounds per square inch	175.7
Barometer, pounds per square inch	14.9
Vacuum in condenser, inches of mercury	27.25
Pressure in high and intermediate jacket and reheaters, pounds per square inch	175.7
Pressure in low-pressure jacket, pounds per square inch	99.6
Horse-power	575.7
Steam per horse-power per hour, pounds	11.2
Thermal units per horse-power per minute	204.3
Thermal efficiency of engine, per cent	20.8
Efficiency for non-conducting engine, per cent	28.0
Ratio of efficiencies, per cent	74
Coal per horse-power per hour, pounds	1.146
Duty per 1,000,000 B.T.U.	141,855,000
Efficiency of mechanism, per cent	89.5

The Sulzer engine at Augsburg has four cylinders in all, a high-pressure, an intermediate, and two low-pressure cylinders. The high-pressure cylinder and one low-pressure cylinder are in line, with their pistons on one continuous rod, and the intermediate

cylinder is arranged in a similar way with the other low-pressure cylinder. The engine has two cranks at right angles, between which is the fly-wheel, grooved for rope-driving. Each cylinder has four double-acting poppet-valves, actuated by eccentrics, links, and levers from a valve-shaft. The admission-valves are controlled by the governors. Four tests were made on this engine, as recorded in Table XII.

Table XII.

TRIPLE-EXPANSION HORIZONTAL MILL-ENGINE.

CYLINDER DIAMETERS 29.9, 44.5, AND TWO OF 51.6 INCHES; STROKE 78.7 INCHES.

Built by Sulzer of Winterthur, *Zeitschrift des Vereins Deutscher Ingenieure*, vol. xl, p. 534.

	I	II	III	IV
Duration, minutes	306	322	272	327
Revolutions per minute	56.23	56.28	56.18	56.18
Steam-pressure, pounds per square inch	145.4	147.9	148.4	149.0
Vacuum, inches of mercury	27.24	27.20	27.20	27.19
Horse-power	1872	1835	1850	1823
Steam per horse-power per hour, pounds	11.53	11.49	11.49	11.33
Mean for four tests 11.46				
Coal per horse-power per hour, pounds	1.37	1.36	1.29	1.19
Mean for four tests 1.30				
Steam per pound of coal	8.78	8.49	8.97	9.62

The test on the experimental engine at the Massachusetts Institute of Technology is quoted here because its efficiency and economy are chosen for discussion in Chapter VIII. Taking its performance as a basis, it appears on page 148 that with 150 pounds boiler-pressure and 1.5 pounds absolute back-pressure such an engine may be expected to give a horse-power for 11.5 pounds of steam, from which it appears that under the same conditions its performance compares favorably with the Sulzer engine or even the Leavitt engine.

TABLE XIII.

MARINE-ENGINE TRIALS.

By Professor ALEXANDER B. W. KENNEDY, *Proc. Inst. Mech. Engrs.*, 1889–1892; summary by Professor H. T. BEARE, 1894, p. 33.

	Fusi Yama.	Colchester.	Ville de Douvres.	Meteor.	Iona.
Triple or compound	C.	C.	C.	T.	T.
Diameter high-pressure cylinder, inches	27.4	30	50.1	29.4	21.9
Diameter intermediate cylinder, inches	....	...	...	44	34
Diameter low-pressure cylinder, inches	50.3	57	97.1	70.1	57
Stroke, inches	33	36	72	48	39
Duration of trial, hours	14	10.9	9	17	16
Number of expansions	6.1	6.1	5.7	10.6	19.0
Revolutions per minute	55.6	86	36	71.8	61.1
Steam-pressure above atmosphere, pounds per square inch	56.8	80.5	105.8	145.2	165
Pressure in condenser, absolute, pounds per square inch	2.32	2.51	4.72	2.73	0.70
Back-pressure, absolute, pounds per square inch	3.8	3.4	6.0	3.3	1.8
Horse-power	371	1022	2977	1994	645
Steam per horse-power per hour, pounds	21.2	21.7	20.8	15.0	13.4
Thermal units per horse-power per minute	380	398	367	265	250
Coal per horse-power per hour, pounds	2.66	2.9	2.3	2.01	1.46
Steam evaporated per pound of coal	7.96	7.49	8.97	7.46	9.15
Weight of machinery per horse-power, pounds	603	448	272	439	701

The engines of the S. S. *Iona* have an unusually large expansion and give a correspondingly good economy. The engines of the *Meteor* and of the *Brookline* give the usual economy to be expected from medium-sized marine engines. Table XIII gives details of tests on the engines of the first two ships mentioned, together with tests on compound marine engines. Table XIV gives tests on the engine of the *Brookline*. It appears probable that the relatively poor economy of marine engines compared with stationary engines is due to the smaller degree of expansion, which is accepted to avoid using large and heavy engines.

Table XIV.

TESTS ON THE ENGINE OF THE S. S. *BROOKLINE.*

CYLINDER DIAMETERS 23, 35, AND 57 INCHES; STROKE 36 INCHES.
By F. T. Miller and R. G. B. Sheridan, Thesis, 1895, M.I.T.

	I	II	III	IV	V
Duration, hours	2	2	1	$3\frac{1}{2}$	$2\frac{1}{2}$
Revolutions per minute	94.6	93.6	93.6	93	93
Steam-pressure, pounds per square inch above atmosphere	155	155	154	145	148
Vacuum, inches of mercury	21.6	21.0	22.2	21.7	20.9
Horse-power	1242	1221	1136	1137	1148
Steam per horse-power per hour, pounds	17.2	16.9	15.5	17.0	16.3
Coal per horse-power per hour, pounds	2.22	2.17	1.99	2.18	2.09
B.T.U. per horse-power per minute	292	288	263	288	277

The horizontal mill-engine which heads the list of compound-engines in Table X, is a tandem engine for which particulars are given in Table XXVI on page 273. Its performance with superheated steam is the best among the engines named, and with saturated steam is a trifle superior to that of the Louisville engine.

Table XV.

COMPOUND LEAVITT PUMPING-ENGINE AT LOUISVILLE, KENTUCKY.

CYLINDER DIAMETERS 27.2 AND 54.1 INCHES; STROKE 10 FEET.
By F. W. Dean, *Trans. Am. Soc. Mech. Engrs.*, vol. xvi, p. 169.

Duration, hours	144
Revolutions per minute	18.6
Pressures, pounds per square inch:	
Barometric	14.6
Boiler above atmosphere	140
At engine above atmosphere	137
Back-pressure, l.p. cylinder	0.95
Total expansions	20
Moisture in steam, per cent	0.55
Horse-power	643.4
Steam per horse-power per hour, pounds	12.2
B.T.U. per horse-power per minute	222
Thermodynamic efficiency, per cent	19
Mechanical efficiency, per cent	93

This engine has two cylinders, each jacketed with steam at boiler-pressure on barrels and heads, and steam at the same pressure is used in a tubular reheater. Each cylinder has four gridiron valves actuated by as many cams on a cam-shaft.

TABLE XVII.

ENGINES OF THE U. S. REVENUE STEAMERS *RUSH* AND *GALLATIN*.

	Rush.	Gallatin.
Diameters of cylinders, inches	24 and 38	34.1
Stroke, inches	27	30
Duration, hours	55	24
Revolutions per minute	71	51
Steam-pressure by gauge, pounds	69.1	65.4
Vacuum, inches of mercury	26.5	25.1
Total expansions	6.2	4.5
Horse-power	266.5	260.5
Steam per horse-power per hour, pounds	18.4	22

The details of the tests on the U. S. Revenue Steamers *Rush* and *Gallatin* are given in Table XVII, as made about 1875 by a board of naval engineers to determine the advantages of compounding and using steam-jackets. Three other engines were tested at the same time, but they were of older types and are less interesting.

A remarkably complete and important series of tests was made in 1884 by M. F. Delafond. These tests are recorded in Tables XXX and XXXI, from which there are quoted in Table X four results with and without condensation and with and without steam in the jackets.

TABLE XVIII.

AUTOMATIC CUT-OFF ENGINES.

CYLINDER DIAMETERS 18 INCHES; STROKE 4 FEET.

By J. W. HILL.

(First Millers' International Exhibition, Cincinnati, 1880.)

	Condensing.			Non-condensing.		
	R.	H.	W.	R.	H.	W.
Duration	10	10	10	9	10	10
Cut-off	0.124	0.119	0.131	0.160	0.136	0.170
Revolutions per minute	75.4	75.8	74.5	75.3	75.8	76.1
Boiler-pressure above atmos., lbs. per sq. in.	95.8	96.1	96.3	96.6	96.3	96.3
Barometer, inches of mercury	29.7	29.6	29.4	29.8	29.6	29.5
Vacuum, inches of mercury	25.5	25.7	24.0	...	...	...
Back-pressure, absolute, lbs. per sq. in.	4.5	3.4	4.7	15.5	14.0	15.5
Horse-power	143.2	145.1	143.9	121.7	119.7	126.7
Steam per horse-power per hour, pounds	20.6	19.4	19.5	25.9	23.9	24.9
B.T.U. per horse-power per hour	372	349	343	433	400	415

The details of the tests on the Harris-Corliss engine at Cincinnati, together with tests on two similar engines, are given in Table XVIII.

Table XIX.

DUPLEX DIRECT-ACTING FIRE-PUMP AT THE MASSACHUSETTS INSTITUTE OF TECHNOLOGY.

TWO STEAM-CYLINDERS 16 INCHES DIAMETER, 12 INCHES STROKE.

Technology Quarterly, vol. viii, p. 19.

Single strokes per minute.	Length of stroke. West.	Length of stroke. East.	Steam-pressure by gauge.	Horse-power. Steam-cylinders.	Horse-power. Water-cylinders.	Steam per horse-power per hour.	B.T.U. per horse-power per minute.	Duty. (Foot-pounds per 1,000,000 B.T.U.)
99	11.40	10.10	58.5	6.78	...	125	2070	13,920,000
114	11.70	11.07	55.6	12.48	...	101	1674	17,540,000
119	11.49	11.07	51.4	12.18	...	109	1809	16,980,000
135	11.60	11.10	53.8	18.24	...	92	1530	19,850,000
156	10.90	10.26	47.2	21.00	19.80	98	1619	18,280,000
193	10.09	10.31	45.6	32.95	...	78	1291	23,730,000
175	11.77	11.79	45.6	39.55	...	66	1083	27,980,000
180	11.74	11.66	46.5	41.20	...	67	1110	27,030,000

Table XX.

TESTS OF AUXILIARY STEAM MACHINERY OF THE U. S. S. *MINNEAPOLIS.*

By P. A. Engineer W. W. White, U. S. N., *Journal Am. Soc. Naval Engrs.*, vol. x.

Engine or pump tested.	Number of steam-cylinders.	Diam. of steam-cylinders, inches.	Diam. of water-cylinders, inches.	Nominal stroke, inches.	Actual stroke, inches.	Double strokes or revolutions per minute.	Duration of test.	Indicated horse-power.	Steam per horse-power per hour.
Centre circulating-pump:									
Full power	2	10	36	6	6	171.6	3–7	18.9	95
Reduced power*	2	10	36	*6	6	90	2–50	4.1	76
Starboard circulating-pump:									
Reduced power	2	10	36	6	6	82	3–28	2.0	125
Starboard air-pump	2	16	31.5	21	...	16.6	2–58	6.5	183
Centre air-pump †	2	16	1.5	21	...	15.2	3–2	25.2	78
Water-service pump	2	7.5	4.5	10	7.5	40.9	2–59	1.04	205
Fire- and feed-pump	2	12	8.5	12	10.9	12.7	3–31	0.78	319
Do.	2	12	8.5	12	12.0	37.3	1–46	6.4	156
Do.	2	12	7.5	12	10	11.0	2–23	8.8	91
Do.	2	12	7.5	12	10.8	2.6	3–27	1.6	243
Fire-and bilge-pump	2	14	9	12	11.2	27.7	2–2	2.5	171
Blower-engine	2	5	...	4	4	595	1–24	16.3	77
Dynamo-engine	2	10.5	...	5	5	425	1–10	22.9	65
Do.	2	10.5	...	5	5	425	0–26	35.2	56
Ice-machine engine	1	7	...	10	10	73.1	5–12	6.0	70

* One cylinder only supplied with steam.

† Pump loaded with three times the power developed during official trial, when main engine indicated 7219 H.P.

The two tests on the direct-acting fire-pump at the Massachusetts Institute of Technology are taken from Table XIX, and the tests on the feed- and fire-pump on the *Minneapolis* are given in Table XX. Both sets of tests show the extravagant consumption of steam by such pumps when running at reduced powers. The latter table is most interesting on account of the light that it throws on the way that coal is consumed by a war-vessel when cruising at slow speeds or lying in harbor.

Methods of Improving Economy. — The least expensive type of engine to build is the simple non-condensing engine with slide-valve gear; this type is now used only where economy is of little importance, or where simplicity is thought to be imperative. Starting with this as the most wasteful type of engine, improvements in economy may be sought by one or more of the following devices:

1. Increasing steam-pressure.
2. Condensing.
3. Increasing size.
4. Expansion.
5. Compounding.
6. Steam-jackets.
7. Superheating.
8. The binary engine.

An investigation of the conditions under which these various devices can be used to advantage, of the gain to be expected, and of their limitations, is one of the most interesting and important problems for the engineer. For the student the process of such an investigation is even of more importance than the conclusions, because by it he may learn to form his own opinions and may take account of other tests as they may be presented. The order chosen is to some extent arbitrary, and cannot be adhered to strictly, as the tests on which the investigation is based were made for various purposes, and combine the several devices in various manners.

Of these devices the first two and the last are clearly methods of extending the temperature-range, and are indicated directly

by the ideas that have been presented in the general discussion of thermodynamics, and in particular by the adiabatic theory of the steam-engine; the fourth (expansion) may almost be included in this category as a means of making the extension of temperature-range effective. It has been seen that the necessity of making the cylinder of metal which is a good conductor and has an energetic action on the steam in the cylinder, interferes with our attempts to approach the efficiency that can be computed for non-condensing engines, and places limitations on the advantages to be gained by increasing the temperature-range. The other devices enumerated (increase of size, compounding, steam-jackets, and superheating) are various methods which have been applied to diminish the influence of the cylinder walls, and allow us to take advantage of a large temperature-range. It appears at first sight that superheating should be included in the first category, as it clearly does increase the temperature-range between the steam-pipe and the exhaust-pipe of the engine, but the steam in the cylinder is seldom superheated at cut-off, and it is better to consider this device as a means of reducing cylinder condensation.

It is interesting to consider that condensation, expansion, and steam-jackets were used by Watt for his earliest engines, and that he was limited in pressure by the condition of the art of engineering, so that there was no occasion for compounding; his cylinders also had considerable size, though the powers of the engines would not now appear to be large. In the course of his development of the true steam-engine from the atmospheric engine, which had the steam condensed in the cylinder by spraying in water, Watt's attention was especially directed to the influence of the cylinder walls; he also made experiments on the properties of saturated steam within the range of available pressures, and had such an appreciation of the conditions of his problem that little was left to his successors except to learn how to use the higher steam-pressures which the developments of metallurgy and machine-shop practice made possible. The fact that our theory of the steam-engine was developed after his time, and

that the theory has sometimes been misapplied, has given an erroneous opinion that the steam-engine has been developed without or in spite of thermodynamics. And further, his use of all the advantages then available has had a tendency to obscure their importance, and makes it the more desirable to state the several methods categorically as given above.

It is now commonly considered that the steam-engine has been brought to full development, and that there is little if any substantial improvement to be expected; in fact, this condition was reached a decade or two ago, when the triple engine using steam at 150 to 175 pounds by the gauge, was perfected. The most recent change is the use of superheated steam at high pressures, now that effective and durable superheaters have been devised. Experiment and experience have settled fairly well the limitations for the various methods of improving economy and allow of a fair and conservative presentation to which there will probably be few exceptions. We will, therefore, state the general conclusions as briefly as may be, and give the tests on which they may be based.

In order to bring out the advantage to be obtained by a certain device, such as compounding, we will compare only the best performance of the simple engine with the best performance of the compound engine, each being given all the advantages that it can use. The fact that marine compound engines have a worse economy than stationary simple-engines, has no other meaning for our present purpose, than that engines on shipboard are subject to unfavorable limitations.

Effect of Raising Steam-Pressure. — A glance at the table on page 148 which gives the efficiency for Carnot's cycle, will show that if we begin with a low steam-pressure, there is a large advantage from increasing the pressure and consequently the temperature-range, but that this advantage becomes progressively less marked. This conclusion is of course immediately evident from the efficiency for Carnot's cycle, which may be written

$$e = \frac{T - T'}{T}.$$

If t' is taken to be 100° F., and if t is made successively 200°, 300°, and 400°, the values of the efficiency are 0.15, 0.26, and 0.35. But the influence of the cylinder quickly puts a stop to this improvement unless we resort to compounding, as will be seen by studying Delafond's tests in Table XXI, page 250, and by Figs. 57 and 58 on pages 252 and 253, in which the steam-consumption is plotted as ordinates on the fraction of the stroke at cut-off, each curve being lettered with the steam-pressure which was maintained while a series of tests was made. Fig. 57 represents tests without steam in the jackets, and Fig. 58, tests with steam in the jackets. Those curves bearing the letter C were with condensation, and those bearing the letter N were non-condensing. Inspection of Fig. 57 shows a progressive reduction in steam-consumption, as the pressure is increased from 35 pounds by the gauge to 60 pounds for the condensing engine without a steam-jacket, but raising the pressure from 60 pounds to 80 and 100 pounds gives a marked increase in steam-consumption. The same figure indicates that 100 pounds is probably the limit for non-condensing, unjacketed engines. The curves on Fig. 58 are not quite so conclusive; but we may from both figures give the following as the best pressures to be used with simple engines of good design and automatic valve-gear:

Desirable Pressures for Simple Engines.

Condensing, without steam-jackets,	60 pounds gauge.
Condensing, with steam-jackets,	80 pounds gauge.
Non-condensing, without steam-jackets,	100 pounds gauge.
Non-condensing, with steam-jackets,	125 pounds gauge.

Delafond's Tests. — In 1883 an extensive and important investigation was made by Mons. F. Delafond on a horizontal Corliss engine at Creusot to determine the conditions under which the best economy can be obtained for such an engine. The engine had a steam-jacket on the barrel, but was not jacketed on the ends. Steam was supplied to the jacket by a branch from the main steam-pipe, and the condensed water was drained through a steam-trap into a can, so that the amount of steam

used in the jacket could be determined. The engine was tested with and without steam in the jacket, both condensing and non-condensing, and at various pressures from 35 to 100 pounds above the pressure of the atmosphere. The effective power and the friction of the engine were also obtained by aid of a friction-brake on the engine-shaft.

The piping for the engine was so arranged that steam could be drawn either from a general main steam-pipe or from a special boiler used only during the test. Before making a test the engine, which had been running for a sufficient time to come to a condition of thermal equilibrium, was supplied with steam from the general supply. At the instant for beginning the test the general supply was shut off and steam was taken from the special boiler during and until the end of the test, and then the pipe from that boiler was closed. The advantage of this method was that at the beginning and end of the test the water in the boiler was quiescent and its level could be accurately determined. At the end of a test the water-level was brought to the height noted at the beginning. The water required for feeding the special boiler during the test and for adjusting the water-level at the end was measured in a calibrated tank. As the steam-pressure in the general-supply main and in the special boiler was the same, there was little danger of leakage through the valves for controlling the steam-supply; the regularity and consistency of results shown by the curves of Figs. 57 and 58 attest to the skill and accuracy with which these tests were made.

Table XXI gives the results of tests made with condensation, and Table XXII gives the results of tests without condensation. All the tests both with and without condensation, but during which no steam was used in the jackets, are represented by the several curves of Fig. 57, while Fig. 58 represents tests made with steam in the jackets. The curves are lettered to show the mean steam-pressure for the series represented and the condition, whether with or without condensation. Thus on Fig. 57 the lowest curve 60*C* represents tests made without steam in the jackets and with condensation, while the highest curve on Fig.

58 represents tests with steam in the jackets and without condensation, at 50 pounds boiler-pressure. The abscissæ for the curves are the per cents of cut-off, and the ordinates are the steam-consumptions in pounds per horse-power per hour. The

TABLE XXI.

HORIZONTAL CORLISS ENGINE AT CREUSOT.

CYLINDER DIAMETER 21.65 INCHES; STROKE 43.31 INCHES; JACKET ON BARREL ONLY; CONDENSING.

BY F. DELAFOND, *Annales du Mines*, 1884.

Number of test.	Duration, minutes.	Revolutions per minute.	Cut-off in per cent of stroke.	Steam-pressure, pounds per sq. in.	Vacuum, inches of mercury.	Steam used in jacket, per cent.	Indicated horse-power.	Steam per horse-power per hour, pounds.
1	60	60.0	4	96.3	27.1	...	109	23.2
2	105	58.6	6	98.8	27.1	...	128.5	22.2
3	75	59.4	9	100	27.0	...	161	21.4
4	36	57.7	12.5	99.1	27.0	...	186	22.0
5	73	58.8	5.5	104	27.4	?	141	17.1
6	55	61.5	6.7	102.4	27.1	?	159.5	16.7
7	80	59.9	6.7	103.8	27.4	2.9	155	16.5
8	39	58.1	12.5	105.2	26.8	3.2	212	17.6
9	120	59.8	7.5	79.8	27.1	...	126	21.2
10	100	59.3	8.3	81.1	27.4	...	134	21.1
11	90	59.8	10.5	80.1	27.1	...	150	20.8
12	55.5	58.0	14	85.5	27.1	...	175	19.9
13	50	59.1	18	84.8	26.5	...	194	20.4
14	94	59.6	5	85.1	27.4	3.0	112	17.7
15	102	59.6	5.5	83.3	27.6	3.1	124	17.3
16	40	59.4	11.5	84.1	27.1	1.2	176	16.9
17	40	60	14	84.1	27.0	1.5	193	17.5
18	91	58.3	5.9	60.5	28.0	...	85.3	20.4
19	90	59.5	9	55.8	27.6	...	115	19.1
20	75	59.0	15.5	61.2	27.8	...	150	18.1
21	75	58.3	22.7	58.3	27.6	...	172	18.4
22	31	59.2	25	61.2	27.1	...	186	18.8
23	115	59.9	6	59.9	27.8	2.5	91.7	18.5
24	92	59.6	9	59.9	27.4	2.5	117	17.6
25	90	58.8	15.5	60.9	27.1	1.8	150	17.3
26	71	59.1	20	61.9	26.8	1.5	175	17.7
27	50	59.0	25	62.3	26.4	1.6	194	18.6
28	70	60.7	6	45.0	28.0	...	75.6	20.7
29	80	58.8	9.5	48.9	28.1	...	94.3	19.4
30	111	60.4	15	47.9	27.6	...	120	18.8
31	54	58.8	21	47.8	27.6	...	140	19.0
32	55	59.4	29	47.6	27.1	...	165	19.8
33	98	60.3	5	45.8	28.0	2.6	68.8	19.3
34	63	57.6	10	51.6	27.6	2.3	95.5	18.5
35	60	59.7	14.3	49.1	28.1	1.4	120	18.2
36	74	60.1	22	48.6	27.8	1.4	152	18.9
37	50	59.5	29	50.2	26.8	1.2	179	19.7
38	85	60.3	18.2	33.1	27.8	...	106	20.5
39	68	61.1	43	34.7	26.5	...	160	22.7
40	42.5	61.0	56.7	36.3	26.0	...	181	25.3
41	20	60.0	100	31.7	25.2	...	182	35.9
42	73	60.7	19	32.0	27.6	1.6	111	19.8
43	80	61.9	42	33.0	26.5	1.1	162	22.1
44	40	61.1	58	35.1	26.0	0.6	180	25.4
45	25	60.4	100	34.7	25.2	0.2	199	33.0

results for individual tests are represented by dots, through which or near which the curves are drawn. As there are only a few tests in any series, a fair curve representing the series can be drawn through all the points in most cases. The exceptions

TABLE XXII.

HORIZONTAL CORLISS ENGINE AT CREUSOT.

CYLINDER DIAMETER 21.65 INCHES; STROKE 43.31 INCHES; JACKET ON BARREL ONLY; NON-CONDENSING.

By F. DELAFOND, *Annales des Mines*, 1884.

Number of test.	Duration, minutes.	Revolutions per minute.	Cut-off in per cent of stroke.	Steam-pressure, pounds per square inch.	Steam used in jacket, per cent.	Indicated horse-power.	Steam per horse-power per hour, pounds.
1	78	61.7	13	96.3	...	147.5	28.4
2	55	61.4	17	100.2	...	181.5	26.8
3	25	63.6	20	102.0	...	217	25.8
4	80	60.8	11	98.1	2.5	143	22.8
5	60	62.0	13	103.8	3.4	177.5	22.1
6	36	62.0	16	103.0	3.1	194	22.4
7	30	62.7	20	103.5	2.0	237	21.5
8	66	62.0	15.5	73.7	...	121	27.6
9	60	60.9	18	77.0	...	136	26.7
10	60	60.0	24.5	76.7	...	178	24.6
11	30	60.6	32	77.5	...	209	24.2
12	70	61.1	16.5	77.0	1.7	137	23.7
13	50	61.6	23.5	75.8	1.2	180	21.8
14	30	60.5	30	78.0	1.3	204	22.0
15	71	61.4	24.5	50.8	...	108	27.3
16	70	61.1	37	51.2	...	147	27.2
17	50	60.9	58	50.5	...	173	30.2
18	25	6c.6	100	34.9	...	145	46.8
19	70	60.5	23	52.6	1.5	108	25.3
20	60	60.5	34	51.8	1.1	141.5	25.2
21	50	60.3	58	46.2	0.7	168.5	28.7
22	30	61.1	100	33.7	0 3	147.5	46.3

are tests made with condensation for boiler-pressure of 80 and 100 pounds per square inch. The forms of the curves 80*C* and 100*C*, Fig. 57, were made to correspond in a general way to the curves 50*C* and 60*C*. The discrepancies appear large on account of the large scale for ordinates, but they are not really of much importance; the largest deviation of a point from the curve 100*C* is half a pound out of about 22, which amounts to little more than two per cent. On Fig. 58 the curve 80*C* is drawn through the points, but though its form does not differ

radically from the curves $60C$ and $50C$, so marked a minimum at so early a cut-off is at least doubtful. Considering that the probable error of determining power from the indicator is about

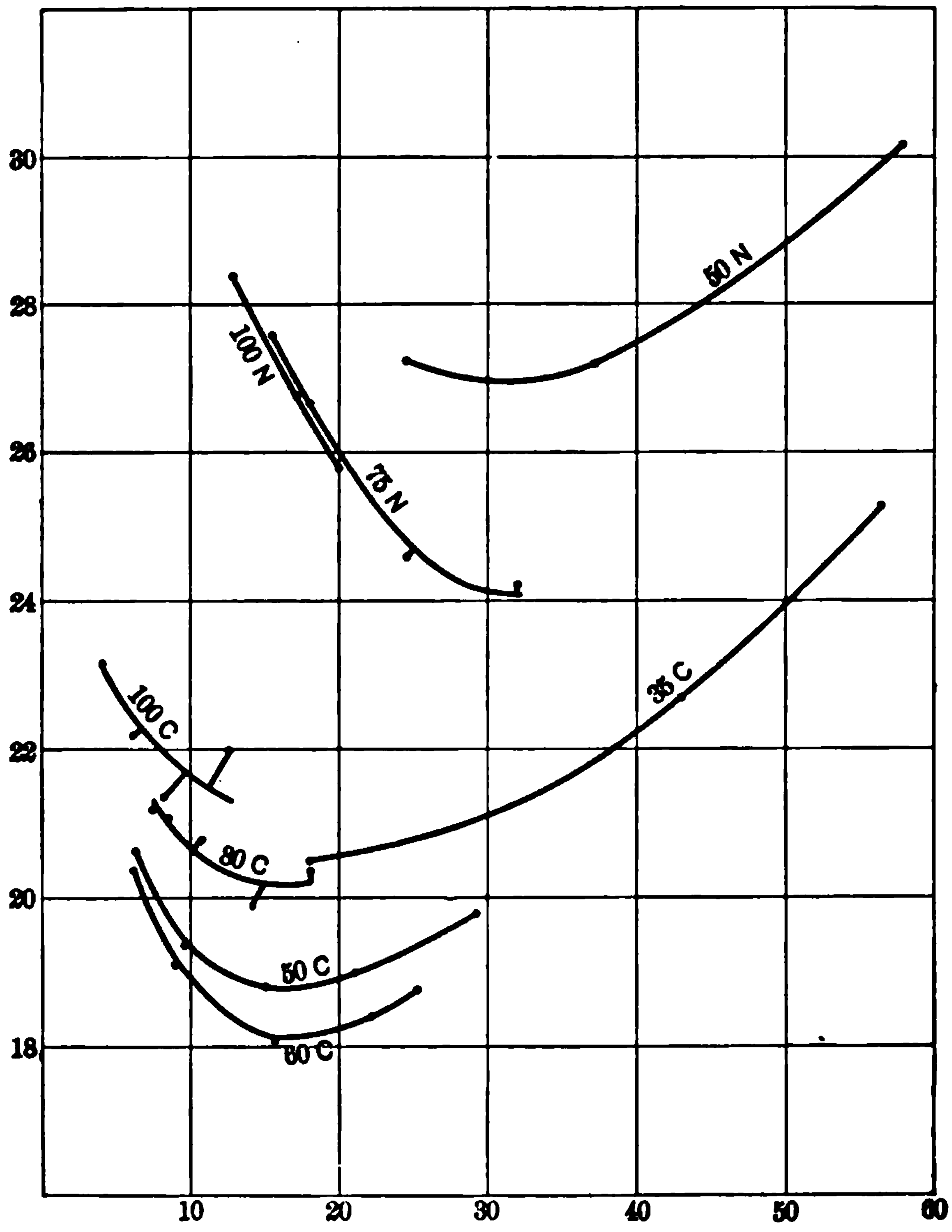

FIG. 57.

two per cent, it would not be difficult to draw an acceptable curve in place of $80C$ which should correspond to the forms of $60C$ and $50C$.

The results of the four tests made with steam in the jacket and with condensation, and which are numbered 5, 6, 7, and 8, in Table XXII, are represented by dots inside of small circles

on Fig. 58. It does not appear worth while to try to draw a curve to represent these tests.

Condensation. — The complement of raising the steam-pressure

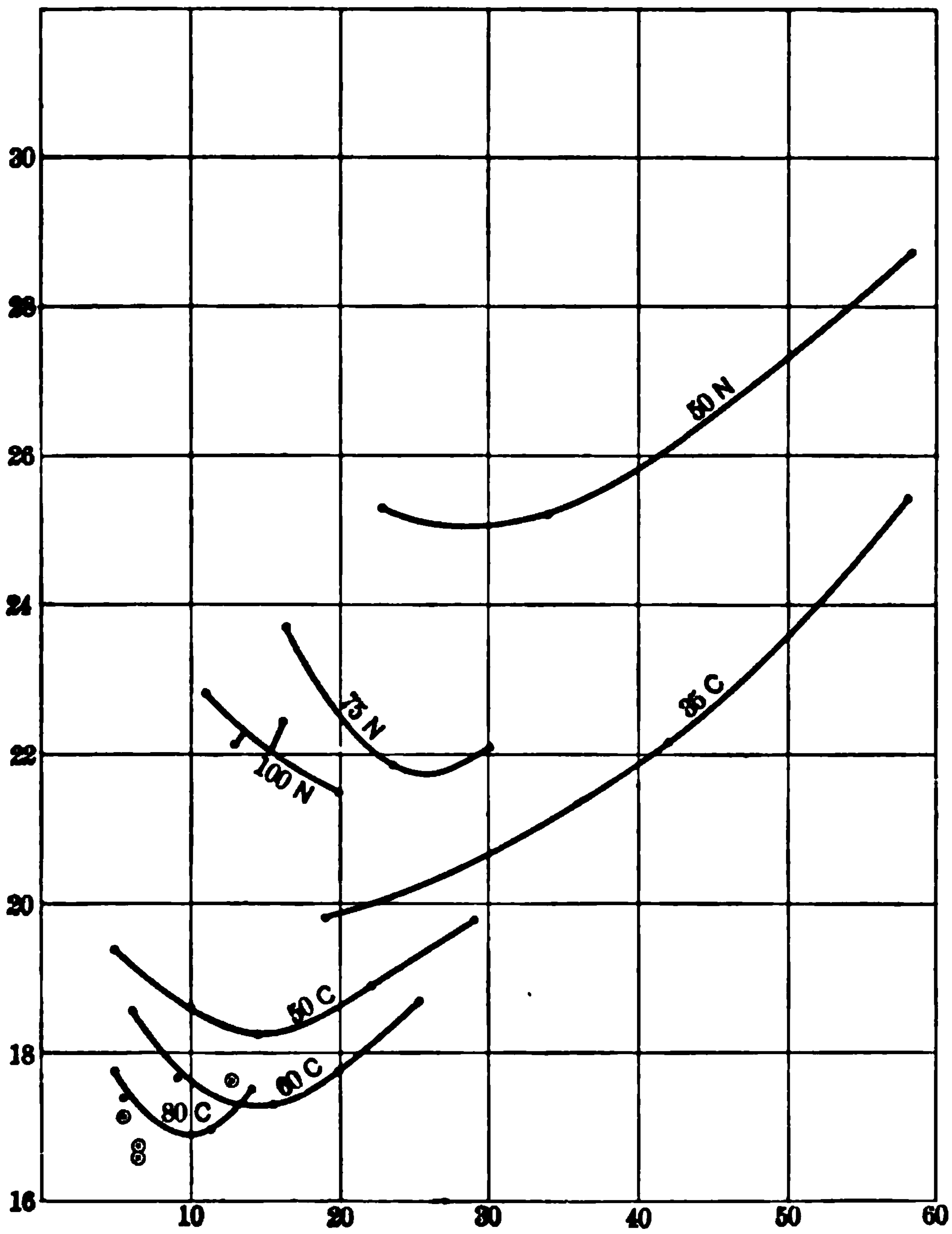

FIG. 58.

is the use of a condenser with a good vacuum. The advantage to be obtained by this means can be determined from Delafond's tests by aid of Figs. 57 and 58; taking the best conditions as already recorded in Table X, the engine without a jacket and without a vacuum used 24.2 pounds of steam per horse-power per hour, and with a vacuum it used 18.1 pounds; with steam in the jackets the results were 21.5 and 16.9. A direct comparison

of either pair of results would appear to give a saving of about 25 per cent, which would be manifestly misleading. The results of brake tests for this engine on page 292, show that the mechanical efficiency when running non-condensing was 0.90, but that it was only 0.82 when running condensing. The steam per brake horse-power per hour can be obtained by dividing the indicated steam by the mechanical efficiency, so that the above pairs of results became for the engine *without* steam in the jacket, non-condensing 26.9, and condensing 22.1, and for the engine *with* steam in the jacket, 23.9 and 20.6; so that the real gain from condensation was

$$\frac{26.9 - 22.1}{26.9} = 0.18 \text{ or } \frac{23.9 - 20.6}{23.9} = 0.14.$$

The gain from condensation will vary with the type of engine and the conditions of service, and may be estimated from ten to twenty per cent. Clearly the gain is greater with a good vacuum than with a poor vacuum. There is, however, another feature which should be considered, namely, the mean effective pressure; when the conditions of service are such that the mean effective pressure is large, the gain from condensation and the advantage of maintaining a good vacuum are not so great as when the mean effective pressure is small. This feature can be best illustrated with examples of triple-expansion engines, which are able to work advantageously with a large total expansion, and for them we may deal with the reduced mean effective pressure, meaning by that expression the result obtained by the following process: the mean effective pressure for the high-pressure cylinder is to be multiplied by the area of that piston and divided by the area of the low-pressure piston; the mean effective pressure for the intermediate cylinder is to be treated in a similar way; the two results are then to be added to the mean effective pressure for the low-pressure cylinder; clearly this sum, which is called the reduced mean effective pressure, if it were applied to the low-pressure piston would develop the actual power of the engine. Now the reduced mean effective

pressure for a pumping-engine or mill-engine may be as low as 18 pounds per square inch, and a difference of one inch of vacuum (or half a pound of back-pressure) will be equivalent to nearly three per cent in the power; on the other hand, a naval engine is likely to have a reduced mean effective pressure of forty pounds per square inch, and compared with it a difference of one inch of vacuum is equivalent to a little more than one per cent. In any case the gain in economy due to a small improvement in vacuum is approximately equal to the reduction in the absolute pressure in the condenser, divided by the reduced mean effective pressure.

A very important matter is brought out in this discussion of the gain from condensation, namely, that the real gain is determined by comparing the engine consumption for the net or brake horse-powers. The only reason for using the indicated power (as is most commonly done) is that the brake-power is often difficult to determine and sometimes impossible. As was pointed out on page 144, a true basis of comparison is the heat-consumption of the engines compared in B.T.U. per horse-power per hour. But that quantity was not determined for the tests by Delafond, and since the comparisons are for two pairs of tests, one pair with and the other without jackets there is no objection to it in the cases discussed.

Increase of Size. — Since the failure to attain the economy computed for the non-conducting engine is due mainly to the action of the cylinder walls, and since the volume of the cylinder are proportional to the cube of a linear dimension, while the surface is only proportional to the square, a great advantage might be expected by simply increasing the size of the engine. Such an advantage is indicated by the comparison of the small Harris-Corliss engine at the Massachusetts Institute of Technology with the Corliss engine at Creusot, the steam-consumption without condensation or steam-jackets being 33.5 pounds and 24.2 pounds per horse-power per hour, and the gain from increase of size being

$$\frac{33.5 - 24.2}{33.5} = 0.28.$$

In this case the larger engine has about twelve times the cylinder capacity of the smaller one. This feature appears to depend on the absolute size of the engine, because, as will appear later, there is little if any advantage in speed of rotation within the usual limits of practice.

But the advantage from increase of size soon reaches a limit, as will be apparent from the consideration that the best results in Table X are for engines of moderate power, judged by modern standards. These engines have the advantages of compounding, and of the use of steam-jackets or superheated steam; the advantages from jacketing or superheating decrease with the size, and such devices are possibly of little advantage to massive engines.

Expansion. — There are two limits to the amount of expansion that can be advantageously used for a given engine: one limit is imposed by the action of the cylinder walls, and the other is imposed by the friction of the engine. Simple engines have the most advantageous point of cut-off determined by the first limit, which can be clearly determined by aid of Delafond's experiments; compound and triple-expansion engines so divide up the temperature-range that any desirable expansion can be employed. The terminal pressure at the end of expansion for a stationary, triple, or compound engine may be made as low as five pounds absolute; and as the back-pressure is likely to be a pound or a pound and a half, so that the terminal effective pressure is three and a half or four pounds, and as it takes about two pounds per square inch to drive the piston and connected parts, there is evidently little to be gained in economy by further expansion.

As for simple engines, an inspection of Figs. 57 and 58 on pages 252 and 253 shows that the best point of cut-off for non-condensing engines is one-third stroke, and for condensing engines about one sixth-stroke; if the engine has a steam-jacket, the cut-off may be a little earlier than one-sixth stroke, but there probably is little advantage from such an increase of expansion if we deal with the net or brake horse-power.

The total expansion for a compound or triple engine can be obtained in two ways: we may use a large ratio of the large cylinder to the small cylinder, or we may use a short cut-off for the high-pressure cylinder. The two methods may be illustrated by the two Leavitt engines mentioned in Table X; the ratio of the large to the small cylinder of the compound engine at Louisville, is a trifle less than four, and the cut-off for the high-pressure cylinder is a little less than one-fifth stroke; on the other hand, the triple engine at Chestnut Hill has a little more than eight for the extreme ratio of the cylinders, and has the cut-off for the high-pressure cylinder at a little more than four-fifths. So large an extreme ratio as eight would not be convenient for a compound engine, but ratios of five or six have been used, though not with the best results.

Marine engines usually have comparatively little total expansion both for compound and for triple engines, and consequently are unable to work with an economy equal to that for stationary engines; the type of valve-gear which the designers feel constrained to use is also little adapted to give the best results. There is some question whether there is not room for improvement in both these directions.

Compounding. — The most efficacious method which has been devised to increase the amount of expansion of steam in an engine, and at the same time to avoid excessive cylinder-condensation, is compounding; that is, passing the steam in succession through two or more cylinders of increasing size. An engine with two cylinders, a small or high-pressure cylinder and a large or low-pressure cylinder, is called a compound engine. An engine with three cylinders, a high-pressure cylinder, an intermediate cylinder, and a low-pressure cylinder, is called a triple-expansion engine. A quadruple engine has a high-pressure cylinder, a first and a second intermediate cylinder, and a low-pressure cylinder. Any cylinder of a compound or multiple-expansion engine may be duplicated, that is, may be replaced by two cylinders which are usually of the same size. Thus, at one time a compound engine with one high-pressure

and two low-pressure cylinders was much used for large steamships. Many triple engines have two low-pressure cylinders, which with the high-pressure and the intermediate cylinders make four in all. Again, some triple engines have two high-pressure cylinders and two low-pressure cylinders and one intermediate cylinder, making five in all.

Two questions arise: (1) Under what conditions should the several types of engines be used? and (2) What gain can be expected by using compound or triple expansion?

Neither question can be answered explicitly.

From tests already discussed and for which the main results are given in Table X, it appears that with saturated steam, the best results were attained with the following pressures: for triple engines about 175 pounds by the gauge, for compound engines 145 pounds, and for simple engines with about 80 pounds, all for engines with condensation. Nearly as good results were obtained for a compound engine with 135 pounds pressure, and on the other hand the simple engine could use 100 pounds with equal advantage. The information concerning the simple engine is sufficient to serve as a reliable guide, but there is at least room for discretion concerning the best pressures for compound and triple engines. There will probably be little chance of serious disappointment if the following table is used as a guide in designing engines, all being with condensation and with steam-jackets.

Best Gauge-Pressures for Steam-Engines.

Simple	80
Compound	140
Triple	175

If for any reason it is desired to use a higher or lower pressure in any case, a variation of 20 pounds either way may be assumed without much loss of efficiency; this, however, cannot be stated quantitatively at the present time.

For non-condensing simple engines the pressure should preferably be 100 pounds without a steam-jacket, and 125

pounds with a steam-jacket; with an allowable variation of twenty pounds. For a non-condensing compound engine we may take as the preferred pressure about 175 pounds, but our tests do not include this case, and the figure is open to question. There is little, if any, occasion for using triple-expansion non-condensing engines.

About ten years ago an attempt was made to introduce quadruple-expansion engines, using steam at about 250 pounds for marine purposes in conjunction with water-tube boilers, which can readily be built for high-pressures; but more recent practice has been to adhere to triple engines even where the designer has chosen a high-pressure for sake of developing a large power per ton of machinery, or for any other purpose.

For convenience in trying to determine the gain from compounding, the following supplementary table has been drawn off.

Data and Results.	Simple Corliss at Creusot.	Compound Mill-Engine.	Triple Leavitt at Chestnut Hill.
Revolutions per minute	60	127	50.6
Steam-pressure above atmosphere, pounds	84	148	176
Total expansion	9	20	21
Steam per horse-power per hour, pounds	16.9	11.8	11.2
B.T.U. per horse-power per minute	...	220	204

Gain from compounding,

$$\frac{16.9 - 11.8}{16.9} = 0.30.$$

Gain from using triple engine in place of simple engine,

$$\frac{16.9 - 11.2}{16.9} = 0.34.$$

Gain from using triple engine in place of compound engine

$$\frac{11.8 - 11.2}{11.8} = 0.05.$$

Compound and triple engines have been found well adapted to marine work, where for various reasons a short cut-off cannot well be used. Taking the engines of the three ships mentioned in the following supplementary table to represent good practice, we can determine the gain from compounding.

Data and Results.	Simple Galatin.	Compound Rush.	Triple Meteor.
Revolutions per minute	51	71	72
Steam pressure by gauge	65	69	145
Total expansion	4.5	6.2	10.6
Steam per horse-power per hour, pounds	22	18.4	15

Gain from compounding,

$$\frac{22 - 18.4}{22} = 0.16.$$

Gain from using triple engine instead of simple engine,

$$\frac{22 - 15}{22} = 0.32.$$

Gain from using triple engine instead of compound engine,

$$\frac{18.4 - 15}{18.4} = 0.18.$$

Two things are to be noted: first, that the total number of expansions is very moderate even for the triple engine; and, second, that the steam-consumption is correspondingly large as compared with that for stationary engines.

A notable exception in marine practice is the engine of the *Iona*, which was relatively much larger than can commonly be placed in a steamer; it had the advantage of 165 pounds steam-pressure and 19 total expansions, and had a steam-consumption of only 13 pounds per horse-power per hour.

Properly the comparison for finding the gain from compounding should be based on thermal units per horse-power per minute, but the data for such a comparison are not given for all the engines, and as all the engines have steam-jackets, the comparison of steam-consumptions is not much in error.

Steam-Jackets. — As has already been pointed out in the discussion of the influence of the cylinder walls, the beneficial action of a steam-jacket is to dry out the cylinder during exhaust, without unduly reducing the temperature of the cylinder walls, and thus check the condensation during admission. The steam-jacket does indeed supply some heat during expansion, but that effect is of secondary importance, and the heat is applied with a thermodynamic disadvantage. The principal effect is thus to supply heat which is thrown out in the exhaust, which is all lost in case of a simple engine; in case of a compound engine the heat supplied by a jacket during exhaust from the high-pressure cylinder is intercepted by the low-pressure cylinder, and is not entirely lost. It would clearly be much more advantageous to make the cylinders of non-conducting material, if that were possible. A clear grasp of the true action of the steam-jacket has a natural tendency to prejudice the mind against that device, and this prejudice has in many cases been strengthened by the confusion that has come from indiscriminate comparison of many tests made to determine the advantage from the use of steam-jackets.

There are two series of tests that appear to dispose of this question, — those by Delafond on the Corliss engine at Creusot, and those made at the Massachusetts Institute of Technology on a triple-expansion experimental engine; the former has already been given, and the latter will now be detailed; afterward the gain from the use of the jacket will be discussed.

Experimental Engine at the Massachusetts Institute of Technology. — This engine, which was added to the equipment of the laboratory of steam-engineering of the Institute in 1890, is specially arranged for giving instruction in making engine-tests. It has three horizontal cylinders and two intermediate receivers,

the piping being so arranged that any cylinder may be used

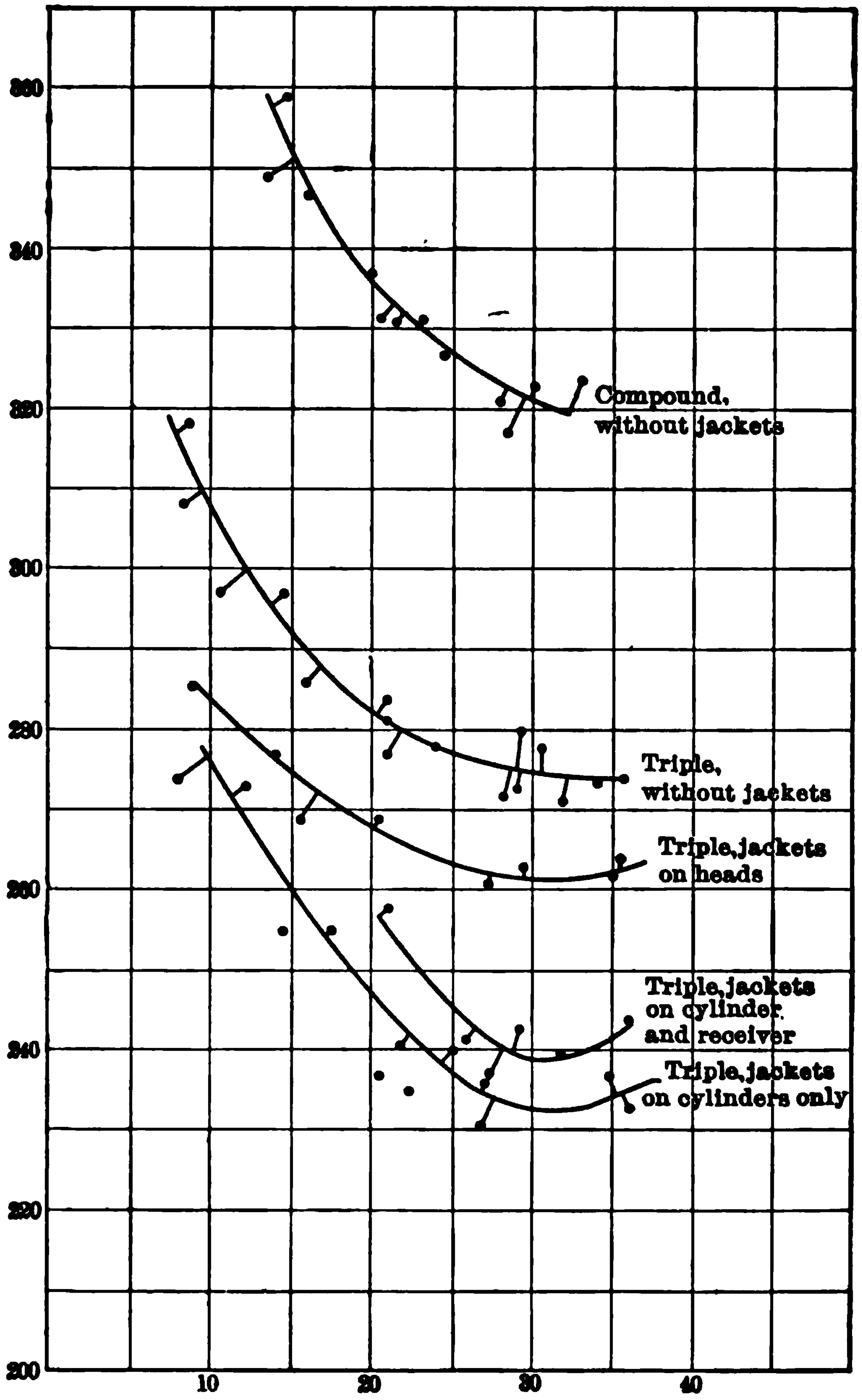

FIG. 59

singly or may be combined with one or both of the other cylinders to form a compound or a triple engine. Each cylinder has

steam-jackets on the barrel and the heads, and steam may be supplied to any or all of these jackets at will. The steam condensed in the jackets of any one of the cylinders is collected under pressure in a closed receptacle and measured. Originally the receivers were also provided with steam-jackets; now they are provided with tubular reheaters so divided that one-third, two-thirds, or all the surface of the reheaters can be used. The steam condensed in the reheaters is also collected and measured in a closed receptacle.

The valve-gear is of the Corliss type with vacuum dash-pots which give a very sharp cut-off. The high-pressure and intermediate cylinders have only one eccentric and wrist-plate, and consequently cannot have a longer cut-off than half stroke under the control of the drop cut-off mechanism. The low-pressure cylinder has two eccentrics and two wrist-plates, and the admission valves can be set to give a cut-off beyond half stroke. The governor is arranged to control the valves for any or all of the cylinders. Each cylinder has also a hand-gear for controlling its valves. For experimental purposes the governor is set to control only the high-pressure valve-gear, when the engine is running compound or triple-expansion. The hand-gear is used for adjusting the cut-off for the other cylinder or cylinders; usually the cut-off for such cylinder or cylinders is set to give a very small drop between the cylinders. This arrangement throws a very small duty on the governor, so that by the aid of a large and heavy fly-wheel the engine can be made to give nearly identical indicator-diagrams for an entire test during which the load and the steam-pressure are kept constant.

The main dimensions of the engine are as follows:

Diameter of the high-pressure cylinder	9	inches.
" " intermediate "	16	"
" " low-pressure "	24	"
" " piston-rods	$2\frac{3}{16}$	"
Stroke	30	"

Clearance in per cent of the piston displacements:

High-pressure	cylinder,	head end,	8.83;	crank	end,	9.76
Intermediate	"	"	"	10.4	"	"	10.9
Low-pressure	"	"	"	11.25	"	"	8.84

Results of tests on the engine with the cylinders arranged in order to form a triple-expansion engine are given in Table XXIII, and are represented by the diagram Fig. 60 with the cut-off of the high-pressure cylinder for abscissæ and with the consumptions of thermal units per horse-power per minute as ordinates.

The most important investigation which has been made on this engine is of the advantage to be obtained from the use of steam in the jackets. Four series of tests were made for this purpose: (1) with steam in all the jackets of the cylinders and receivers, (2) with steam in the jackets of the cylinders, both heads and barrels, (3) with steam in the jackets on the heads of the cylinders only, and (4) without steam in any of the jackets.

The most economical method of running the engine was with steam in all the jackets on the cylinders, but without steam in the receiver-jackets, as shown by the lowest curve on Fig. 59. There is a small but distinct disadvantage from using steam in the receiver-jackets also. This fact could not be surely determined from any pair of tests, for the difference is not more than two per cent, and is therefore not more than the probable error for such a pair of tests, but a comparison of the two curves on Fig. 59, representing tests under the two conditions, gives conclusive evidence with regard to this point. It may not be improper in this connection to call attention to the three points below the lowest curve and not connected with it; they represent tests which were made after the nine tests represented by points joined to the curve, and when some additional non-conducting covering had been applied to the piping and valves of the engine. Here the slight gain from reduced radiation is made manifest, though it is too small to be taken into account in making comparisons of the different conditions of running the engine.

TABLE XXIII.

TRIPLE-EXPANSION EXPERIMENTAL ENGINE AT THE MASSACHUSETTS INSTITUTE OF TECHNOLOGY.

Trans. Am. Soc. Mech. Engs., 1892–1894; *Technology Quarterly*, 1896.

	Condition	Revolutions per minute.	Per cent of cut-off, high-pressure cylinder.	Boiler-pressure by gauge.	Vacuum in condenser, inches of mercury.	Barometer, inches of mercury.	Steam used in jackets, per cent.					Horse-power.	Steam per horse-power per hour.	B.T.U per H.P. per minute, actual.	B.T.U. per H.P. per min. reduced to 26" vacuum.
							High-pressure cylinder.	First receiver.	Intermediate-pressure cylinder.	Second receiver.	Low-pressure cylinder.				
1	Jackets on cylinders.	89.93	36.1	146.2	24.1	29.8	3.2	...	8.6	...	6.3	140.8	13.8	240	233
2		90.60	35.0	147.0	24.7	30.3	2.5	...	8.8	...	5.4	138.0	13.9	241	237
3		91.93	27.3	146.9	24.5	29.9	2.5	....	8.5	....	7.2	125.4	13.7	237	231
4		91.55	27.0	146.7	25.4	30.1	3.2	....	9.8	....	8.1	123.9	13.7	239	236
5		92.37	25.0	146.6	24.5	30.7	3.4	....	10.4	....	10.1	114.7	14.3	247	240
6		84.87	21.9	145.2	24.3	30.1	3.5	....	11.3	....	8.7	105.3	14.5	250	241
7		93.15	17.4	146.0	26.0	30.2	3.5	....	10.7	....	11.6	103.5	14.7	255	255
8		86.70	12.0	147.0	27.4	30.5	6.1	....	15.2	....	12.2	78.3	15.1	261	273
9		87.55	8.3	146.7	26.0	30.1	6.5	....	15.3	....	13.0	67.4	16.0	274	274
10	Ditto.	84.23	13.5	145.2	26.1	30.0	5.3	....	11.3	....	12.1	77.8	14.7	253	255
11		82.50	20.5	144.5	26.2	29.9	4.5	....	9.1	....	9.9	101.9	13.5	235	237
12		82.13	23.6	145.3	26.4	30.1	3.1	....	8.5	....	9.8	104.2	13.3	232	235
13	Jackets on cyl'drs and receivers.	91.20	36.1	143.7	24.7	30.2	2.6	4.7	6.4	5.4	5.9	154.2	14.4	249	244
14		91.40	32.8	143.6	25.0	30.2	2.9	6.4	7.1	4.3	6.4	145.1	14.1	244	240
15		91.82	29.3	143.2	25.2	30.5	3.0	5.6	7.6	4.9	6.1	137.0	14.3	246	243
16		91.83	27.5	147.1	24.7	30.3	1.4	4.7	8.9	3.1	7.3	128.8	14.1	242	237
17		92.17	25.9	145.5	25.5	30.4	3.2	4.5	8.2	4.7	5.7	125.8	14.1	243	241
18		92.57	21.9	143.7	26.4	30.6	3.4	6.8	7.1	4.1	7.7	120.2	14.6	256	258
19	Jackets on heads.	84.95	9.1	145.8	25.6	30.0	2.9	....	7.7	....	8.7	55.9	16.6	290	285
20		84.03	13.9	144.5	26.4	29.9	2.1	....	7.2	....	8.6	69.4	15.5	273	277
21		83.35	15.6	144.9	25.6	29.8	2.2	....	6.8	....	8.0	72.8	15.5	273	269
22		82.40	20.7	145.3	26.7	30.3	1.4	....	6.6	....	8.0	84.2	15.1	269	269
23		81.40	27.3	144.2	24.7	29.7	1.3	....	7.7	....	5.6	97.4	15.2	267	261
24		81.05	29.7	143.4	25.4	29.9	1.4	....	5.3	....	6.8	101.5	15.0	265	263
25		80.28	34.9	143.1	25.5	30.2	1.2	....	5.0	....	6.4	109.4	15.0	265	262
26		80.32	35.6	144.0	25.0	29.9	1.1	....	4.6	....	7.4	114.1	15.2	267	264
27	No jackets.	85.60	8.4	152.8	26.1	29.7	....	....	....	....	....	53.2	17.3	318	318
28		85.62	8.3	153.3	26.1	....	....	....	....	....	....	55.7	16.9	306	308
29		85.60	10.6	152.1	26.1	29.9	....	....	....	....	....	60.6	16.2	296	297
30		84.22	15.8	152.8	25.9	29.8	....	....	....	....	....	74.9	15.4	287	286
31		83.03	21.3	152.0	26.3	....	....	....	....	....	....	85.8	15.1	276	277
32		82.92	21.2	152.4	26.09	30.15	....	....	....	....	....	86.9	15.4	281	281
33		82.55	21.0	153.0	26.02	30.0	....	....	....	....	....	87.8	15.2	284	284
34		83.32	24.1	152.0	25.70	....	....	....	....	....	....	91.1	15.5	280	278
35		82.67	29.5	151.9	25.6	30.0	....	....	....	....	....	99.9	15.5	283	280
36		81.78	29.1	152.0	25.7	....	....	....	....	....	....	100.5	15.2	275	273
37		82.92	28.7	152.5	26.0	29.9	....	....	....	....	....	102.4	15.0	272	272
38		81.52	30.7	151.5	26.1	29.9	....	....	....	....	....	106.0	15.2	278	278
39		81.57	31.8	152.0	26.0	....	....	....	..	....	....	108.2	14.9	271	271
40		81.40	35.6	152.0	26.04	30.1	....	....	....	....	....	111.2	14.3	274	274
41		81.50	33.8	151.9	25.9	30.26	....	.	....	....	....	112.2	15.1	274	274

Table XXIV gives tests made on this engine without steam in the jackets and *with* steam supplied to the tubular reheaters; the results of these tests will be discussed later.

Table XXIV.

TRIPLE-EXPANSION EXPERIMENTAL ENGINE AT THE MASSACHUSETTS INSTITUTE OF TECHNOLOGY WITH TUBULAR REHEATERS.

	Condition.	Revolutions per minute.	Per cent of cut-off, high-pressure cylinder.	Boiler-pressure by gauge.	Vacuum in condenser, inches of mercury.	Barometer, inches of mercury.	Per cent of steam used in reheaters: First.	Per cent of steam used in reheaters: Second.	Horse-power.	Steam per horse-power per hour.	B.T.U. per H.P. per minute, actual.	B.T.U. per H.P. per min. reduced to 26″ vacuum.
1	Without steam in reheaters.	81.8	27	146.7	26.4	30.6	......	......	88.6	16.0	290	288
2		81.8	27	147.5	26.1	30.4	......	......	87.5	16.0	291	28
3		81.6	29	147.0	25.9	30.5	......	......	89.7	16.0	290	28
4		81.2	36	148.2	25.9	30.2	......	......	103.3	15.5	282	281
5	Steam in first reheater.	85.5	10	147.2	25.5	30.0	......	13	66.5	15.7	277	273
6		83.5	19	146.9	23.8	30.2	......	14	84.9	15.9	277	262
7		81.4	31	146.1	25.8	30.2	......	12	112.4	15.0	266	264
8	Steam in both reheaters.	85.0	8	147.3	26.6	30.3	10	7	61.5	15.5	269	274
9		84.5	10	146.9	26.2	30.3	12	8	74.8	14.9	261	260
10		82.4	21	147.1	25.3	30.4	10	6	95.7	14.7	258	252
11		81.9	27	147.7	25.4	30.1	6	9	105.9	14.7	259	254
12		82.0	28	146.6	25.7	30.2	7	8	107.0	14.5	256	254

Gain from Steam-Jackets. — Much of the difference of opinion concerning the advantage to be derived from the use of steam-jackets is to be ascribed to indiscriminate comparison of tests on various engines, or to the failure to obtain any advantage from jackets which were not applied with discrimination. Should any engine when properly tested and computed, show no advantage from the use of a steam-jacket, it will be better to omit that device in future constructions for the same conditions unless there are constructive reasons for retaining it.

In order to obtain definite conclusions from tests made to determine the advantage of the use of steam-jackets, such tests should be made in definite series in which only one property is varied at a time, and from these tests the best results under

the most favorable conditions should be chosen when the engine has steam in the jackets, and in like manner the best result without steam in the jackets should be selected; a comparison of two such selected tests has more weight than a haphazard comparison of individual tests, however great the number of such tests may be. An investigation of Delafond's tests in Tables XXI and XXII and represented by Figs. 57 and 58, gives such a comparison. The tests selected are those given in Table X and give two pairs, with condensation and without. Thus the best result with steam in the jacket and with condensation is 16.9 pounds, and without steam in the jacket is 18.1; the gain is

$$\frac{18.1 - 16.9}{18.1} = 0.07.$$

Without condensation the best results are 21.5 with steam in the jackets and 24.2 without steam in the jackets; the gain is

$$\frac{24.2 - 21.5}{24.2} = 0.11.$$

These results are probably too small, as the steam used in the jackets should be collected and returned to the boiler with only a moderate reduction of temperature below the temperature of the steam in the boiler. The drip from the jackets was passed through a trap, and as reported is probably too small, this being the most questionable result from the tests.

Data for a similar comparison for compound engines are not at hand, but the tests described on page 265 seem to be conclusive for the triple engine.

From the diagram Fig. 59 the best results with steam in all the jackets of the cylinders and without steam in any of the jackets are 233 and 274 B.T.U. per horse-power per minute, and the gain from the use of the steam in the jacket is

$$\frac{274 - 233}{274} \times 100 = 15 \text{ per cent.}$$

These heat-consumptions correspond to 13.8 and 15.2 pounds of steam per horse-power per hour, so that on the basis of steam-consumption the gain from the use of steam in the jackets would appear to be only 9 per cent, instead of the actual gain of 15 per cent. This large difference is due to the large percentage of steam used in the jackets, amounting in all to 17 or 18 per cent of the total steam-consumption. The steam used in an individual jacket is, however, not excessive, being about 2.5 per cent in the jackets of the high-pressure cylinder and 7 or 8 per cent in the jackets of each of the other two cylinders.

The effect of jacketing the heads of the cylinders only is surprisingly small, as from the diagram the best result is 262 B.T.U. per horse-power per minute, which compared with the best result without steam in any of the jackets gives a gain of only

$$\frac{274 - 262}{274} \times 100 = 4 \text{ per cent.}$$

The correspondence between this result and the experiments by Callendar and Nicolson on the action of the cylinder walls, has already been pointed out.

From the tests just discussed and compared it appears conservative to say that about ten per cent can be gained by using steam-jackets on simple and compound engines and that fifteen per cent can be gained by their use on triple-expansion engines; provided that these conclusions shall not be applied to engines of more than 300 horse-power. The saving on massive engines of 1000 horse-power or more is likely to be smaller, and very large engines may derive no benefit from steam-jackets. On the other hand, a saving of 25 per cent may be obtained from jackets on small engines of five or ten horse-power. Such trivial engines are never provided with jackets unless for experimental purposes, and the results of such experiments are of little value.

Intermediate Reheaters. — Many compound and triple-expansion engines have some method of reheating the steam on its way from one cylinder to another. Notable examples

are the Leavitt pumping-engines, for which results are given in Table X. The fact that these engines give the best economies recorded for engines using saturated steam lead to the inference that such reheaters may be used to advantage. The only direct evidence, however, is not so favorable, for, as has been pointed out on page 264, there was found a small but distinct disadvantage from using steam in double walls or jackets on the intermediate receivers of the experimental engine at the Massachusetts Institute of Technology. It appears that this engine gives the best economy when steam is supplied to the jackets on the cylinders and not to the jackets on the reheaters, and, further, that when steam is used in the receiver-jackets the steam in the low-pressure cylinder shows signs of superheating, which may be considered to indicate that the use of the steam-jacket is carried too far.

After the tests referred to were finished the engine was furnished with reheaters made of corrugated-copper tubing, so arranged that one-third, two-thirds, or all of the reheating-surface can be used, when desired. Table XXIV, page 266, gives the results of tests made on the engine with and without steam in the reheaters; in these tests the entire reheating-surface was used when steam was supplied to a reheater.

For some reason the heat-consumption when no steam was used in the reheaters is somewhat greater than that given in Table XXIV for the engine without steam in any of the jackets; the difference, however, is not more than two or two and a half per cent and cannot be considered of much importance. It is clear from the table that there is advantage from using one reheater, and still more from using two. If the heat-consumption for the engine without steam in the jackets and without steam in the reheaters (taken from Table XXIV) is assumed to be 274 B.T.U. per minute, then the gain from using the reheaters appears to be

$$\frac{274 - 252}{274} \times 100 = 8 \text{ per cent,}$$

which is scarcely more than half the gain from using steam in the jackets. These tests cannot be considered conclusive, as they are too few and refer only to one engine.

Superheating. — The most direct and effective way of reducing the interference of the cylinder walls and of improving steam-engine economy is by the use of superheated steam. About 1863–64 a number of naval vessels were supplied with superheaters by Chief Engineer Isherwood, and when tested by him showed a marked advantage which led to the adoption of superheated steam for stationary and marine practice both in America and in Europe. But the superheaters which were exposed to dry steam on one side and to the flue gases on the other, rapidly deteriorated, and after an experience lasting ten or fifteen years the use of superheated steam was abandoned in favor of compound and triple engines with high-pressure steam.

More recently improved forms of superheaters have been introduced in Great Britain and Germany, which show good endurance, and superheated steam appears to have been used successfully for sufficient times to warrant the conclusion that the application of superheated steam has been accomplished. Two series of tests will be discussed, namely, some early tests on a simple engine, and some recent tests on compound engines. There appears to be no reason for extending the application of superheated steam to triple engines.

Dixwell's Tests. — A small Harris-Corliss engine was fitted up for making tests on superheated steam at the Massachusetts Institute of Technology by Mr. George B. Dixwell. Six tests with superheated and saturated steam were made on this engine in 1877 in the presence of a board of engineers of the United States Navy.

Table XXV.

DIXWELL'S TESTS ON SUPERHEATED STEAM.

CYLINDER DIAMETER 8 INCHES; STROKE 2 FEET.

Proceedings of the Society of Arts, Mass. Inst. Tech., 1887–88.

	Saturated Steam.			Superheated Steam.		
	I	II	III	IV	V	VI
Duration, minutes	127	83	63	180	108	75
Cut-off	0.217	0.443	0.689	0.218	0.439	0.672
Revolutions per minute	61.5	60.4	58.0	61.0	61.4	59.5
Boiler-pressure above atmosphere, pounds per square inch	50.4	50.2	50.3	50.4	50.0	50.2
Back-pressure, absolute, pounds per sq. in.	15.4	15.7	15.8	15.2	15.4	15.5
Temperatures Fahrenheit:						
Near engine	302	303	303	478	441	406
In cylinder by pyrometer	278–297	279–296	282–300	313	316	315
Per cent of water in cylinder:						
At cut-off	52.2	35.9	27.9	27.4	13.6	8.9
At end of stroke	32.4	29.3	23.9	18.3	13.6	11.5
Horse-power	7.65	12.7	15.68	6.83	12.37	15.63
Steam per horse-power per hour, pounds	48.2	42.2	45.3	35.2	31.7	35.8
B.T.U. per horse-power per minute	796	696	747	631	546	621

A metallic thermometer or pyrometer was placed in a recess in the head of the cylinder. When saturated steam was used this pyrometer showed a large fluctuation, but when superheated steam was used its needle or indicator was at rest. Even if a part of the apparent change of temperature with saturated steam is attributed to the vibration of the needle and the multiplying mechanism, it is very clear that the use of superheated steam reduces the change of temperature of the cylinder-head in a remarkable manner. The effect of superheating on the action of the cylinder walls is also indicated by the per cent of water in the cylinder at cut-off and release.

The apparent gain by comparing the amounts of steam used per horse-power per hour in favor of superheated steam is but

$$\frac{42.2 - 31.7}{42.2} \times 100 = 25 \text{ per cent};$$

this result is of course misleading, since the superheating required additional coal. As the coal-consumption was not determined,

we must compare instead the B.T.U. per horse-power per minute, giving a real gain of

$$\frac{696 - 546}{696} \times 100 = 19 \text{ per cent.}$$

This same Harris-Corliss engine afterwards showed a heat-consumption of 548 B.T.U. per horse-power per minute when supplied with saturated steam at 77 pounds pressure, which shows why the earlier attempts at the use of superheated steam were so easily set aside when it was found expedient to raise the steam-pressure.

Though we have no tests with high-pressure steam and with condensation on engines of two or three hundred horse-power, it is probable that a very material saving could be made by the use of superheated steam under such conditions; if the saving in heat were as much as fifteen per cent, it would reduce the steam-consumption to a larger degree, perhaps by twenty per cent, and would be likely to give from 14.5 to 15 pounds of superheated steam per horse-power per hour.

The best results obtained from the application of superheated steam in compound engines are reported by Professor Schröter, in Table XXVI, for a tandem-engine with poppet-valves built in Ghent. Five tests were made with varying cut-off and with saturated steam, and five others also with varying cut-off and with steam that was superheated about 250° F., the absolute *initial* pressure in the cylinder being about 145 pounds, so that the boiler-pressure was probably between 130 and 135 pounds by the gauge.

This engine gave a remarkable economy both with saturated steam and with superheated steam, its steam and heat-consumption being only five per cent more than that of the triple-expansion Leavitt engine recorded in Table X. The gain from using superheated steam appears to be

$$\frac{213 - 199}{213} = 0.06,$$

which places it a little beyond the performance of the triple engine mentioned. But since the uncertainty of the determination of power by the indicator is probably two per cent, we may reasonably conclude that the effect of using superheated steam in a compound engine is to place it on a level with a triple engine, and the question is to be decided in practice by the relative expense and trouble of supplying and using a superheater instead of a third cylinder and higher steam-pressure.

It is somewhat remarkable that steam was supplied to the jackets during the superheating tests, but not at all surprising that for those tests the jackets had a small effect, as is made evident by noting the percentages of steam condensed in them.

TABLE XXVI.

COMPOUND HORIZONTAL MILL-ENGINE.

CYLINDER DIAMETERS 12.8 AND 22 INCHES; STROKE 33.5 INCHES.

By Professor M. SCHRÖTER, *Mitteilungen über Forschungsarbeiten, Heft* 19, 1904.

	Saturated.					Superheated.				
	I	II	III	IV	V	VI	VII	VIII	IX	X
Horse-power	299	263	211	160	112	303	258	112	161	115
Duration, minutes	60	61	57.5	55	50	48	60	51	64.5	55
Revolutions per minute	126	126	126.5	127	128	126	126	126.5	127	128
Cut-off, high-pressure cylinder	0.38	0.31	0.22	0.15	0.10	0.41	0.33	0.26	0.16	0.10
Total expansions	7.9	9.7	13.5	20	30	7.3	9.1	11.5	18.7	30
Initial pressure, absolute pounds per sq. in.	148	146	147	141	142	148	149	149	146	146
Back-pressure, absolute pounds per sq. in.	1.3	1.1	1.1	1.1	1.1	1.3	1.0	1.1	1.1	1.1
Superheating, degrees F.	...	...	...	...	...	246	257	258	256	256
Steam per horse-power per hour, pounds	13.6	12.8	12.3	11.8	12	10.9	10.4	10	9.7	9.6
Per cent condensed in jackets	10.9	11.8	12.9	13.7	14.4	2.1	3.5	3.8	4.4	4.6
B.T.U. per horse-power per min.	246	232	222	213	216	223	215	206	201	199
Mechanical efficiency	0.901	0.891	0.872	0.842	0.786	0.902	0.890	0.872	0.842	0.790

Cut-off and Expansion. — It has already been pointed out on page 256 in connection with Delafond's tests that the best point of cut-off for a simple engine, whether jacketed or not, is about

one-third stroke when the engine is non-condensing, and it is about one-sixth stroke when condensing. In general, other tests on simple engines such as those on the *Gallatin*, and on the small Corliss engine at the Massachusetts Institute of Technology, confirm these conclusions.

The term *total expansion* for a compound or a triple engine can properly have only a conventional significance; it is usually taken to be the product of the ratio of the large to the small cylinder by the reciprocal of the fraction of the stroke at cut-off for the high-pressure cylinder. This conventional total expansion is about 20 for all the tests on triple engines quoted in Table X, except those on marine engines, which show a relatively poor economy. It may therefore be concluded that it is not advisable to use much more expansion for any triple engine, and that less expansion should be used only when the conditions of service (for example, at sea) prevent the use of large expansion.

The stationary compound engines given in Table X also have about 20 expansions, and experience shows conclusively that for highest economy such a degree of expansion is required. In practice somewhat less may frequently be found advisable.

Variation of Load. — In general, an engine should be so designed that it may give a fair economy for a considerable range of load or power. Very commonly the engine will have sufficient range of power with good economy if designed to give the best economy at the normal load. In general, however, it is well to assign a less expansion and consequently a longer cut-off to the engine than would be determined from a consideration of the steam- (or heat-) consumption alone. For, in the first place, the best brake or dynamic economy is always attained for a little longer cut-off than that which gives the best indicated economy, and in the second place the economy is less affected by lengthening than by shortening the cut-off. The first comes from the fact that the frictional losses of the engine increase less rapidly than the power, as will be shown

in the next chapter; and the second is evident from consideration of curves of steam-consumption as given by Fig. 59, page 262, and Figs. 57 and 58, pages 252–253.

The allowable range of power for a simple engine is greater than for a compound or a triple engine. Comparisons for a simple and a triple engine may be made by aid of Figs. 58 and 59. The Corliss engine at Creusot when supplied with steam at 60 pounds pressure, with condensation and with steam in the jacket, developed 150 horse-power and used 17.3 pounds of steam per horse-power per hour. If the increase be limited to 10 per cent of the best economy, that is, to 19 pounds per horse-power per hour, the horse-power may be reduced to about 92, giving a reduction of nearly 40 per cent from the normal power. The triple engine at the Massachusetts Institute of Technology with steam at 150 pounds pressure and using steam in all the cylinder-jackets developed 140 horse-power and used 233 B.T.U. per horse-power per minute. Again, limiting the increased consumption to 10 per cent or to 254 B.T.U., the power may be reduced to about 104 horse-power, giving a reduction of 26 per cent from the normal power. The effect of increasing power for these engines cannot be well shown from the tests made on them, but there is reason to believe that the simple engine would preserve its advantage if a comparison could be made. Though the tests which we have on compound engines do not allow us to make a similar investigation of the effect of changing load, there is no doubt that it is intermediate in this respect between the simple and the triple engine.

When the power developed by a compound engine is reduced by shortening the cut-off of the high-pressure cylinder, the cut-off of the low-pressure cylinder must be shortened at the same time to preserve a proper distribution of power and division of the range of temperature between the cylinders. If this is not done the work will be developed mainly in the high-pressure cylinder, which will be subjected to a large fluctuation of temperature, and the engine will lose the advantages sought from compounding. A compound non-condensing engine, if the cut-off for the large

cylinder is fixed, is likely to have a loop on the low-pressure indicator-diagram due to expansion below the atmosphere, if the power is reduced by shortening the cut-off of the high-pressure cylinder. Such a loop is always accompanied by a large loss of economy; if the loop is large the engine may be more wasteful than a simple engine, for the high-pressure piston develops nearly all the power and may have to drag the low-pressure piston, which is then worse than useless.

There is seldom much difficulty in running a simple engine at any desired reduced power by shortening the cut-off or reducing the steam-pressure, or by a combination of the two methods. But a compound engine sometimes gives trouble when run at very low power (even when attention is given to the cut-off of the low-pressure cylinder), which usually takes the form just discussed; i.e., the power is developed mainly in the high-pressure cylinder. Triple engines are even more troublesome in this way. A compound or triple engine running at much reduced power is subject not only to loss of economy and to irregular action, but the inside surface of the low-pressure cylinder is liable to be cut or abraded.

Automatic and Throttle Engines. — The power of an engine may be regulated by (1) controlling the steam-pressure, or (2) by adjusting the cut-off. Usually these two methods are used separately, but in some instances they are used in combination. Thus a locomotive-driver may reduce the power of his engine either by shortening the cut-off or by partially closing the throttle-valve, or he may do both at once. Stationary engines are usually run at a fixed speed and are controlled by mechanical governors, which commonly consist of revolving weights that are urged away from the axis of revolution by centrifugal force and are restrained by the attraction of gravity or by the tension of springs.

The earliest and simplest steam-engine governor, invented by Watt, has a pair of revolving pendulums (balls on the ends of rods that are hinged to a vertical spindle at their upper ends) which are urged out by centrifugal force and are drawn down

by gravity. When the engine is running steadily at a given speed the forces acting on the governor are in equilibrium and the balls revolve in a certain horizontal plane. If the load on the engine is reduced the engine speeds up and the balls move outward and upward until a new position of equilibrium is found with the balls revolving in a higher horizontal plane. Through a proper system of links and levers the upward motion of the balls is made to partially close a throttle-valve in the pipe which supplies steam to the engine and thus adjusts the work of the engine to the load.

Shaft-governors have large revolving-weights whose centrifugal forces are balanced by strong springs. They are powerful enough to control the distribution or the cut-off valve of the engine, which, however, must be balanced so that it may move easily.

Automatic engines, like the Corliss engines, have four valves, two for admission and two for exhaust of steam. The admission, release, and compression are fixed, but the cut-off is controlled by the governor. Usually an admission-valve is attached to the actuating mechanism by a latch or similar device, which can be opened by the governor, and then the valve is closed by gravity by a spring, or by some other independent device. The office of the governor is to control the position of a stop against which the latch strikes and by which it is opened to release the valve.

Corliss and other automatic engines have long had a deserved reputation for economy, which is commonly attributed to their method of regulation. It is true that the valve-gears of such engines are adapted to give an early cut-off, which is one of the elements of the design of an economical simple engine, but their advantage over some other engines is to be largely attributed to the small clearance which the use of four valves makes convenient, and to the fact that the exhaust-steam is led immediately away from the engine, without having a chance to abstract heat after it leaves the cylinder. These engines also are free from the loss which Callendar and Nicolson attribute to direct leakage

from the steam to the exhaust side of slide-valves, and to valves of similar construction.

Every steam-engine should have a reserve of power in excess of its normal power; and again it is convenient if not essential that a single-cylinder engine should be able to carry steam through the greater part of its stroke in starting. These conditions, together with the fact that it is somewhat difficult to design a plain slide-valve engine to give an early cut-off, have led to the use of a long cut-off for engines controlled by a throttle-governor. The tests on the Corliss engine at Creusot (Tables XXI and XXII, pp. 250 and 251) show clearly the disadvantage of using a long cut-off for simple engines. It has already been pointed out that a non-condensing engine should have the cut-off at about one-third stroke. With cut-off at that point and with 75 pounds steam-pressure the engine developed 209 horse-power and used 24.2 pounds of steam per horse-power per hour when running without steam in the jacket and without condensation. If the steam-pressure is reduced to 50 pounds and the cut-off is lengthened to 58 per cent of the stroke, the steam-consumption is increased to 30.2 pounds per horse-power per hour, the horse-power being then 173. The gain from using the shorter cut-off is

$$\frac{30.2 - 24.2}{30.2} \times 100 = 20 \text{ per cent.}$$

A similar comparison for the same engine running with a vacuum and with steam in the jacket shows even a larger difference. Thus in test 16 the steam-pressure is 84 pounds and the cut-off is at 11.5 per cent of the stroke, the horse-power is 176, and the steam-consumption per horse-power per hour is 16.9 pounds, while the consumption for about the same power in test 44 is 25.4 pounds of steam per horse-power per hour, the steam-pressure being 35 and the cut-off at 58 per cent of the stroke; here the gain from using the shorter cut-off is

$$\frac{25.4 - 16.9}{25.4} \times 100 = 33 \text{ per cent.}$$

Considering also that automatic engines are usually well built and carefully attended to, while throttling-engines are often cheaply built and neglected, the good reputation of the one and the bad reputation of the other are easily accounted for.

It is, however, far from certain that an automatic engine will have a decided advantage over a throttle-engine, provided the latter is skilfully designed, well built and cared for, and arranged to run at the proper cut-off. Considering the rapid increase in steam-consumption per horse-power per hour when the cut-off is unduly shortened, it is not unreasonable to expect as good if not better results from a simple throttling-engine than from an automatic engine when both are run for a large part of the time at reduced power.

The disadvantage of running a compound or a triple engine with too little expansion can be seen by comparing the steam-consumptions of marine and stationary engines; on the other hand, the great disadvantage of too much expansion is made evident from the tests on the engine in the laboratory of the Massachusetts Institute of Technology (Table XXIII, page 265). Considering that the allowable variation from the most economical cut-off is more limited for a compound or a triple engine, it appears that there is less reason for using an automatic governor instead of a throttling governor for compound and triple engines than there is with simple engines. Nevertheless the most economical engines (simple, compound, or triple) are automatic engines.

Effect of Speed of Revolution. — Though the condensation of steam on the walls of the cylinder of a steam-engine is very rapid, it is not instantaneous. It would therefore appear that an improvement in economy might be attained by increasing the number of revolutions per minute; but whatever might be thus gained is more than offset by the increase of the dimensions of valves, passages, and clearances that would accompany such a change in speed, for it has already been pointed out that the evil of initial condensation is much aggravated by increasing the

surfaces exposed to steam in clearance spaces. As a matter of fact, all engines which for various reasons have been designed to run at very high rotative speeds have shown relatively poor economy, in part from the reason given, and in part from the fact that piston-valves are commonly used, and they are subject to the kind of leakage described by Callendar and Nicolson on page 234, even when they are in good condition. Very commonly the engine has a fly-wheel governor, which requires the valve to be very free with the chance of excessive leakage. Mr. Willans invented a single-acting triple-expansion engine to run at high rotative speed, and succeeded in getting abundant steam-passages without excessive clearances by using a hollow piston-rod to carry the steam from cylinder to cylinder, all arranged tandem. Tests on this engine (which are not quoted elsewhere in this book) showed that an increase from 100 revolutions to 200 revolutions per minute reduced the steam-consumption from 24.7 to 23.1 pounds per horse-power per hour, and a further increase of speed to 400 revolutions gave a reduction to 21.4 pounds; the engine was then running compound non-condensing. This engine used 12.7 pounds of steam per horse-power per hour, when developing 30 horse-power, at 380 revolutions per minute under 170 pounds gauge-pressure, acting as a triple-expansion condensing engine.

Binary Engine. — On page 180, under the subject "Compound Engines," attention was called to the possibility of extending the range of temperature for vapor-engines by the use of two fluids; the second fluid (for example, sulphur dioxide) being chosen so that a good working back-pressure could be maintained at the temperature of the available condensing water which acts as the refrigerator for the combined engines. Considering only the efficiency of Carnot's cycle for the customary range of temperature for a steam-engine, and the efficiency for the extended range, it appeared that a gain of 20 per cent might be possible.

Recent investigations by Professor Josse on an experimental engine in the laboratory of the Technical High School at Char-

lottenburg give some insight into the possibilities of this method. The engine is of moderate size, developing about 150 horse-power as a steam-engine, and about 200 horse-power as a binary engine, using steam at about 160 pounds by the gauge with 200° F., superheating. The engine is a three-cylinder triple-expansion engine, but can be run also as a compound engine, though it probably is not proportioned to give the best economy under the latter condition.

The general arrangement of the engine is as follows: the three steam-cylinders are arranged horizontally side by side, and the additional cylinder using the volatile fluid (sulphur dioxide) lies on the opposite side of the crank-shaft, to which it is connected by its own crank and connecting-rod. Steam is supplied from the boiler and superheater to the steam-engine, and is exhausted into a tubular condenser which acts as the sulphur dioxide vaporizes; the condensed steam is pumped back into the boiler, and the vacuum is maintained by an air-pump as usual; a vacuum of 20 to 25 inches of mercury was maintained in this condenser. The vaporous sulphur dioxide at a pressure of 120 to 180 pounds by the gauge was led to the proper cylinder, from which it was exhausted at about 35 pounds by the gauge; this exhaust was condensed in a tubular condenser by circulating water with a temperature of about 50° F. at the inlet and about 65° F. at the exit.

The drips from the steam-jackets of the steam-cylinders were piped to the steam-condenser instead of being returned to the boiler, but that cannot be of much importance because the condensation in the jackets was probably less than five per cent of the total steam supplied to the engine. The performance of the engine is given in Table XXVIII in terms of steam per horse-power per hour and in thermal units per horse-power per minute; the latter I have calculated from the total heat of the steam including the superheat, and the heat of the liquid at the vacuum in the steam-condenser. Comparisons must be made in terms of thermal units in order to take account of the superheating.

TABLE XXVII.

BINARY ENGINE, STEAM AND SULPHUR DIOXIDE.

By Professor E. JOSSE, *Royal Technical High School*, Charlottenburg.

	Triple Expansion.								Compound.	
	1	2	3	4	5	6	7	8	9	10
Revolutions per minute	139.6	136.3	143.5	137.4	145	145	148	149	137	148
Steam-Engine:										
Pressure at inlet, h.p. cylinder by gauge pounds	136.5	156.5	158	156.5	156.5	156.5	156.5	165.3	165.3	163.7
Vacuum, inches of mercury	23.9	24.1	20.9	25.4	23.8	20.6	20.5	20.4	21.8	20.7
Superheating, degrees Fahrenheit	175	219	221	214	210	210	221	...	257	247
Horse-power, indicated	132.1	125.2	154.2	101.6	145.3	144.5	161.0	156.3	121.8	140.5
Steam per h.p. per hour, pounds	12.5	11.2	12.2	14.4	13.6	13.8	13.2	16.4	13.5	13.4
Thermal units per h.p. per minute	244	223	240	289	270	270	261	283	271	266
Sulphur-Dioxide Engine:										
Pressure by gauge pounds:										
In vaporizer	132	128	172	111	142	188	186	181	178	183
In condenser	31	34	35	31	36	36	36	38	33	35
Temperature Fahr. at inlet to cylinder	132.0	133.7	151.7	123.7	137.3	157.1	155.1	153.5	152.8	155.4
Temperature Fahr. at outlet from condenser	66.2	65.8	67.6	64.4	68.5	67.6	68.0	70.0	64.6	66.9
of circulating water inlet	49.6	49.9	49.9	50.2	50.2	50.2	50.2	50.2	50.2	50.2
outlet	59.9	60.2	62.4	60.2	63.8	63.8	63.4	65.1	61.2	63.3
Horse-power, indicated	45.3	42.8	56.8	31.0	50.1	57.6	61.3	66.0	48.0	55.6
per cent of steam-engine power	34.4	34.2	37.0	30.3	34.5	40.0	37.9	42.1	39.4	39.5
Combined Engine:										
Horse-power, indicated	177.4	168	211	132.6	195.4	202.1	223.2	222.3	169.8	196.1
Steam per h.p. per hour, pounds	9.7	8.36	8.92	11.05	10.12	9.86	9.55	11.5	9.7	9.6
Thermal units per h.p. per minute	183	167	176	215	200	193	189	205	195	191
Mechanical efficiency	85.5	86.2	83.8	87.5	89.1	87	90.8	90.5	89.8	92

Before comparing the results of these tests to determine the gain from working binary, it is interesting to see that the increased range of temperature in this case appears to give a possible advantage of 9 per cent. Thus, if the engine working as a steam-engine only had a vacuum of 27 inches so that the lower temperature was about 115° F., the efficiency of Carnot's cycle would be

$$\frac{T - T'}{T} = \frac{575 - 115}{575 + 460} = 0.50,$$

in which 575 is the temperature of the superheated steam supplied to the engine. On the other hand, with a back-pressure of

about 35 pounds in the sulphur-dioxide cylinder and a temperature of about 65° F., the efficiency would be

$$\frac{T - T''}{T} = \frac{575 - 65}{575 + 460} = 0.55:$$

and

$$\frac{0.55 - 0.50}{0.55} = 0.09.$$

The results of the tests given in Table XXVIII are somewhat difficult to use as a basis for the discussion of the advantage of the binary system on account of certain discrepancies; for example, tests No. 3 and No. 7 have substantially the same total power, steam-pressure, superheating and vacuum, and nearly the same vapor-pressures in the sulphur-dioxide cylinder; in fact, the advantage appears to lie slightly in favor of No. 7; nevertheless, the latter test is charged with 189 thermal units per horse-power per minute, and the former with 176, giving to it an apparent advantage of about 7 per cent. A comparison of steam per horse-power per hour gives nearly the same result. A comparison of tests No. 2 and No. 4 gives even a more striking discrepancy, though the conditions vary more, and especially the total power of the latter is much greater.

If we take 200 thermal units per horse-power per hour as the best result from a steam-engine, then the result from the second test appears to show a gain of 16 per cent, while the seventh test shows a gain of 6 per cent, and the fourth test is distinctly worse than the standard taken for the steam-engine. Under these conditions it is necessary to await further information.

The last two tests made with the engine running compound gave results that are a trifle better than those for the compound engine using superheated steam but as it probably had not the most favorable proportions the comparison is hardly fair.

Test No. 8 with saturated steam gave a record equivalent to that of the best steam-engine, which is distinctly favorable so far as it goes, as the steam-consumption for the steam-engine is large even making allowance for so poor a vacuum.

Finally it appears probable that the best results for the binary engine could be obtained from a correctly designed compound engine, using superheated steam; or nearly as good results might be expected for saturated steam at about 175 pounds gauge pressure with steam-jackets. Attention has already been called to the fact that steam-jackets accomplish but little with highly superheated steam, and appear to be unnecessary and illogical.

CHAPTER XIII.

FRICTION OF ENGINES.

THE efficiency and economy of steam-engines are commonly based on the indicated horse-power, because that power is a definite quantity that may be readily determined. On the other hand, it is usually difficult and sometimes impossible to make a satisfactory determination of the power actually delivered by the engine. A common way of determining the work consumed by friction in the engine itself is to disconnect the driving-belt, or other gear for transmitting power from the engine, and to place a friction-brake on the main shaft; the power developed is then determined by aid of indicators, and the power delivered is measured by the brake, the difference being the power consumed by friction. Such a determination for a large engine involves much trouble and expense, and may be unsatisfactory, since the engine-friction may depend largely on the gear for transmitting power from the engine, especially when belts or ropes are used for that purpose.

The friction of a pumping-engine may be determined from a comparison of the indicated power of the steam-cylinders with the indicated work of the pumps, or, better, with the work done in lifting water from the well and delivering it to the forcing-main. But the friction thus determined is the friction of both the engine and the pump. Air-compressors and refrigerating machines may be treated in the same way to determine the friction of both engine and compressor. Again, the combined friction of an engine and a directly connected electric generator may be determined by comparing the indicated power of the engine with the electric output of the generator, allowing for electricity consumed or wasted in the generator itself.

The friction of a steam-engine may consume from 5 to 15 per

cent of the indicated horse-power, depending on the type and condition of the engine. The power required to drive the air-pump (when connected to the engine) is commonly charged to the friction of the engine. It is usual to consider that seven per cent of the indicated power of the engine is expended on the air-pump. Independent air-pumps which can be driven at the best speed consume much less power; those of some United States naval vessels used only one or two per cent of the power of the main engines. But as independent air-pumps are usually direct-acting steam-pumps, much of the apparent advantage just pointed out is lost on account of the excessive steam-consumption of such pumps.

Mechanical Efficiency. — The ratio of the power delivered by an engine to the power generated in the cylinder is the mechanical efficiency; or it may be taken as the ratio of the brake to the indicated power. The mechanical efficiency of engines varies from 0.85 to 0.95, corresponding to the per cent of friction given above.

The following table gives the mechanical efficiencies of a number of engines, determined by brake-tests, or, in case of the

TABLE XXIX.

MECHANICAL EFFICIENCIES OF ENGINES.

Kind of Engine.	Horse-Power.	Efficiency.
Simple engines:		
Horizontal portable	24	0.86
Horizontal portable Hoadley	80	0.91
High-speed, straight-line	56	0.96
Corliss condensing	160	0.81
Corliss non-condensing	100	0.86
Compound:		
Portable	78	0.88
Semi-portable	60	0.88
Horizontal	59	0.90
Horizontal mill-engine	288	0.86
Schmidt, superheated steam	110	0.92
Leavitt pumping-engine	643	0.93
Triple-expansion Leavitt pumping-engine	576	0.90

pumping-engines, by measuring the work done in pumping water.

Initial Friction and Load Friction. — A part of the friction of an engine, such as the friction of the piston-rings and at the stuffing-boxes of piston-rods and valve-rods, may be expected to remain constant for all powers. The friction at the cross-head guides and crank-pins is due mainly to the thrust or pull of the steam-pressure, and will be nearly proportional to the mean effective pressure. Friction at other places, such as the main bearings, will be due in part to weight and in part to steam-pressure. On the whole, it appears probable that the friction may be divided into two parts, of which one is independent of the load on the engine, and the other is proportional to the load. The first may be called the initial friction, and the second, the load friction. Progressive brake-tests at increasing loads confirm this conclusion.

Table XXX gives the results of tests made by Walther-Meunier and Ludwig * to determine the friction of a horizontal-receiver compound engine, with cranks at right angles and with a fly-wheel, grooved for rope-driving, between the cranks. The piston-rod of each piston extended through the cylinder-cover and was carried by a cross-head on guides, and the air-pump was worked from the high-pressure piston-rod. The cylinders each had four plain slide-valves, two for admission and two for exhaust; the exhaust-valves had a fixed motion, but the admission-valves were moved by a cam so that the cut-off was determined by the governor.

The main dimensions of the engine were:

Stroke	40.2	inches.
Diameter: small piston	21.2	"
large piston	31.6	"
piston-rods	3.2	"
Diameter, air-pump pistons	14.2	"
Stroke, air-pump	18.8	"
Diameter, fly-wheel	24.1	"

* *Bulletin de la Soc. Ind. de Mulhouse*, vol. lvii, p. 140.

TABLE XXX.

FRICTION OF COMPOUND ENGINE.

WALTHER-MEUNIER and LUDWIG, *Bulletin de la Soc. Ind. de Mulhouse*, vol. lvii, p. 140.

	Condition.	Horse-Powers—Chevaux aux Vapeur.			Friction.	Efficiency.
		Indicated.	Effective.	Absorbed by Engine.		
1	Compound condensing with air-pump.	288.5	249.0	39.5	0.137	0.863
2		276.9	238.9	38.0	0.138	0.862
3		265.6	228.9	36.7	0.139	0.861
4		243.7	208.8	34.9	0.144	0.856
5		222.7	188.7	34.0	0.153	0.847
6		201.5	168.6	32.9	0.164	0.836
7		180.4	148.5	31.9	0.178	0.822
8		158.1	128.4	29.7	0.189	0.811
9		136.1	108.3	27.8	0.205	0.795
10	High-pressure cylinder only. Condensing with air-pump.	153.1	128.4	24.7	0.161	0.839
11		142.0	118.3	23.7	0.167	0.833
12		130.9	108.3	22.6	0.173	0.827
13		120.1	98 2	21.9	0.182	0.818
14		109.0	88.2	20.8	0.191	0.809
15		97.5	78.1	19.4	0.199	0.801
16		86.3	68.1	18.3	0.212	0.788
17		75.7	58.0	17.7	0.234	0.766
18		65.5	48.0	17.5	0.267	0.733
19		55.2	37.9	17.3	0.313	0.687
20	High-pressure cylinder only. Non-condensing, no air-pump.	145.9	128.4	17.5	0.120	0.880
21		135.7	118.3	17.4	0.129	0.871
22		125.2	108.3	16.9	0.135	0.865
23		114.4	98.2	16.2	0.142	0.858
24		103.9	88.2	15.7	0.152	0.848
25		93.0	78.1	14.9	0.160	0.840
26		82.0	68.1	13.9	0.170	0.830
27		71.7	58.0	13.7	0.191	0.809
28		61.6	48.0	13.6	0.221	0.779
29		51.3	37.9	13.4	0.262	0.738

The engine during the experiments made 58 revolutions per minute. The air-pump had two single-acting vertical pistons.

Each experiment lasted 10 or 20 minutes, during which the load on the brake was maintained constant, and indicator-diagrams were taken. The experiments with small load on the

brake (numbers 9, 18, 19, 28, and 29) were irregular and uncertain.

The first nine tests were made with the engine working compound. Tests 10 to 19 were made with the high-pressure cylinder only in action and with condensation, the low-pressure connecting-rod being disconnected. Tests 20 to 29 were made with the high-pressure cylinder in action, without condensation.

The results of these tests are plotted on Fig. 60, using the

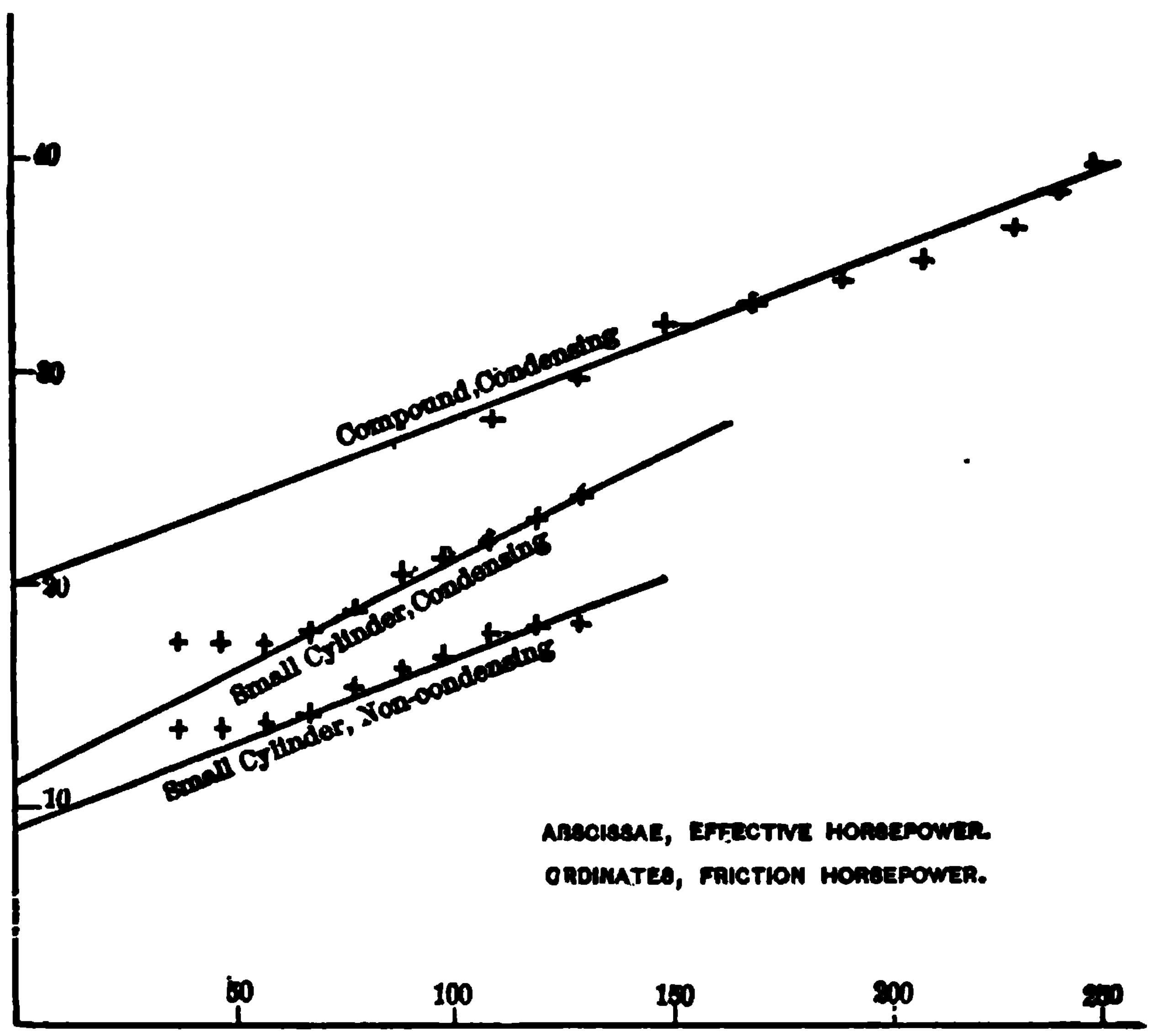

FIG. 60.

effective horse-powers for abcissæ and the friction horse-powers for ordinates. Omitting tests with small powers (for which the brake ran unsteadily), it appears that each series of tests can be represented by a straight line which crosses the axis of ordinates above the origin; thus affording a confirmation of the assumption that an engine has a constant initial friction, and a load friction which is proportional to the load.

Now the initial friction which depends on the size and construction of the engine may be assumed to be proportional to the

normal net or brake horse-power, P_n, which the engine is designed to deliver, and may be represented by

$$aP_n,$$

where a is a constant to be determined from a diagram like Fig. 60. If P is the net horse-power delivered by the engine at any time, then the load friction corresponding is

$$bP,$$

where b is a second constant to be determined from experiments. The total friction of the engine will be

$$F = aP_n + bP,$$

so that the indicated power of the engine will be

$$\text{I.H.P.} = P + aP_n + bP = aP_n + (1 + b)P.$$

The mechanical efficiency corresponding will be

$$e_m = \frac{\text{I.H.P.} - F}{\text{I.H.P.}} = \frac{P}{\text{I.H.P.}}.$$

The compound condensing engine for which tests are represented by Fig. 60 developed 290 I.H.P. and delivered 250 horse-power to the brake, so that 40 horse-power were consumed in friction. The diagram shows also that the initial friction was 20 horse-power, and consequently the load friction was 20 horse-power. The values of a and b are consequently

$$a = 20 \div 250 = 0.07;$$
$$b = (40 - 20) \div 250 \doteq 0.07.$$

The indicated horse-power for a given load P is

$$\text{I.H.P.} = 0.07P_n + 1.07P.$$

Similar equations can be deduced for the engine with steam supplied to the small cylinder only; but as the engine is not then in normal condition they are not very useful.

The maximum efficiency of this engine is

$$250 \div 290 = 0.86;$$

but at half load (125 horse-power) the indicated horse-power is

$$\text{I.H.P.} = 0.07 \times 250 + 1.07 \times 125 = 151,$$

and the efficiency is

$$125 \div 151 = 0.83.$$

TABLE XXXI.

FRICTION OF CORLISS ENGINE AT CREUSOT.

By F. DELAFOND, *Annales des Mines*, 1884.

Condensing with air-pump, tests 1–33.
Non-condensing without air-pump, tests 34–46.

	Cut-off Fraction of Stroke	Pressure at Cut-off, Kilos per Sq. Cm.	Revolutions per Minute.	Horse-Power — Cheval à Vapeur.		
				Indicated.	Effective.	Absorbed by Engine.
1	0.039	0.64	64.0	27.8	16.3	11.5
2	0.044	2.40	68.5	60.0	37.6	22.4
3	0.044	2.90	65.0	67.2	45.2	22.0
4	0.065	4.90	64.0	117.0	88.7	28.3
5	0.065	6.20	61.0	138.5	106.3	32.2
6	0.065	7.10	64.0	163.2	129.2	34.0
7	0.065	7.60	64.0	185.0	144.6	40.4
8	0.100	0.16	58.0	21.0	10.6	10.4
9	0.106	1.55	60.0	61.9	42.3	19.6
10	0.100	2.82	57.3	82.7	61.0	21.7
11	0.090	4.80	58.3	135.3	106.7	28.6
12	0.128	4.82	58.3	154.5	124.8	29.7
13	0.142	0.76	62.0	42.3	28.4	13.9
14	0.137	0.71	60.6	44.3	28.7	15.6
15	0.132	2.50	54.0	79.5	59.8	19.7
16	0.147	2.60	61.6	100.0	78.2	21.8
17	0.155	4.65	60.0	177.2	145.0	32.2
18	0.167	0.22	61.0	40.2	27.9	12.3
19	0.197	2.55	57.2	110.8	83.3	27.5
20	0.273	0.40	62.3	50.2	33.8	16.4
21	0.264	1.57	63.3	89.1	61.8	27.3
22	0.240	1.64	62.0	87.2	63.1	24.1
23	0.245	3.25	56.0	145.0	116.0	29.0
24	0.260	4.76	58.0	209.4	178.0	31.4
25	0.335	0.25	59.0	47.2	32.5	14.7
26	0.339	1.94	58.3	111.7	90.0	21.7
27	0.338	2.97	61.0	161.8	133.0	28.8
28	1	0.47	59.3	81.3	67.2	14.1
29	1	0.47	61.0	80.8	67.9	12.9
30	1	1.60	61.6	148.5	128.4	20.1
31	1	2.70	61.5	216.5	191.0	25.5
32	1	2.70	61.5	215.5	191.0	24.5
33	0.50	0.70	61.5	15.8	0.0	15.8
34	0.120	6.00	60.0	132.5	107.5	25.0
35	0.106	7.00	53.0	125.0	103.0	22.0
36	0.120	7.50	62.0	172.0	148.0	24.0
37	0.150	4.57	55.0	102.3	86.5	15.8
38	0.262	4.50	59.0	149.2	132.3	16.9
39	0.293	4.55	59.0	171.8	153.8	18.0
40	0.371	4.40	60.0	195.3	177.2	18.1
41	0.348	2.75	58.0	85.1	73.1	12.0
42	0.348	2.75	58.5	84.8	71.1	13.7
43	0.440	3.48	62.0	151.0	134.3	16.7
44	0.111	3.30	62.0	12.8	0.0	12.8
45	0.50	1.20	62.0	12.3	0.0	12.3
46	1	0.50	62.0	10.45	0.0	10.45

Table XXXI gives the results of a large number of brake-tests made on a Corliss engine at Creusot by M. F. Delafond, both with and without a vacuum, and with varying steam-pressures and cut-off. The tests with a vacuum are plotted on Fig. 61, and those without a vacuum are given in Fig. 62. In both figures the abscissæ are the indicated horse-powers, and the ordinates are the friction horse-powers. Most of the tests are represented by dots; those tests which were made with the most economical cut-off (one-sixth for the engine with conden-

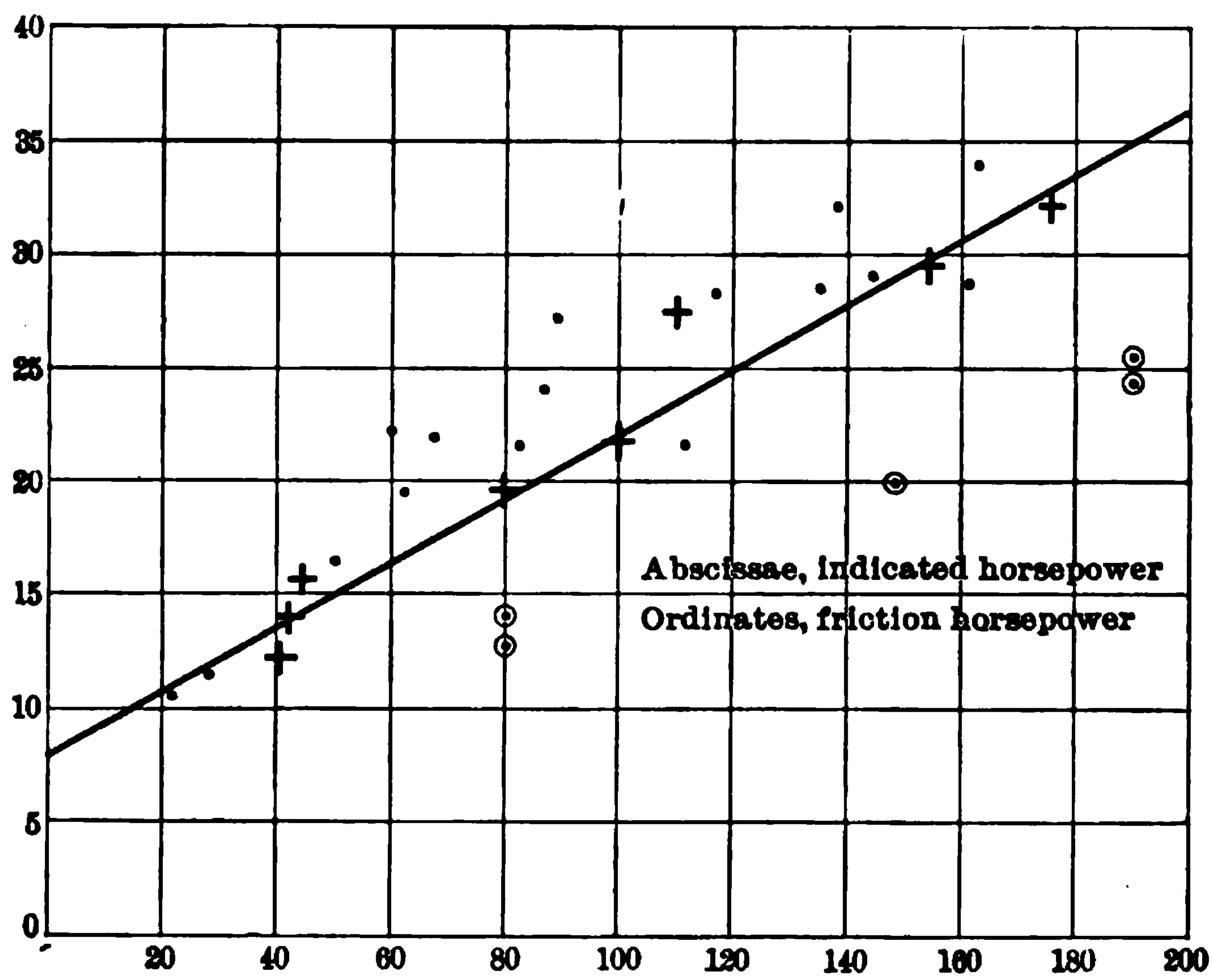

FIG. 61.

sation and one-third without) are represented by crosses. A few tests with very long cut-off, on Fig. 61, are represented by circles. The straight lines on both figures are drawn to represent the tests indicated by crosses. In general the points representing tests with short cut-off and high steam-pressure lie above the lines, and points representing tests with long cut-off and low steam-pressure lie below the lines, though there are some notable exceptions to this rule. The circles on Fig. 61, representing tests with cut-off near the end of the stroke, show much less

friction than the other tests. The tests on this engine show clearly that both initial and load friction are affected by the cut-off and the steam-pressure, and that friction tests should be made at the cut-off which the engine is expected to have in service.

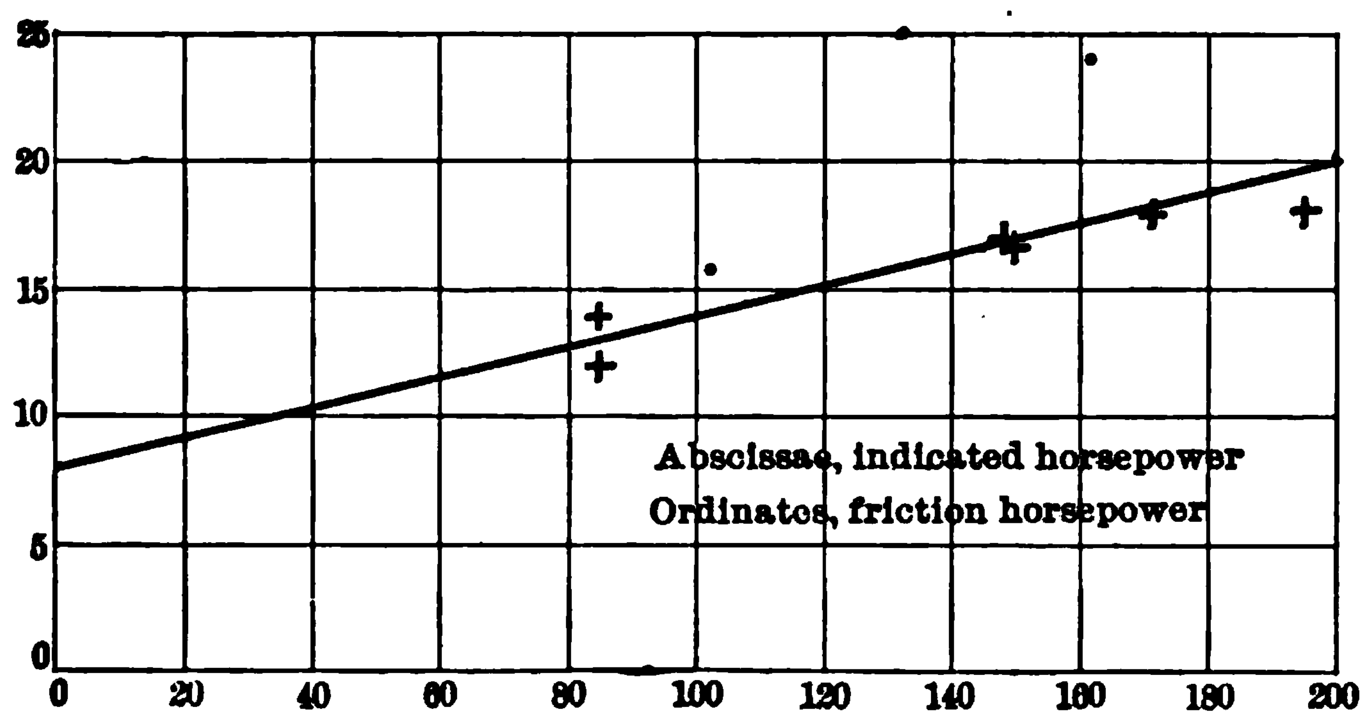

FIG. 62.

The initial friction was eight horse-power both with and without condensation. But p. 250 shows that the engine with condensation gave the best economy when it indicated 160 horse-power; the friction was then 30 horse-power, so that the net horse-power was 130, which will be taken for the normal horse-power P_n. Consequently

$$a = 8 \div 130 = 0.06;$$
$$b = (30 - 8) \div 130 = 0.17.$$
$$\therefore \text{I.H.P.} = 0.062P_n + 1.17P.$$

In like manner Fig. 62 shows the best economy without condensation, for about 200 indicated horse-power, for which the friction is 20 horse-power, leaving 180 for the normal power of the engine. Consequently

$$a = 8 \div 180 = 0.045;$$
$$b = (20 - 8) \div 180 = 0.07.$$
$$\therefore \text{I.H.P.} = 0.045P_n + 1.07P.$$

This engine with condensation had 36 horse-power expended

in friction, when developing 200 horse-power; without condensation it had 20; consequently the air-pump can be charged with

$$(36 - 20) \div 200 = 0.08$$

of the indicated power. The large percentage is probably due to the high vacuum maintained.

Thurston's Experiments. — As a result of a large number of tests on non-condensing engines, made under his direction or with his advice, Professor R. H. Thurston * concluded that, for engines of that type, the friction is independent of the load, and that it can, in practice, be determined by indicating the engine without a load.

TABLE XXXII.

FRICTION OF NON-CONDENSING ENGINE.

STRAIGHT-LINE ENGINE, 8 INCHES DIAMETER, 14 INCHES STROKE.

No. of Diagram.	Boiler-Pressure.	Revolutions.	Brake H.P.	I.H.P.	Frictional H.P.
1	50	232	4.06	7.41	3.35
2	65	229	4.98	7.58	2.60
3	63	230	6.00	10.00	4.00
4	69	230	7.00	10.27	3.27
5	73	230	8.10	11 75	3.65
6	77	230	9.00	12.70	3.70
7	75	230	10.00	14.02	4.02
8	80	230	11.00	14.78	3.78
9	80	230	12.00	15.17	3.17
10	85	230	13.00	15.96	2.96
11	75	230	14.00	16.86	2.86
12	70	230	15.00	17.80	2.80
13	72	231	20.10	22.07	1.97
14	75	230	25.00	28.31	3.31
15	60	229	29.55	33.04	3.40
16	58	229	34.86	37.20	2.34
17	70	229	39.85	43.04	3.19
18	85	230	45.00	47.79	2 78
19	90	230	50.00	52.60	2.60
20	85	230	55.00	57.54	2.54

Table XXXII gives the details of one series of tests. The friction horse-power is small in all the tests, and the variations are small and irregular, and appear to depend on the state of

* *Trans. of the Am. Soc. of Mech. Engrs.*, vols. viii, ix, and x.

lubrication and other minor causes rather than on the change of load.

Distribution of Friction. — As a consequence of his conclusion in the preceding section, Professor Thurston decided that the friction of an engine may be found by driving it from some external source of power, with the engine in substantially the same condition as when running as usual, but without steam in its cylinder, and by measuring the power required to drive it by aid of a transmission dynamometer. Extending the principle, the distribution of friction among the several members of the engine may be found by disconnecting the several members, one after another, and measuring the power required to run the remaining members.

The summary of a number of tests of this sort, made by Professor R. C. Carpenter and Mr. G. B. Preston, are given in Table XXXIII. Preliminary tests under normal conditions showed that the friction of the several engines was practically the same at all loads and speeds.

The most remarkable feature in this table is the friction of the main bearings, which in all cases is large, both relatively and absolutely. The coefficient of friction for the main bearings, calculated by the formula

$$f = \frac{33{,}000 \text{ H.P.}}{pcn},$$

is given in Table XXXIV. p is the pressure on the bearings in pounds for the engines light, and *plus* the mean pressure on the piston for the engines loaded; c is the circumference of the bearings in feet; n is the number of revolutions per minute, and H.P. is the horse-power required to overcome the friction of the bearings.

The large amount of work absorbed by the main bearings and the large coefficient of friction appear the more remarkable from the fact that the coefficient of friction for car-axle journals is often as low as one-tenth of one per cent, the difference being probably due to the difference in the methods of lubrication.

TABLE XXXIII.

DISTRIBUTION OF FRICTION.

Parts of Engine.	Percentages of Total Friction.				
	Straight-line 6×12 Balanced Valve.	Straight-line 6″×12″ Unbalanced Valve.	7″×10″ Lansing Iron Works—Traction Locomotive Valve-Gear.	12″×18″ Lansing Iron Works—Automatic Balanced Valve.	21″×20″ Lansing Iron Works—Condensing Balanced Valve.
Main Bearings	47.0	35.4	35.0	41.6	46.0
Piston and Rod	32.9	25.0	21.0	49.1	21.8
Crank Pin	6.8	5.1	13.0		
Cross Head and Wrist Pin	5.4	4.1			
Valve and Rod	2.5	26.4	22.0	9.3	21.0
Eccentric Strap	5.3	4.0			
Link and Eccentric . .	...	...	9.0	. .	...
Air-Pump	...	...	...	...	12.0
Total	100.0	100.0	100.0	100.0	100.0

TABLE XXXIV.

COEFFICIENT OF FRICTION FOR THE MAIN BEARINGS OF STEAM-ENGINES.

Engine.	F.H.P. due to Main Journal.	Weight on Journals in Pounds.	Diameter of Journal in Inches.	Coefficient of Friction, Engine Light.	Coefficient of Friction, Engine Loaded.	Revolutions of Journal per Minute.
6″×12″ Straight-line . . .	0.85	1500	3	.10	.06	230
*12″×18″ Automatic (L. I. W.)	3.70	2600	5	.19	.05	190
7″×10″ Traction (L. I. W.) .	0.68	500	2¾	.31	.08	200
21″×20″ Condensing (L. I. W.)	3.30	4000	5½	.09	.04	206

* The 12″×18″ automatic engine was new, and gave, throughout, an excessive amount of friction as compared with the older engines of the same class and make.

The second and obvious conclusion from Table XXXIII is that the valve should be balanced, and that nine-tenths of the friction of an unbalanced slide-valve is unnecessary waste.

The friction of the piston and piston-rod is always considerable, but it varies much with the type of the engine, and with differences in handling. It is quite possible to change the effective power of an engine by screwing up the piston-rod stuffing-box too tightly. The packing of both piston and rod should be no tighter than is necessary to prevent perceptible leakage, and is more likely to be too tight than too loose.

CHAPTER XIV.

INTERNAL-COMBUSTION ENGINES.

RECENT advances in the generation of power from heat have been found in the development of internal-combustion engines and of steam-turbines; the latter will be treated in Chapter XIX. When first introduced the only convenient fuel for internal-combustion or gas-engines was illuminating-gas, which limited their use to small sizes, for which convenience and small cost of attendance offset the cost of fuel. Twenty years ago an engine of fifty horse-power was a large though not an unusual size. At that time Mr. Dowson had succeeded in generating gas from anthracite coal and from coke in his producer. Ten years ago engines of 400 horse-power were built to use Dowson producer gas, but as they had four cylinders the horse-power per cylinder was only twice that of single-cylinder engines of a decade earlier; the fuel used in the producer was a cheap grade of anthracite. At the present time, gas-engines are in use which develop as much as 1500 horse-power per cylinder; these engines are of the two-cycle double-acting type. The application of gas-engines to marine propulsion may now be considered to be fairly under way, though as yet the vessels so propelled have been of small displacement; certain British firms of shipbuilders have plans matured for the application of such engines to the propulsion of large ships.

Hot-air Engines. — Though the attempt to develop hot-air engines on a large scale appears to be definitely abandoned, and though the interest of this type of engine is mainly historical a brief discussion of them has some advantage, for, after all, the internal-combustion engine is a hot-air engine in which heat is applied by burning fuel in the cylinder.

In the discussion of the second law of thermodynamics (see

page 39) it was pointed out that to obtain the maximum efficiency all the heat must be added at the highest practicable temperature, and the heat rejected must be given up at the lowest temperature. The hot-air engine is the only attempt to follow the example of Carnot's engine by supplying heat to and withdrawing heat from a constant mass of working substance (air). An attempt to obtain the diagram of Carnot's cycle from such an engine would involve the difficulty that the acute angle at which the isothermal and adiabatic lines for air cross, gives a very long and attenuated diagram that could be obtained only by an excessively large working cylinder, with so much friction that the effective power delivered by the engine would be insignificant. This is illustrated by Problem 20, page 75. To obviate this difficulty Stirling invented the economizer or regenerator which replaced the adiabatic lines by vertical lines of constant volume, and thus obtained a practical machine. His type of engine is still employed, but only for very small pumping-engines which are used for domestic purposes, as they are free from danger and require little attention.

Stirling's Engine. — This engine was invented in 1816, and was used with good economy for a few years, and then rejected because the heaters, which took the place of the boiler of a steam-engine, burned out rapidly; the small engines now in use have little trouble on this account. It is described and its performance given in detail by Rankine in his "Steam-Engine." An ideal sketch is given by Fig. 63. E is a displacer piston filled with non-conducting material, and working freely in an inner cylinder. Between this cylinder and an outer one from A to C is placed a regenerator made of plates of metal, wire screens, or other material, so arranged that it will readily take heat from or yield heat to air passing through it. At the lower end both cylinders have a hemispherical head; that of the outer cylinder is exposed to the fire of the furnace, and that of the

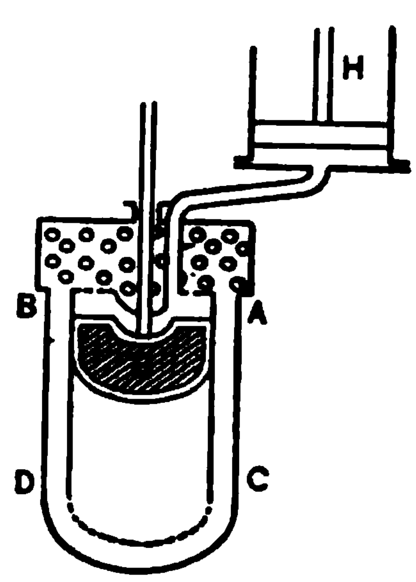

FIG. 63.

inner is pierced with holes through which the air streams when displaced by the plunger. At the upper end there is a coil of pipe through which cold water flows. The working cylinder H has free communication with the upper end of the displacer cylinder, and consequently it can be oiled and the piston may be packed in the usual manner, since only cool air enters it.

In the actual engine the cylinder H is double-acting, and there are two displacer cylinders, one for each end of the working cylinder.

If we neglect the action of the air in the clearance of the cylinder H and the communicating pipe, we have the following ideal cycle. Suppose the working piston to be at the beginning of the forward stroke, and the displacer piston at the bottom of its cylinder, so that we may assume that the air is all in the upper part of that cylinder or in the refrigerator, and at the lowest temperature T_2, the condition of one pound of air being represented by the point D of Fig. 64. The displacer piston is then moved quickly by a cam to the upper end of the stroke; while the working piston moves so little that it may be considered to be at rest. The air is thus all driven from the upper end of the displacer cylinder through the regenerator, from which it takes up heat abandoned during the preceding return stroke, thereby acquiring the temperature T_1, and enters the lower end of that cylinder. During this process the line AD of constant volume is described on Fig. 64. When this process is complete, the working cylinder makes the forward stroke, and the air expands at constant temperature, this part of the cycle being represented by the isothermal AB of Fig. 64. At the end of the forward stroke the displacer piston is quickly moved down, thereby driving the air through the regenerator, during which process heat is given up by the air, into the upper part of the displacer cylinder; this is accompanied by a cooling at constant volume, represented by the line BC. The working piston then makes the return stroke, compressing the air at con-

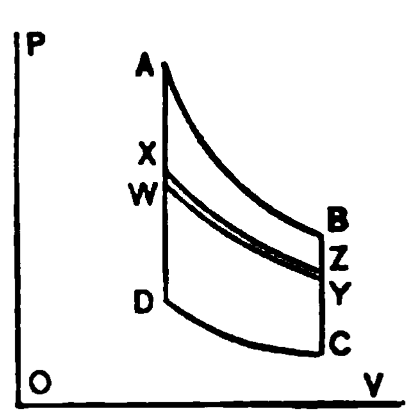

FIG. 64.

stant temperature, as represented by the isothermal line CD, and completing the cycle.

To construct the diagram drawn by an indicator, we may assume that in the clearance of the cylinder H, the communicating pipe, and refrigerator there is a volume of air which flows back and forth and changes pressure, but remains at the temperature T_2. If we choose, we may also make allowance for a similar volume which remains in the waste spaces at the lower end of the displacer cylinder, at a constant temperature T_1.

In Fig. 65, let $ABCD$ represent the cycle of operations, without any allowance for clearance or waste spaces; the minimum volume will be that displaced by the displacer piston, while the maximum volume is larger by the volume displaced by the working piston. Let the point E represent the maximum pressure, the same as that at A; and the united volumes of the clearance at one end of the working cylinder, of the communicating pipe,

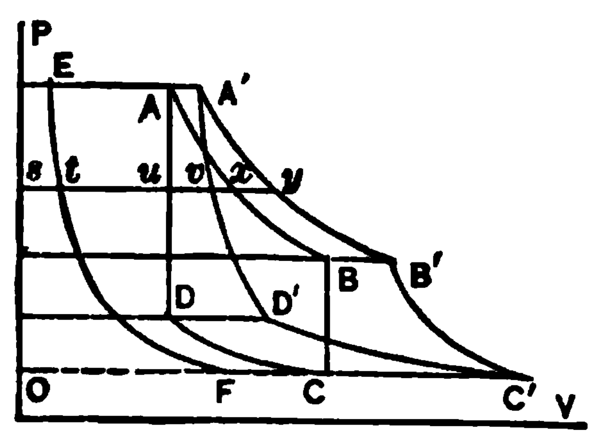

FIG. 65.

of the clearance at the top and bottom of the displacer cylinder, and the volume in the refrigerator and regenerator. Each part of this combined volume will have a constant temperature, so that the volume at different pressures will be represented by the hyperbola EF. To find the actual diagram $A'B'C'D'$, draw any horizontal line, as sy, cutting the true diagram at u and X, and the hyperbola EF at t; make uv and xy equal to st; then v and y are points of the actual diagram. The indicator will draw an oval similar to $A'B'C'D'$ with the corners rounded.

The diagram in Fig. 66 was reduced from an indicator-diagram from a hot-air engine made on the same principle

as Stirling's hot-air engine. To avoid destruction of the lubricant in the working cylinder Stirling found it advisable to connect only the cool end of the displacer cylinder with the working cylinder, and had two displacer cylinders for one working cylinder. It has been found that a good mineral oil can be used to lubricate the displacer piston, and that the hot end also of the displacer cylinder can be advantageously connected with the working cylinders, of which there are two. Thus each working cylinder is connected with the hot end of one displacer cylinder and with the cool end of the other displacer cylinder.

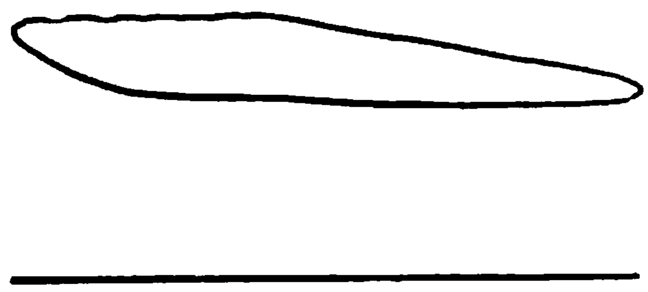

FIG. 66.

The distortion of the diagram Fig. 66 is due in part to the large clearance and waste space, and partly to the fact that the displacer pistons are moved by a crank at about 70 degrees with the working crank.

A test on the engine mentioned by Messrs. Underhill and Johnson * showed a consumption of 1.66 of a pound of anthracite coal per horse-power per hour; but the friction of the engine is large, so that the consumption per brake horse-power is 2.37 pounds. This engine, like the original Stirling engine, appears to have given much difficulty from the burning of the heaters. The difficulty is likely to be more serious with large than with small engines, as the volume of the displacer cylinders increases more rapidly than the heating surface.

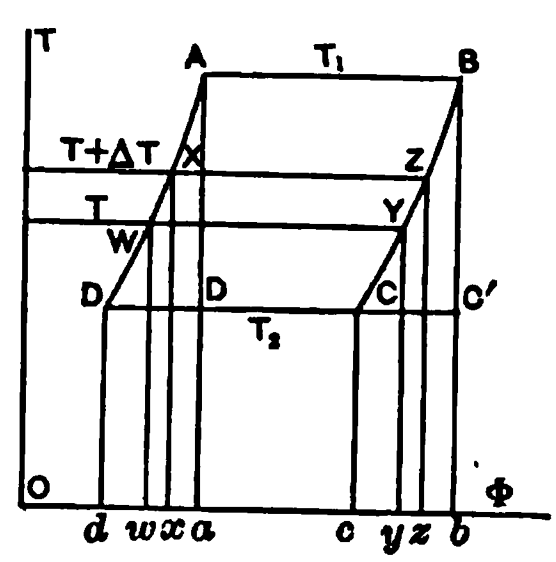

FIG. 67.

The action of the regenerator may be best explained by redrawing the diagram Fig. 64 on the temperature-entropy plane as shown in Fig. 67, where *AB* and *CD* are constant temperature lines representing

* Thesis, M. I. T. 1889.

isothermal expansion, and DA and BC take the place of the constant volume lines on Fig. 64. To show that these lines are properly drawn, we may consider the equation

$$d\phi = c_v \frac{dt}{T} + (c_p - c_v) \frac{dv}{v}$$

which was deduced on page 67. For the lines DA and BC the volumes are constant, so that the equation reduces to

$$d\phi = c_v \frac{dt}{T};$$

or transposing,

$$\frac{dt}{d\phi} = \frac{T}{c_v};$$

but this last expression represents the tangent of the angle between the axis $O\Phi$ and the tangent to the curve. This angle increases (but with a diminishing ratio) with the temperature, and as c_v is constant for a gas, the angle depends only on the temperature T, so that the curve BC is identical in form with the curve AD, and is merely set off further to the right; in consequence, parts like WX and ZY between a pair of constant temperature lines are identical except in their positions with regard to the axis OT.

Suppose now that the material of the regenerator has the temperature T_1 at the lower end and T_2 at the upper end, and that the temperature varies regularly from bottom to top. Suppose further that the air when giving heat to the regenerator (or receiving heat from it) differs from it by only an inappreciable amount. Then the diagram of Fig. 67 will represent this ideal action correctly, and it is easy to show that its efficiency is the same as that of Carnot's cycle $ABC'D'$. For the amount of heat acquired by the regenerator during the operation represented by BC, corresponding to the down stroke of the displacer piston, is measured by the area $bBCc$; and the heat yielded during the up stroke DA, is represented by the area $dDAa$; and these two areas are manifestly equal.

Moreover, the small amount of heat gained during the operation ZY at the temperature T is exactly counterbalanced by the heat yielded during the operation XW at the same temperature, so that there is no loss of efficiency; the small amounts of heat mentioned are represented by the equal areas $zZYy$ and $wWXx$.

It can be shown that one of the curves like DA may be drawn at random, provided that the other curve like BC is made identical and set off further to the right; but the matter is not of importance enough to warrant its discussion.

In practice a regenerator must be at an appreciably lower temperature than the air from which it receives heat, and at a higher temperature than that to which it yields heat, as the flow of air is rapid. The loss of heat stored and restored per cycle of the original Stirling engine was estimated at five per cent to ten per cent. It may be proper before passing from the subject state that regenerators are not applicable to gas-engines in use at the present day.

Gas-Engines. — The chief difficulty with hot-air engines is to transmit heat to and from the working substance. In gas-engines this difficulty is removed by mixing the fuel with the air (so that heat is developed in the working substance itself), and by rejecting the hot gases after they have done their work. The fuel may be illuminating-gas, fuel-gas, or vapor of a volatile liquid like gasoline. It will be shown that the specific volume and the specific heat of the mixture of air and gas, both before and after the heat is developed by combustion, are not very different from the same properties of air. The general theory of gas-engines may therefore be developed on the assumption that the working substance is air, which is heated and cooled in such a manner as to produce the ideal cycles to be discussed, as is done by Clerk.*

Experience has shown that in order to work efficiently, the mixture of gas and air supplied to a gas-engine must be compressed to a considerable pressure before it is ignited. This may be done either by a separate compressor or in the cylinder of the

* *The Gas and Oil Engine:* Dugald Clerk.

engine itself; the second type of engines, of which the Otto engine is an example, is the only successful type at the present time; the other type has some advantages which may lead to its development.

Gas-Engine with Separate Compressor. — This engine has a compressor, a reservoir, and a working cylinder. When run as a gas-engine a mixture of gas and air is drawn into a pump or compressor, compressed to several atmospheres, and forced into a receiver. On the way from the receiver to the working cylinder the mixture is ignited and burned so that the temperature and volume are much increased. After expansion in the working cylinder the spent gases are exhausted at atmospheric pressure.

The ideal diagram is represented by Fig. 68. *ED* represents the supply of the combustible mixture to the compressor, *DA* is the adiabatic compression, and *AF* represents the forcing into the receiver. *FB* represents the supply of burning gas to the working cylinder, *BC* represents the expansion, and *CE* the exhaust. In practice this type of engine always has a release, represented by *GH*, before the expansion has reduced the pressure of the working substance to that of the atmosphere.

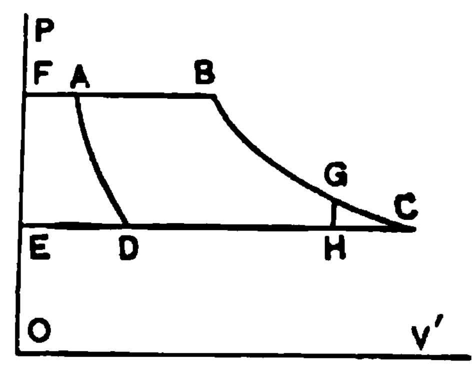

Fig. 68.

This type of engine has been used as an oil-engine by supplying the fuel in the form of a film of oil to the air after it has been compressed. In such case the compressor draws in air only, and there is not an explosive mixture in the receiver. The Brayton engine when run in this way could burn crude petroleum, or, after it was started, could burn refined kerosene. Its chief defect appears to have been incomplete combustion and consequent fouling of the cylinder with carbon.

The effective cycle may be considered to be represented by the diagram *ABCD* (Fig. 68), and may be assumed to be produced in one cylinder by heating the air from *A* to *B*, by cooling it from *C* to *D*, and by the adiabatic expansion and compression from *B* to *C* and from *D* to *A*. If T_a and T_b are the absolute

temperatures corresponding to the points A and B, then the heat added from A to B is

$$c_p\,(T_b - T_a),$$

and the heat withdrawn from C to D is

$$c_p\,(T_c - T_d),$$

so that the efficiency of the ideal cycle is

$$e = \frac{c_p\,(T_b - T_a) - c_p\,(T_c - T_d)}{c_p\,(T_b - T_a)} = 1 - \frac{T_c - T_d}{T_b - T_a}. \quad (177)$$

But since the expansion and compression are adiabatic,

$$\frac{T_c}{T_b} = \left(\frac{p_c}{p_b}\right)^{\frac{\kappa-1}{\kappa}}, \quad \frac{T_d}{T_a} = \left(\frac{p_d}{p_a}\right)^{\frac{\kappa-1}{\kappa}};$$

but $p_c = p_d$ and $p_b = p_a$, therefore

$$\frac{T_c}{T_b} = \frac{T_d}{T_a} \text{ and } \frac{T_c - T_d}{T_b - T_a} = \frac{T_d}{T_a} = \left(\frac{p_d}{p_a}\right)^{\frac{\kappa-1}{\kappa}};$$

so that the equation for efficiency becomes

$$e = 1 - \left(\frac{p_d}{p_a}\right)^{\frac{\kappa-1}{\kappa}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (178)$$

This discussion of ideal efficiency is due to Dugald Clerk,* and has the advantage of replacing an exceedingly complex operation by a simple ideal operation which has approximately the same efficiency. How far the ideal cycle can be used to determine the probable advantages of certain conditions depends on the degree of approximation, — a matter which will be referred to later. It must be admitted that the divergence of the actual from the real cycle is much greater than the divergence of the steam-engine cycle from that of a non-conducting cylinder.

For example, with the pressure in the reservoir at 90 pounds

* *The Gas Engine*, 1886; *The Gas and Oil Engine*, 1896.

above the atmosphere the efficiency is

$$e = 1 - \left(\frac{14.7}{14.7 + 90}\right)^{\frac{1.405 - 1}{1.405}} = 0.43.$$

When the cycle is incomplete the expression for the efficiency is not so simple, for it is necessary to assume cooling at constant volume from G to H (Fig. 68), and cooling at constant pressure from H to D; so that the heat rejected is

$$c_v\ (T_g - T_h) + c_p\ (T_h - T_d),$$

and the efficiency becomes

$$e = 1 - \frac{\frac{1}{\kappa}(T_g - T_h) + (T_h - T_d)}{T_b - T_a} \quad . \quad . \quad . \quad . \quad (179)$$

For example, let it be assumed that the pressure at A is 90 pounds above the atmosphere, that the temperature at B is 2500° F., and that the volume at G is three times the volume at B.

First, the temperature at A is

$$T_a = T_d\left(\frac{p_a}{p_d}\right)^{\frac{\kappa - 1}{\kappa}} = (60 + 460)\left(\frac{14.7 + 90}{14.7}\right)^{\frac{0.405}{1.405}} = 917,$$

provided that the temperature of the atmosphere is 60° F.

The temperature at G is

$$T_g = T_b\left(\frac{v_b}{v_g}\right)^{\kappa - 1} = 2960\ \left(\frac{1}{3}\right)^{0.405} = 1897,$$

and the pressure at G is

$$p_g = p_b\left(\frac{v_b}{v_g}\right)^{\kappa} = (14.7 + 90)\left(\frac{1}{3}\right)^{1.405} = 22.4 \text{ pounds},$$

so that the temperature at H is

$$T_h = T_g\frac{p_h}{p_g} = 1897 \times \frac{14.7}{22.4} = 1247,$$

and finally the efficiency is

$$e = 1 - \frac{\frac{1}{1.405}\ (1897 - 1247) + 1247 - 520}{2960 - 917} = 0.42.$$

Gas-Engines with Compression in the Cylinder. — All successful gas-engines of the present day compress the explosive mixture in the working cylinder. Very commonly they take gas at one end of the cylinder only, and require four strokes to complete the cycle, so that there is one explosion for two revolutions when working at full power. Such engines are commonly known as *four-cycle* engines. Some engines have the exhaust and filling of the cylinder accomplished in some other way, and are known as *two-cycle* engines; they have an explosion for every revolution when single-acting. Both four-cycle and two-cycle engines have been made double-acting in large sizes. The first forward stroke of the piston from the head of the cylinder draws in the mixture of gas and air, which is compressed on the return stroke; at the completion of this return stroke the mixture is ignited and the pressure rises very rapidly; the next forward stroke is the working stroke, which is succeeded by an exhaust-stroke to expel the spent gases. In almost all engines these four strokes are of equal length, for the advantage of making them of unequal length, as required for the best ideal cycle, is more than counterbalanced by the mechanical difficulty of producing unequal strokes.

The most perfect ideal cycle, represented by Fig. 69, has four strokes of unequal length so arranged that the piston starts from the head of the cylinder when gas is drawn in, and the pressure in the cylinder is reduced to that of the atmosphere before the exhaust stroke. Thus there is the filling stroke, represented by *EC*; the compression stroke, represented by *CD*; the working stroke, represented by *AB*; and the exhaust stroke, represented by *BE*.

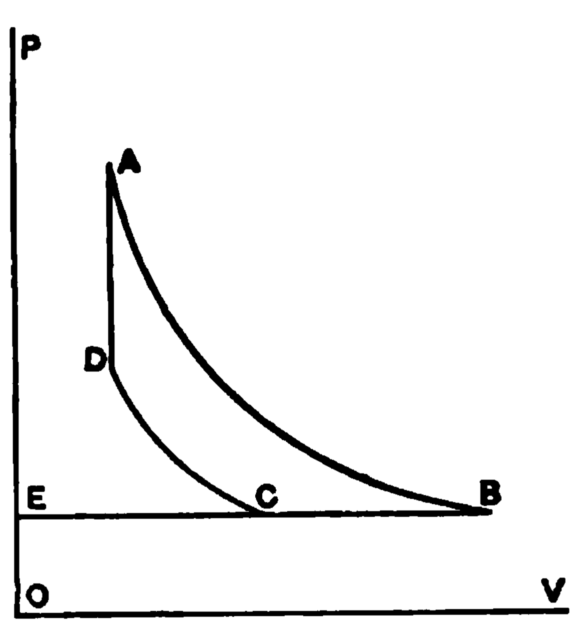

FIG. 69.

The effective cycle is *ABCD*, which may be considered to be performed by adding heat at constant volume from *D* to *A*,

and withdrawing heat at constant pressure from B to C, together with the adiabatic expansion and compression AB and CD.

The heat added under this assumption is

$$c_v(T_a - T_d),$$

and the heat rejected is

$$c_p\,(T_b - T_c),$$

so that the efficiency is

$$e = \frac{c_v\,(T_a - T_d) - c_p\,(T_b - T_c)}{c_v\,(T_a - T_d)} = 1 - \kappa\,\frac{T_b - T_c}{T_a - T_d}\,. \qquad (180)$$

If the temperature at A and the pressure at D are assumed, then it is necessary to make preliminary calculations of the temperatures at D and at B before using equation (180). Thus, adiabatic compression from C to D gives for the temperature at D

$$T_d = T_c\left(\frac{p_d}{p_c}\right)^{\frac{\kappa - 1}{\kappa}} \quad \ldots\ldots \quad (181)$$

in like manner adiabatic expansion from A to B gives

$$T_b = T_a\left(\frac{p_b}{p_a}\right)^{\frac{\kappa - 1}{\kappa}} \quad \ldots\ldots \quad (182)$$

in which the value of p_a may be calculated by the equation

$$p_a = p_d\,\frac{T_a}{T_d} \quad \ldots\ldots \quad (183)$$

since the pressure rises with the temperature at constant volume from D to A.

For example, if the pressure at the end of compression is 90 pounds above the atmosphere, and the temperature at the end of the explosion is 2500° F., then

$$T_d = (60 + 460)\left(\frac{14.7 + 90}{14.7}\right)^{\frac{0.405}{1.405}} = 917,$$

provided that the temperature of the atmosphere is 60° F.

$$p_a = 104.7 \frac{2500 + 460}{917} = 338 \text{ pounds;}$$

$$T_b = (2500 + 460)\left(\frac{14.7}{338}\right)^{\frac{0.405}{1.405}} = 1199;$$

$$e = 1 - 1.405 \frac{1199 - 520}{2960 - 917} = 0.55.$$

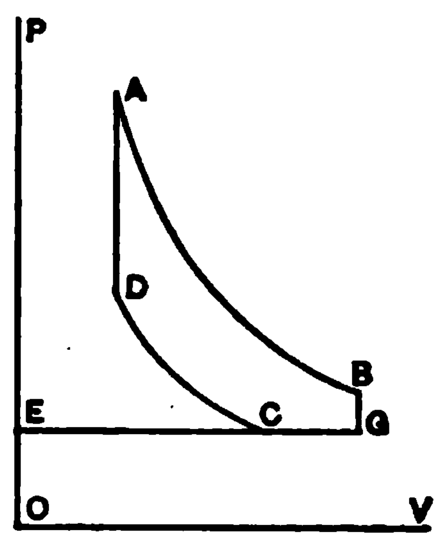

Fig. 70.

If the expansion is not carried to the atmospheric pressure, then the diagram shows a release at the end of the stroke, as in Fig. 70, and the cycle must be considered to be formed by adding heat as before at constant volume, but by withdrawing heat at constant volume to cause a loss of pressure from B to G, and by withdrawing heat at constant pressure, during the process represented by GC. The heat rejected becomes, therefore,

$$c_v(T_b - T_g) + c_p(T_g - T_c),$$

and the efficiency is

$$e = \frac{c_v(T_a - T_d) - c_v(T_b - T_g) - c_p(T_g - T_c)}{c_v(T_a - T_d)}$$

$$= 1 - \frac{T_b - T_g + \kappa(T_g - T_c)}{T_a - T_d}. \quad . \quad . \quad . \quad . \quad . \quad (184)$$

Assuming, as before, the pressure at D and the temperature at A, it becomes necessary to find the temperatures at B and at G as well as the temperature at D; this last may of course be found by equation (181). If the pressure at B is assumed also, then equations (182) and (183) may be used as before to find T_b; and T_g may be found by the equation

$$T_g = T_b \frac{p_g}{p_b} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (185)$$

For example, let it be assumed that the expansion ceases when the pressure becomes 20 pounds above the atmosphere, the other conditions being as in the previous example. Then

$$T_b = (2500 + 460)\left(\frac{14.7 + 20}{338}\right)^{\frac{0.405}{1.405}} = 1536;$$

$$T_c = 1536\,\frac{14.7}{34.7} = 650;$$

and

$$e = 1 - \frac{1536 - 650 + 1.405\,(650 - 520)}{2960 - 917} = 0.48.$$

Though not essential to the solution of the example, it is interesting to know that the volume at C is

$$\left(\frac{90 + 14.7}{14.7}\right)^{\frac{1}{\kappa}} = 4 +$$

times the volume at D, and that the volume at B is

$$\left(\frac{338}{34.7}\right)^{\frac{1}{\kappa}} = 5 +$$

times the volume at A.

When, as in common practice, the four strokes of the piston are of equal length, the diagram takes the form shown by Fig. 71; the effective cycle may be considered to be equivalent to heating at constant volume from D to A and cooling at constant volume from B to C, together with adiabatic expansion and compression from A to B and from C to D

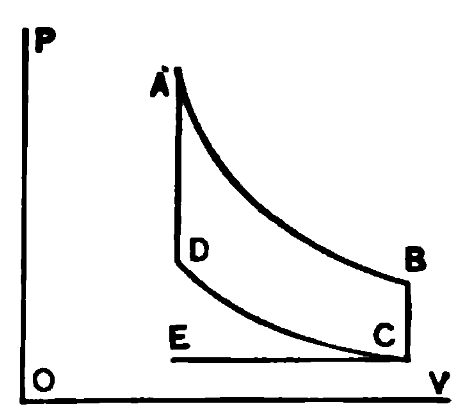

FIG. 71.

The heat applied is

$$c_v\ (T_a - T_d),$$

and the heat rejected is

$$c_v\ (T_b - T_c),$$

so that the efficiency is

$$e = \frac{c_v\,(T_a - T_d) - c_v\,(T_b - T_c)}{c_v(T_a - T_d)} = 1 - \frac{T_b - T_c}{T_a - T_d}. \quad (186)$$

Since the expansion and compression are adiabatic, we have the equations

$$T_b\, v_b^{\kappa - 1} = T_a v_a^{\kappa - 1}, \quad \text{and} \quad T_c v_c^{\kappa - 1} = T_d\, v_d^{\kappa - 1};$$

but the volumes at A and D are equal, as are also the volumes at B and C; consequently by division

$$\frac{T_b}{T_c} = \frac{T_a}{T_d};$$

consequently

$$\frac{T_b - T_c}{T_a - T_d} = \frac{T_b}{T_a} = \frac{T_c}{T_d} = \left(\frac{v_d}{v_c}\right)^{\kappa - 1},$$

and the expression for efficiency becomes

$$e = 1 - \left(\frac{v_d}{v_c}\right)^{\kappa - 1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (187)$$

which shows that the efficiency depends only on the compression before explosion.

For example, if the volume of the clearance or compression space is one-third of the piston displacement, so that v_d is one-fourth of v_c, then the efficiency is

$$e = 1 - \left(\frac{1}{4}\right)^{0.405} = 0.43.$$

The pressure at the end of compression is

$$p_d = p_c\left(\frac{v_c}{v_d}\right)^{\kappa} = 14.7\left(\frac{4}{1}\right)^{1.405} = 103.1$$

pounds absolute, or 88.4 pounds by the gauge. The calculated efficiency is therefore not much less than the efficiencies found for other examples; it is notable that the efficiency is nearly the same as that calculated on page 307 for an engine with separate compression to 90 pounds by the gauge. For the case in hand, however, the pressure after explosion, which depends on the temperature, may exceed 300 pounds per square inch.

The diagrams from engines of this type * resemble Fig. 72,

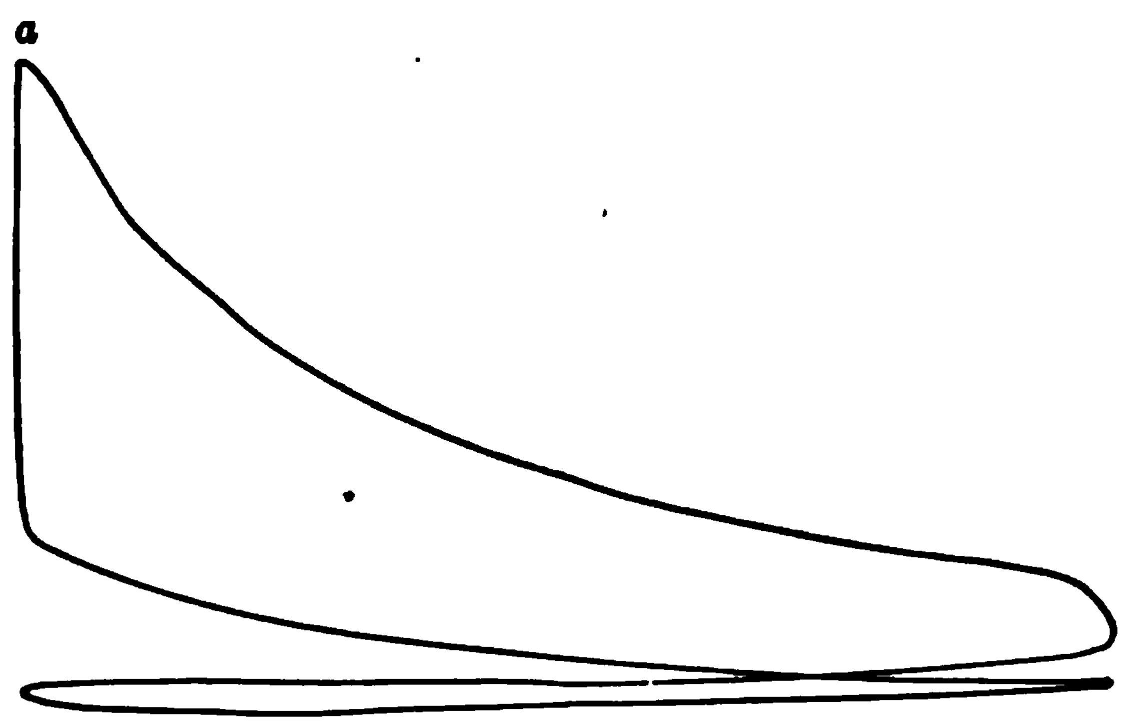

Fig. 72.

which was taken from an Otto engine in the laboratory of the Massachusetts Institute of Technology. During the filling stroke, the pressure in the cylinder is less than that of the atmosphere; the charge is ignited just before the end of the compression stroke, and the explosion though rapid is not instantaneous, as is indicated by the rounding of the corners of the diagram at both the bottom and the top of the explosion line, and by the leaning of that line to the right. Release occurs before the end of the stroke, and there is considerable back pressure during the exhaust stroke. The scale of the diagram is 150 pounds to the inch, and the maximum pressure is 251 pounds. The atmospheric line is omitted to avoid confusion.

In order to show clearly the conditions during the exhaust and filling strokes, the diagram Fig. 73 was taken with a scale

* A description of a four-cycle gas-engine will be found on page 337, and may be read for the first time in this connection.

of 20 to the inch, and with a stop to limit the rise of the indicator-piston; the upper part of the diagram consequently does not appear in the figure. The mean back-pressure is about five pounds, and the reduction of pressure in the cylinder is between

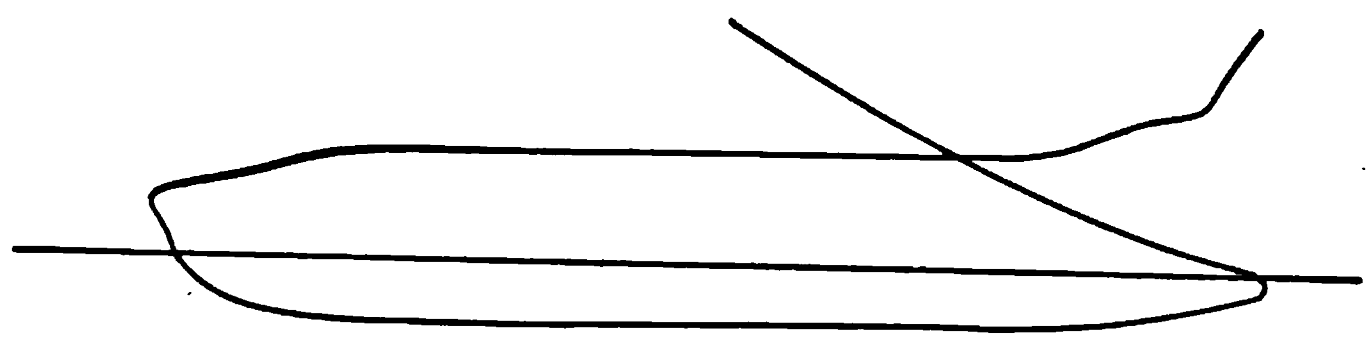

FIG. 73.

three and four pounds below the atmosphere. Reference to the influence of the negative area of Fig. 73 on the effective indicated horse-power will be made later.

The compression line does not differ very much in appearance or in reality from an adiabatic line from air, though the air may be expected to receive heat from the walls of the cylinder during the first part of the compression stroke, and may part with heat during the latter part. The expansion line has a resemblance to the adiabatic line for air, but is usually less steep, especially for large engines; but in reality the conditions in the cylinder are very different, for the combustion does not cease at the maximum pressure, but continues more or less during the expansion stroke, and may extend to the release; and at the same time heat is taken up energetically by the walls of the cylinder, which are cooled by a water-jacket to avoid overheating. These two effects, after-burning and loss of heat to the water-jacket, determine the form of the expansion line and its resemblance to an adiabatic line.

Characteristics of Gases. — There are three distinct kinds of gases used in gas-engines: (1) illuminating-gas, (2) producer-gas, (3) blast-furnace gas. Each class has fairly well-marked characteristics, though there is considerable variation in a class. The greatest variation is liable to be found in blast-furnace gas, since the metallurgical operations are of the first importance,

and, if the gas is to be used for generating power, the engines and adjuncts must be adapted to the conditions. Producer-gas is made from coke, anthracite, or from non-caking bituminous coal, and consists mainly of hydrogen and carbon monoxide, diluted with the nitrogen of the air, together with five or ten per cent of carbon dioxide and a small percentage of hydrocarbons especially when bituminous coal is used. Illuminating-gas is now commonly made by the water-gas process, which yields a gas not very unlike producer-gas, but that gas is enriched with hydrocarbons of varying composition; formerly illuminating-gas was distilled from gas-coal, which was a rich bituminous coal yielding a large percentage of hydrocarbons when distilled.

The general characteristics of illuminating-gas are represented by the following analysis of Manchester coal-gas quoted from the first edition of Clerk's *Gas Engine*, and used by him to investigate the effect of combustion on the volume of the gas.

ANALYSIS OF MANCHESTER COAL-GAS. (Bunsen and Roscoe.)

	Vols.	Vols. O required for Combustion.	Products. Vols.
Hydrogen, H	45.58	22.79	45.58, H_2O
Methane, CH_4	34.9	69.8	104.7, CO_2 & H_2O
Carbon monoxide, CO . .	6.64	3.32	6.64, CO_2
Ethylene, C_2H_4	4.08	12.24	16.32, CO_2 & H_2O
Tetrylene, C_4H_8	2.38	14.28	19.04, CO_2 & H_2O_2
Sulphuretted hydrogen, H_2S	0.29	0.43	0.58, H_2O & SO_2
Nitrogen, N	2.46	......	2.46
Carbon dioxide, CO_2 . . .	3.67	......	3.67
Total	100.00	122.86 O	198.99, CO_2,H_2O & SO_2

An analysis of illuminating-gas made by the water-gas process at Boston gave: Hydrogen 27.9, methane 28.9, carbon monoxide 25.3, carbon dioxide 1.9, hydrocarbons 12.0, nitrogen 3.0, oxygen 1.0; the analysis being only proximate does not allow of a calculation of the oxygen required for combustion.

The following composition of producer-gas was taken from a report of tests on a gas-engine by Professor Meyer, for which

COMPOSITION OF PRODUCER-GAS.

	Vols.	Vols. of Oxygen for Combustion.	Products Vols.
Hydrogen, H	13.7	6.8	13.7 H_2O
Methane, CH_4	0.7	1.4	2.1, CO_2 & H_2O
Carbon monoxide, CO	24.6	12.3	24.6, CO_2
Carbon dioxide, CO_2	6.5	0	6.5, CO_2
Oxygen, O	.5	0	0.5, O
Nitrogen, N	54.0	0	54.0, N
	100	20.5	101.4

details are given on page 350. Eight analyses are given in the original paper, which are here averaged.

Rich non-caking bituminous coals may show a considerably larger proportion of hydrogen.

In a paper on the use of blast-furnace gas Mr. Bryan Donkin gives the composition of gases from five furnaces in England, Scotland, and Germany, from which the average values in the following table were deduced:

COMPOSITION OF BLAST-FURNACE GAS.

	Vols.	Vols. of Oxygen for Combustion.	Products. Vols.
Hydrogen, H	2.5	1.3	2.5, H_2O
Carbon monoxide, CO	29.1	19.6	29.1, CO_2
Carbon dioxide, CO_2	7.0	0	7.0, CO_2
Nitrogen, N.	61.4	0	61.4, N
	100	20.9	100

Not only is there much variation in the composition of gases from different blast-furnaces, but the variation with the progress of the metallurgical operations is so marked that it is customary to mingle the gases from several furnaces in order to insure that the gas is proper for use in gas-engines.

The amounts of oxygen required for the combustion of a given volume of any gas can be computed from the formulæ representing the chemical changes accompanying combustion, together with the fact that a compound gas occupies two volumes, if measured on the same volumetric scale as the component gases. Thus two volumes of hydrogen with one volume of oxygen unite to form superheated steam as represented by the formula

$$2H + O = H_2O,$$

and the three volumes after combustion and reduction to the original temperature are reduced to two volumes; in this case, to have the statement hold, the original temperature would need to be very high, to avoid condensation of the steam into water. But in the application to gas-engines this leads to no inconvenience, because the gases after combustion remain at a high temperature till they are exhausted, and the laws of gases can be assumed to hold approximately. A compound gas like methane can be computed as follows:

$$CH_4 + 4O = CO_2 + 2H_2O.$$

Since the compound gas methane occupies two volumes and requires four volumes of oxygen, it is clear that each cubic foot of that gas will demand two cubic feet of oxygen; the total volume may be reckoned as six before combustion, and in like manner there will be six volumes after combustion, namely, two of carbon dioxide and four of steam.

In this way the oxygen required for combustion of the three kinds of gas for which the compositions are given, has been computed, and also the volumes after combustion. For coal-gas the contraction due to the combustion of hydrogen and carbon monoxide is very nearly compensated by the expansion due to the breaking up and combustion of the hydrocarbons. A similar result may be expected for any illuminating-gas. On the other hand, producer-gas if burned in oxygen would show a contraction of

$$\frac{120.5 - 101.4}{120.5} = \frac{19}{120} = 0.16;$$

but in practice the producer-gas is mixed with 1.3 to 1.5 of its volume of air, so that the contraction of 19 volumes takes place in 230 to 250 volumes, and thus is therefore of 7½ to 8 per cent contraction.

Clearly this matter has to do with the question raised on page 306, as to the reliance to be placed on the ideal efficiencies which assume heating of air instead of combustion of fuel. For illuminating-gas that assumption appears unobjectionable, and for producer-gas the discrepancy is not so great as to destroy the value of the method.

Temperature after Explosion. — The most difficult question concerning the theoretical thermal efficiency of gas-engines is the determination of the temperature after explosion. Direct determination is difficult both on account of the high temperature and the very short interval of time during which the maximum temperature can be considered to exist.

A comparatively simple calculation of the temperature after explosion can be made from a diagram like Fig. 72, if the compression can be assumed to be adiabatic, and if the laws of perfect gases can be applied. The pressure on the compression line measured on an ordinate through the point a of maximum pressure, is 61 pounds, or 75.7 pounds absolute. If the temperature of the gases in the cylinder at atmospheric pressure is taken to be 70 degrees, adiabatic compression gives approximately

$$T_a = (70 + 460)\left(\frac{75.7}{14.7}\right)^{\frac{0.4}{1.4}} = 847°.$$

The maximum pressure after explosion is 251 pounds, or about 266 pounds absolute. If the temperature at constant volume is assumed to be proportional to the absolute pressure, we have

$$847 \times \frac{266}{75.7} = 2975,$$

or about 2500° F. This result, which depends on the assumption that the properties of the charge in the cylinder of a gas-engine

are and remain the same as those of gases at ordinary temperatures, can be taken as a first approximation only.

In connection with tests on a gas-engine (see page 350) using illuminating-gas, Professor Meyer makes a careful investigation of the temperature which might be developed in the cylinder of a gas-engine if the charge were completely burned in a non-conducting cylinder. The results only will be quoted here. The composition of the gas will be found on page 316, from which it appears that it was probably coal-gas resembling Manchester gas, and not differing very radically from Boston gas, by use of which Fig. 72 was obtained. The pressure at the end of compression was 69 pounds by the gauge, and after explosion was 220 pounds, so that the conditions were not very different from those of Fig. 72, except that the pressure on the compression line is not on the ordinate for measuring the maximum pressure, and therefore the parallel calculation cannot be made.

On the assumption of constant specific heats Professor Meyer finds that complete combustion should give 4250° F. in a non-conducting cylinder, but using Mallard and Le Chatelier's equation for specific heats at high temperatures he gets 3330° F. Those experimenters report that dissociation of carbon monoxide begins at about 3200° F., and of steam at about 4500° F.; but the dissociation is slight at those temperatures. Though the subject is still obscure, it appears fair to assume that the failure to reach the temperatures which can be computed for complete combustion, can be charged in part to suppression of combustion on account of the high temperature in the cylinder.

After Burning. — Accompanying the suppression of heat on account of the approach to the temperature of dissociation is the development of heat during expansion which extends in some cases to release, as is indicated by a flicker of flame into the exhaust; explosions in the mufflers of automobiles are attributable to this action. The fact that the expansion curve approaches the adiabatic line during expansion is indirect evidence of after-burning, because the water-jacket withdraws heat at the same

time. The actual expansion line is less steep than the adiabatic for gas, and for large gas-engines can approach the condition represented by the equation

$$pv^{1.2} = \text{const.};$$

but a part of this action can be attributed to the presence of carbon monoxide and steam in the products of combustion, which may reduce the exponent of the adiabatic line from 1.405 to 1.37.

Water-Jackets. — All except very small internal-combustion engines have the heads and barrels of the cylinder cooled by water-jackets; large engines commonly have the pistons cooled with water, and double-acting engines have the piston-rods and stuffing-boxes cooled. Not uncommonly the valves of large engines are cooled, and if such engines use rich gases, extra cooling surface is provided in the charging space or cartridge chamber; the latter device is to avoid pre-ignition, and the former is in part for the same purpose.

Primarily, water-jackets are to protect the metal of the cylinder and to make lubrication possible. The use of jackets and other cooling devices has been considered a mechanical necessity, which many inventors have sought to avoid; but it appears likely that it is only a question whether the heat shall be withdrawn by a water-jacket, or whether the heat shall be suppressed by dissociation and thrown out in the exhaust. Large engines, which have less exposed area per cubic foot of cylinder contents, show a less percentage of heat withdrawn by the jacket, but a larger percentage thrown on in the exhaust; the balance is, however, in favor of large engines which show a better economy.

Economy and Efficiency. — It is customary and altogether desirable to rate the economy of gas-engines and other internal-combustion engines in thermal units per horse-power per minute; this was found to be desirable, if not necessary, for studying the means of improving the performance of steam-engines. But as steam-engines are commonly rated in terms of steam per horse-power per hour, so also gas-engines have been rated in terms of cubic feet of gas per horse-power per hour, and gasoline-

and oil-engines have been rated in pounds of fuel per horse-power per hour. The variation in the fuel used for such engines makes the secondary methods less satisfactory than rating engines on steam-consumption, so that it should be employed only when the calorific capacity of the fuel cannot be determined or estimated.

Since the heat-equivalent of a horse-power is 42.42 thermal units per minute, the actual thermal efficiency of an internal-combustion engine can be determined by dividing that figure by the thermal units consumed by the engine per horse-power per minute. For example, the engine tested by Professor Meyer used about 170 thermal units per horse-power per minute, and its thermal efficiency was 0.25, using the indicated horse-power. The ratio of the cartridge space to the whole volume was $\frac{1}{3.84}$, so that equation (187) gives in this case 0.42 for the nominal theoretical efficiency; consequently the ratio of the efficiencies is nearly 0.60.

By a somewhat intricate method Professor Meyer computed the efficiency for two tests on the engine for which details are given on page 350, on the assumption that complete combustion occurred in a non-conducting cylinder. The ratio of gas to air in one test was one to 8.9, and in the other one to 12. Assuming that the specific heat of the mixture in the cylinder before and after explosion, remained constant, he found for the first test an efficiency of 0.398, and for the second 0.403; but making use of Mallard and Le Chatelier's investigations on specific heats at high temperatures, he found for the efficiencies 0.297 and 0.318. The values for constant specific heat differ but little from the nominal theoretical efficiency; in fact, if the exponent be reduced from 0.405 to 0.38, the nominal efficiency becomes 0.40, which is a very close coincidence. But the efficiencies computed from the heat-consumptions for these two tests are 0.253 and 0.249. If then the nominal theoretical efficiency, or the efficiency which Professor Meyer calculated on assumption of constant specific

heat, be taken as the basis of comparison, the engine gave for the ratio of actual to theoretical efficiency,

$$0.253 \div 0.398 = 0.64, \text{ or } 0.249 \div 0.403 = 0.62.$$

If, however, we take his second values with variable specific heat, we have

$$0.253 \div 0.297 = 0.85, \text{ or } 0.249 \div 0.318 = 0.78.$$

Professor Meyer uses these computations to emphasize the importance of better knowledge of the properties of the working substance in the cylinder of an internal-combustion engine; because, if the nominal theoretical efficiency be taken for the basis of comparison, there appears to be room for material improvement in the economy of the engine; whereas, if the second set of computations is taken as the basis, there is little prospect of improvement. In conclusion, attention is called to the fact that these tests were on a small engine which developed only ten brake horse-power.

In the discussion of efficiency we have thus far made use of the heat-consumption per indicated horse-power, which is proper, because the fluid efficiency (or the efficiency of the action of the working substance) should for this purpose be preserved from confusion with the friction and mechanical efficiency of the engine. For the same reason, and also because the power of a steam-engine can be determined satisfactorily by the indicator, we used indicated horse-power in the discussion of steam-engine economy. There is, however, a reason why the indicated power is not a satisfactory basis for the discussion of the economy of internal-combustion engines, namely, the fact that a series of successive diagrams taken without removing the pencil from the paper on the indicator drum, will show a wide dispersion, due to the varying explosive action in the cylinder even when conditions are most favorable. When the engine is governed by omitting explosions, this difficulty is much aggravated on account of the negative work of idly drawing in, compressing, and expelling air.

Fig. 74 shows a diagram taken from the same engine as Fig. 72, page 313, but with a fifty-pound spring and a stop to prevent

the indicator piston from rising too high which exhibits the effects of an idle cycle and other features. A portion of the expansion curve is shown, with oscillations due to the piston suddenly leaving the stop. The exhaust of the spent gases is

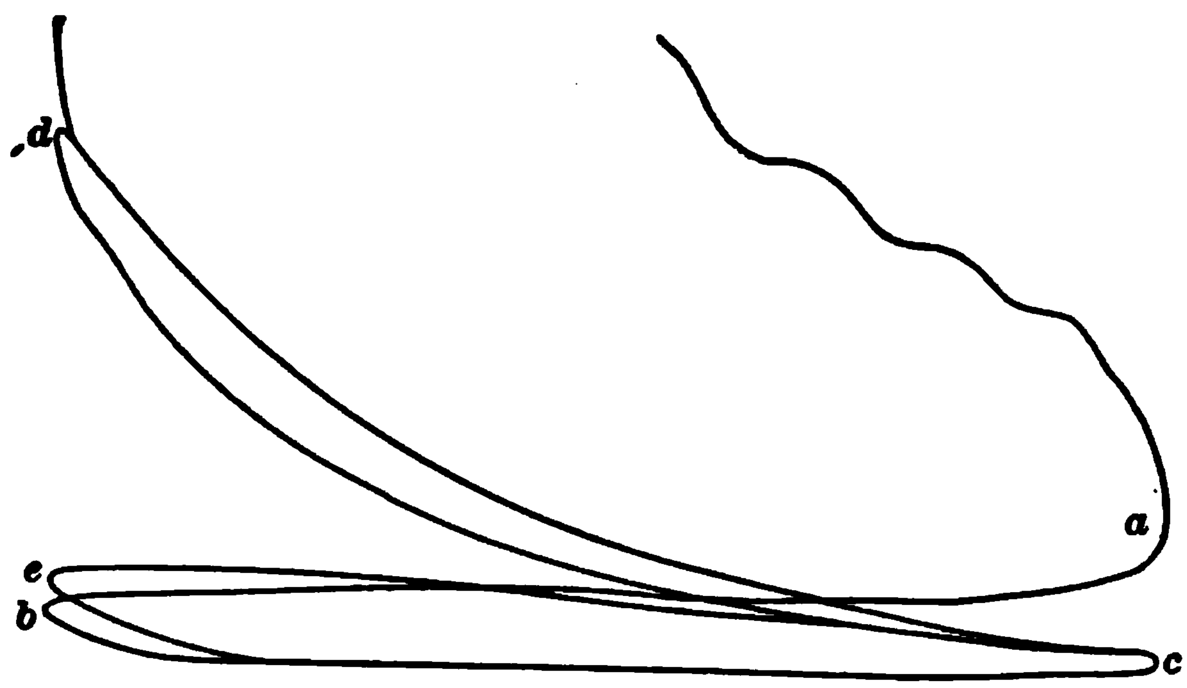

FIG. 74.

shown by the curve *ab*, after which the engine draws a charge of air (without gas) and compresses it on the upper curve from *c* to *d*; on the return stroke the indicator follows the lower curve from *d* to *c*, so that the loop represents work done by the engine; finally the air is exhausted, while the indicator draws the line *ce*. To explain the difference between the exhaust lines *ab* and *ce* with spent gas and with air only, it may be noted that there is a marked drooping of the exhaust line *a* at about one-fifth of the stroke from *b*; this feature is more marked in Fig. 73, which shows the exhaust stroke to a larger scale. This droop may be attributed to the inertia of the column of gas in the exhaust pipe; the smaller volume of air which is exhausted with gradually rising pressure does not happen to develop this feature in such a way as to produce the result shown in Fig. 73. This drop of pressure in the exhaust pipe may be accentuated by adjusting the length of the exhaust pipe so as to give a partial vacuum just before the engine takes its next charge; when this action is obtained, the air-valve is opened before the gas-valve, and fresh air is drawn through the cylinder to produce a scavenging effect before the engine takes a new charge. At one

time considerable importance was given to scavenging to clear out spent gas, but it attracts less importance now for four-cycle engines.

In indicating a gas-engine, allowance is, of course, made for the negative work of exhaust and filling; if an explosion is missed, allowance for the negative work for the operation shown on Fig. 74 should be made for each idle cycle, and when the engine has only a few working cycles the error of not taking proper account of the negative work may be very large. This is, of course, another reason why comparisons are best based on brake horse-power. As can be seen from Table XXXV on page 350, the mechanical efficiency may range from 60 per cent to 80 per cent, depending mainly on the power developed; these figures are for continuous explosions, and the efficiency is liable to be much reduced if explosions are omitted at reduced power.

Two-cycle engines commonly have a compression pump which supplies the mixture of gas and air at a pressure of five or ten pounds above the atmosphere; in such case the work of compression must be determined separately and allowed for, in the measurement of the indicated horse-power.

Valve-Gear. — The supply and exhaust parts for an internal-combustion engine are always separate, so that there are at least two valves (or the equivalent) for each working end of a cylinder; there is also for a gas-engine a separate valve for admitting or controlling the supply of gas. The valves are usually plain disk or mushroom valves with mitered seats; in some cases double-beat valves are used on large engines. Very commonly two-cycle engines exhaust through ports cut through the cylinder walls and opened by the piston itself, which over-runs them near the end of its stroke; in at least one case the exhaust-valves of a four-cycle engine are water-cooled hollow piston-valves, but that construction appears to be exceptional.

The exhaust-valves are always positively controlled, since they must remain closed against pressure in the cylinder until the proper time. The inlet valves may be operated by the pressure of the operating fluid, opening during the suction stroke and

remaining closed during the compression, expansion, and exhaust strokes; but very commonly the admission valves both for air and for gas (when the latter are separate) are positively controlled, and for very high speeds this action is necessary.

From what has been said, it will be evident that the general problem of the design of the valve-gear for an internal-combustion engine resembles that for a four-valve steam-engine, especially that type of steam-engine valve-gear which uses simple lift-valves. The solution which is most evident and most commonly chosen is some form of cam-gear; usually the valves are held shut by springs, and are opened by cams on a cam-shaft either directly or through linkages. This cam-shaft is conveniently placed parallel to the axis of the cylinder and driven from the main shaft through bevel-gears; the four-cycle engine has the gear in the ratio of one to two, so that the cam-shaft makes one revolution for two revolutions of the engine in order to properly time the four principal operations of the cycle. The spring closing a valve must be properly designed not only to give the required pressure to hold the valve shut, but to provide the proper acceleration so that the valves shall remain under the control of the cam when closing. The cam-shaft, in addition to the cams for the normal action of the engine, carries cams which facilitate starting the engine.

Starting Devices. — Since an internal-combustion engine must do the work of drawing in and compressing its charge before energy is developed by explosion, some special device is required to start such an engine, involving the use of power from an external source. It is seldom if ever convenient to apply power sufficient to start an engine under its load, and consequently there must be some disengagement gear to allow the engine to start without load, except in cases where the load is developed only as the engine comes up to speed.

A small engine can be started by hand, by turning the fly-wheel or by working a special hand-gear; the latter should have a ratchet or clutch which will release or throw it out of gear as soon as the engine starts. The engine is driven by hand until

the operations of charging, compressing, and igniting are performed, whereupon the engine should start promptly. Except for very small sizes, there is a special cam that may be thrown into action, and which holds the exhaust-valve open till the piston has completed about half the compression stroke, during which the charge is partially wasted; by this device the labor of compression is much reduced. When an engine is started in this manner the ignition should be delayed until the piston is past the dead-point, otherwise the engine is liable to start backward. The disengagement clutch will not act in such case, and there is great danger of an accident.

When electric or other external power can be substituted for hand-power, this method can be used for starting engines of large size.

A very common device is to start the engine with compressed air from a tank at a pressure of 100 to 200 pounds per square inch. This air is supplied to the tank by a pump driven by the engine when necessary. To start the engine the cylinder is disconnected temporarily from the ordinary gas and air supply, and is worked like a compressed-air engine until well under way, whereupon the compressed air is shut off and the normal action is restored. The air can be supplied from the tank by valves controlled by hand or by a special gear. If the engine has more than one cylinder, compressed air may be supplied to one only, and the other cylinder (or cylinders) may act in the usual manner, except that the compression may be reduced till the engine is started.

At one time gas was withdrawn from the cylinder during the compression stroke, and stood in a reservoir to be used for starting. Such gas could be used at a pressure of 60 to 90 pounds, to start the engine as just described; or the piston could be set beyond the dead-point ready to start, gas could be supplied under pressure and ignited. There is, of course, some objection to the storage of explosive mixtures, though there is no reason why the reservoir should not be made able to endure an explosion.

Governing and Regulating. — There are four ways available

for controlling the power of an internal-combustion engine: (1) by regulating the proportion of air and fuel, (2) by regulating the amount of air and fuel without changing the proportion, (3) by omitting the supply of fuel during a part of the cycles, (4) delaying ignition.

(1) Regulation by controlling the supply of fuel is the normal method for engines working on the Joule or Brayton cycle with compression in a separate cylinder, for which a theoretical discussion is given on page 305. For this cycle there is no explosion, but the gaseous or liquid fuel can be burned during admission in any proportion.

The Brayton engine had a double control for variation in load. In the first place a ball-governor shortened the cut-off for the working cylinder when the speed increased on account of reduction in the load; this had the effect of raising the pressure in the air reservoir into which the air-pump delivered, since that pump delivered nearly the same weight of air per stroke under all conditions. In the second place, there was an arrangement for shortening the stroke of the little oil-pump when the pressure increased; so that indirectly the amount of fuel was proportioned to the load. A similar effect was produced when the engine was designed to use gas.

For the Diesel motor, to be described later, the fuel supply can be adjusted to the power demanded for all conditions of service.

But for gas-engines it has not been found practicable to control the engine by regulating the mixture of gas and air except within narrow ranges. This comes from the fact that very rich or very poor mixtures of gas and air will not explode. Experiments at the Massachusetts Institute of Technology show that illuminating-gas will explode at atmospheric pressure with the ratio of gas to air varying from 1:15 to 1:3.5. Weaker mixtures can be exploded in a gas-engine after compression. Again, gas may be supplied in such a way that the mixture near the point of ignition may be rich enough to explode promptly and fire the remainder of the charge. The ignition of weak mix-

tures should occur before the end of the compression stroke, so that even though the explosion is slow it may be completed near the beginning of the working stroke.

The tests on page 350 show that with the ratio of gas to air varying from 1 : 8 to 1 : 12 the power may vary from 10 to 6 brake horse-power.

This discussion of the possibility of varying the power by varying the mixture of gas and air would appear to show that for many purposes that should be a practicable way of governing a gas-engine. Nevertheless it is used very little if at all, although it was tried early.

(2) The common way of governing large gas-engines is to vary the supply of the mixture without varying its proportions. There are two ways of accomplishing this: in the first place the charge may be throttled so that a less weight is drawn in at a lower pressure; in the second place the admission valve may be closed before the end of the filling stroke, thus cutting off the supply. The effect of throttling is to increase to a marked extent the reduction of pressure during the filling stroke with a corresponding increase in the negative work; the area of the loop like that shown by Fig. 72, page 313, will increase. The effect of closing the inlet-valve before the end of the filling stroke is to produce a diagram similar to Fig. 70, page 310. The charge is drawn in at a pressure a little below that of the atmosphere as far as the point *C*; then the piston goes on to the end of the stroke with an expansion that could be represented by producing the curve *DC*; the return stroke produces a compression that can be represented by retracing the produced part of the curve from *C* and then drawing the true compression curve *CD*. In practice the indicator diagram will show a small negative work due to the expansion and compression caused by the early closing of the supply-valve, but the loss on that account is less than by throttling.

(3) The third way of controlling a gas-engine is to cut off the gas supply so that the engine draws in a charge of air only and makes an idle cycle, represented by Fig. 74, page 323. At

small power the negative work of idle cycles very much reduces the brake economy of the engine. Now, a single-acting four-cycle engine has only one working stroke in four, and must furnish between times the work of expulsion, filling, and compression, and even with a very heavy fly-wheel will show an irregularity in speed of revolution that is very objectionable for many purposes. This difficulty is very much increased if the engine is governed by omitting explosions on the hit-or-miss principle.

(4) Delaying ignition is one of the favorite ways of reducing the power of automobile-engines on account of its convenience; it is little used for other engines, and is very wasteful of fuel, as there is not time for proper combustion.

Ignition. — The ignition of the charge may be produced by one of three methods: (1) by an electric spark, (2) by a hot tube, or (3) by compression in a hot chamber.

(1) The electric spark may be produced in one of two ways, — by the make-and-break method, or by the jump-spark method. For the first method a movable piece is worked inside the cylinder walls, which closes a primary circuit some time before ignition is desired; the slight closing spark has no effect. At the proper time the moving mechanism breaks the circuit, and a good spark is made between the terminals, which are tipped with platinum. A coil in the circuit intensifies or fattens the opening spark. The spark obtained by this method is likely to be better than the jump-spark, but there is the great inconvenience of a moving mechanism in a cylinder exposed to very high pressure, and the motion must be communicated by a piece which enters the cylinder through a stuffing-box.

The jump-spark between two platinum terminals in an insulated spark-plug, screwed through the cylinder wall, is a high-tension spark in a secondary circuit made by a circuit-breaker outside of the cylinder. The movable parts in this case are under observation and can be adjusted, and the spark-plug can be easily withdrawn for examination or renewal. Frequently there are two plugs that can be worked individually or together, or both make-and-break and jump-sparks may be supplied.

The circuit may be supplied by a primary battery, or may be generated by a small dynamo driven by the engine, or may be supplied from any convenient source. When a dynamo is supplied, the engine is usually started by aid of a battery.

The electric method of ignition was the earliest used in the history of the gas-engine, and though it was at one time neglected, now tends to become universal.

(2) The hot tube requires only a small iron tube, which is kept red-hot by a Bunsen burner or other heating flame. The tube comes out horizontally from the cylinder, and sometimes is turned upward for convenience in heating. At the proper time the explosive mixture in the cylinder is admitted to the tube by a valve which is worked by the engine. Sometimes the tube has an inlet-valve at the outer end to ventilate the tube with air drawn in during the filling stroke. This method has been widely used in Great Britain, where the electrical method has met with little favor, though the prejudice against it is passing away.

(3) Ignition by compressing the charge in a hot chamber is used exclusively in oil-engines, and is an ingenious example of taking advantage of a condition that at first sight appears to be undesirable. The mixture of air and kerosene oil in engines of this class is produced by spraying oil into a chamber attached to the cylinder and unprovided with a water-jacket, so that it is maintained by the explosion at a red heat. The charge thus produced is more likely to be exploded than a mixture of gas and air, when it comes in contact with a hot surface, and under the conditions stated explosion cannot be avoided. Much ingenuity has been expended in adjusting sizes and proportions of parts, and frequency of explosion, to obtain the explosion when it is desired.

The tendency to work large gas-engines with high compression, in order to obtain great power without undue bulk and cost, is likely to lead to the danger of premature explosion, especially when rich gas is used. Any projecting part (a bolt-head or part of a valve) may become sufficiently heated to cause explosion; or a spongy spot in a casting may act in the

same way. Premature explosion in a small engine after it is started may be an inconvenience, but in a large engine it may lead to an accident.

Gas-Producers. — A gas-producer is essentially a furnace which burns coal or other fuel with a restricted air supply, so that the combustion is incomplete and the products of combustion are capable of further combustion. In its simplest form a gas-producer will deliver a mixture of carbon monoxide and nitrogen together with small percentages of carbon dioxide oxygen and hydrogen. If a proper proportion of steam is supplied with the air, its decomposition in contact with the incandescent fuel will yield free hydrogen, and the gas will give a higher pressure when exploded, and develop more power in the engine cylinder.

When gas is produced on a large scale in a stationary plant, intricate devices may be used to rectify the gas and save the by-products, which are likely to be so important as to control the methods employed. The most important by-product at the present time appears to be ammonium sulphate, which is used as a fertilizer, and for this reason a coal is preferred which has a relatively large proportion of nitrogen. At a certain station a coal containing three per cent of nitrogen produced crude ammonium sulphate that could be sold for half the price of the coal. This branch of chemical engineering is a specialty of growing importance, and an adequate treatment of it would demand a separate treatise. Such plants, especially when the gas is used for heating furnaces as well as for power, are worked under pressure, the air and steam being blown into the furnace.

When a producer supplies gas for power only, there is a great gain in simplicity and in certainty of control, if the producer is worked by suction, the engine being allowed to draw its charge directly from the producer. During the suction stroke there must be a sufficient vacuum in the engine cylinder to work the producer; this amounts to about two pounds below the atmosphere. There is no attempt in this case to save by-products, and the fuel must be chosen so that comparatively simple rectifying devices will give a gas that will not clog the engine. At

the present time the fuels used are coke, anthracite, and non-caking bituminous coal. At the Louisiana Purchase Exposition, at St. Louis in 1904, a very large variety of fuels, including caking bituminous coal and lignite, were used in an experimental plant, and it is likely that all kinds of fuel will eventually be used in practice.

Fig. 75 gives the section of a Dowson suction producer, in which *A* is the grate carrying a deep coal fire; at *B* is the charging hopper with double doors, so that the vacuum is not lost during charging; at *C* is a vaporizer filled with pieces of fire-brick, which are heated by the hot gases from the furnace; water is sprayed on to the fire-brick through holes in a circular water-pipe *D*, and flashes into steam which mingles with the air supply; the air for combustion enters at *F*, and passing through the vaporizer is charged with steam and then flows through the pipe *L* to the ash-pit. In the normal working of the engine the gas passes through the pipe *G* and the water-seat at *J* to the scrubber *K*, which is filled with coke sprayed with water. From *K* the gas passes directly to the engine. To start the producer, kindling is laid on the grate and the furnace is filled; the fire is lighted through a side door, and air is blown in by a fan driven by hand. At first the gas is allowed to escape through the pipe *I*, until gas will burn well at the testing-cock at *H*; then the pipe *I* is shut off, and the gas is blown through the scrubber and wasted at a pipe near the engine until it appears to be in good condition when tested at that place. The engine is then started and the fan is stopped.

Fig. 75.

The producer described is intended to burn coke or anthra-

cite; those that burn bituminous coal must have some method of dealing with tarry matter. Sometimes this is accomplished by passing the gas through a sawdust cleaner; sometimes a centrifugal extractor is added. Some makers remove the tar by care in cooling before the gas comes in contact with water. Others pass the distillate through the fire, and thus change it into light gas or burn it; with this in view, some producers work with a down-draught. It is probable that different kinds of fuel will need different treatments.

Blast-furnace Gas. — From the composition of blast-furnace gas on page 316, it is evident that it differs from producer-gas only in that it contains very little hydrogen, and therefore is like the gas that would be made in a producer working without steam. During the operation of the furnace the composition is liable to vary and the gas may become too weak; to remedy this difficulty, it is desirable to mingle the gases from two or more furnaces. Since the gas available from a furnace may be equivalent to 2000 horse-power, it is evident that installations to develop power from that source must be on a very large scale.

The gas from a blast-furnace is charged with a large amount of dust, some of which is metallic oxide, and readily falls out, and the remainder is principally silica and lime which is very fine and light. To remove this fine dust the gas should be passed through a scrubber, which has the additional advantage of cooling the gas.

Other Kinds of Gas. — Any inflammable gas that can be furnished with sufficient regularity can be used for developing power. The gas from coke-ovens is a rich gas resembling producer-gas in its general composition. Natural gas consists of 90 to 95 per cent of methane (CH_4) with a small percentage of hydrogen and nitrogen and traces of other gases. This gas for complete combustion requires an equal volume of oxygen and consequently about five times its volume of air; it is probable that ten or twelve volumes of air can be used to advantage with this gas in a gas-engine.

Gasoline. — The lighter distillates of petroleum, known as gasoline, are readily vaporized at atmospheric pressure, and provide the most ready means of supplying fuel to small engines; engines of several hundred horse-power developed in several cylinders have been built for small torpedo-boats, but, in general, the use of gasoline has been limited by its price to comparatively small craft and to automobiles; in both cases, whether for pleasure or for business, other things than cost of fuel determine the selection of the engines. The same is true for the engines of relatively small power used for stationary plants.

The most vital feature of the gasoline-engine is the vaporizer, or carburetor, and this device has received much attention, especially for automobile-engines which are run at very high speed.

There are three types of carburetors that have been used for gasoline-engines: (1) those depending on direct vaporization, (2) those that depend on aspiration with a float, and (3) those depending on aspiration without a float. The earliest types depended on direct vaporization as air was drawn through the mass of the fluid, or through or over fibrous material or a surface of wire gauze; some of the latter devices depended on such a regulation of feed that nearly all the fluid vaporized as it was supplied, leaving only a remnant to return to the tank. But in any case there was a chance of fractional vaporization which resulted in the production of a heavier and less tractable fluid.

The more recent carburetors depend on aspiration, the air supply being drawn past an orifice (or orifices) to which gasoline is supplied, and from which it can be drawn by the air more or less in proportion as required. For stationary and marine engines the supply of gasoline to the aspirator can be nicely regulated by a float which keeps a small chamber filled just to the level of the aspirating orifices, so that the inrush of air may draw out the gasoline in proper proportion. This device has been tried on automobiles, but the shaking of the machine disturbs the proper action of the float.

A third form of carburetor is illustrated by Fig. 76. Here the gasoline is supplied by a pipe *E* to a valve that may be set to give good average action. Below is a fine conical valve at the end of a vertical rod which is held up by a light spring; at the middle of the spindle is a disk-valve which fit sloosely in a sleeve. At *aa* are air-inlet valves, and at *A* is the entrance to the cylinder. During the suction or filling stroke the spindle is drawn down, opening the valve at the top of the spindle and allowing the air to draw gasoline by aspiration. Some of the hot products of combustion from the exhaust are circulated around the aspirating chamber to prevent undue reduction of temperature. This type of carburetor works well enough at moderate speeds, but at very high speeds the inertia of the spindle and disk-valve cannot be overcome rapidly enough by the air, which is consequently throttled, so that there is not the increase of power which might properly be expected at such speeds.

FIG. 76.

It is alleged that this type of vaporizer, or carburetor, can be made to deal with kerosene oil and alcohol.

Kerosene Oil. — The use of kerosene oil has been developed to the greatest extent in England, on account of former restrictions on the transportation and storage of gasoline. It has been used in America where there is objection to gasoline.

There is much difficulty in vaporizing or spraying kerosene oil so that it can be properly mixed with air at the temperature for the supply to an engine. On the other hand, any attempt to vaporize the oil at a high temperature results in the deposit of a hard graphitic material.

One of the most successful English engineers frankly accepts the latter alternative. The essential feature of the carburetor

of this engine is shown in Fig. 77, which gives a vertical section of the cylinder-head and of the vaporizer; the remainder of the engine differs in no essential particular from any horizontal gas-engine. This vaporizer, which has a constricted neck, is bolted to the cylinder-head; the forward end is jacketed with water, as is also the cylinder of the engine; but the after end, which is ribbed internally, is not jacketed; it consequently remains at a red-heat when the engine is running. The oil for each explosion is delivered into this hot end of the vaporizer, and is vaporized and mingles with the hot spent gases; toward the end of the compression stroke the charge of air which has been drawn in and compressed enters the vaporizer and an explosion occurs. When the vaporizer-head has become clogged, after 24 to 200 hours running, depending on the kind of oil used, it is taken off and the hard adherent deposit is removed; to avoid delay a second head is put on for a corresponding run. This engine is governed by controlling the oil supply; the governor opens a bypass-valve on the oil supply-pipe and allows a part to return to the tank. The hit-or-miss principle is not applicable, as the vaporizer would become too cool. Before starting, the vaporizer must be heated to a dull red by aid of a kerosene or gasoline torch. The engine can burn also crude petroleum, or an unrefined distillate resembling kerosene.

Fig. 77.

Alcohol. — The demand for gasoline maintains the price at a point which makes it possible in some countries to use alcohol, if it can be relieved from special taxation. To make alcohol unfit for any but mechanical purposes it is mixed with a little wood-alcohol and benzine; this process, called denaturizing, has little if any effect on its combustion. For combustion the amount of water brought over during distillation should be limited to a small percentage. The use of alcohol for power in this country has only recently been made possible under the internal-revenue laws, so that we have no experience with it. There

appears to be no reason why there should be trouble in the use of some form of carburetor like those used for gasoline engines.

The Four-cycle Engine. — Fig. 78 gives a vertical section of a Westinghouse four-cycle gas-engine built in various sizes, up to 85 horse-power with one cylinder, and up to 360 with three cylinders. Massive engines of this type are horizontal with double-acting pistons, having two cylinders tandem or four twin-tandem. It is somewhat curious that while massive steam-engines tend towards the upright construction, large gas-engines appear to be all horizontal; it may be for the convenience of the tandem arrangement. In Fig. 78 the frame of the engine is arranged to form an inclosed crank-case, which is somewhat unusual for gas-engines. The piston is in the form of a plunger, so that no cross-head is needed; a common arrangement for all except massive gas-engines. The cylinder barrel and head are water-jacketed, the inlet and exits being at *H* and *K*. Gas and air enter the mixer-chamber *M* by separate pipes (not shown) and pass by *N* to the inlet-valve *J*; the engine is controlled by a throttle-valve directly connected to a ball-governor beneath the chamber *M*, but omitted from the figure. The valve is a piston-valve with separate air and gas passages, which works in a sleeve

Fig. 78.

that can be moved by hand; this sleeve may be set by hand to give any desired mixture, and the proportion of the inlet areas for gas and air having been once set, the relative areas remain unchanged, while the governor adjusts the piston-valve to give the amount of mixture that may be demanded by the load on the engine. The inlet-valve *J* and the exhaust-valve *E* are each moved by cams at *B* and at *A* as indicated, the cams making one revolution for each double revolution of the engine required for the four-stroke cycle. Large sizes have the exhaust-valve water-cooled, to prevent burning the valve, and to avoid danger of pre-ignition. Near *A* there is a handle for shifting into action the starting-cam which reduces compression when the engine is started. At *F* are two low-tension make-and-break ignitors, either of which can be thrown into action; they are worked by cams on the shaft that operates the valve *J*.

Two-cycle Engines. — The two strokes of a four-cycle engine which exhaust the spent charge and draw in the new charge are performed with a pressure in the cylinder only a little higher or lower than that of the atmosphere, and could be omitted with advantage provided the operations could be performed in some other way. The first successful attempt at a two-stroke cycle was that by Dugald Clerk, who made the following changes: (1) he cut a ring of exhaust ports through the cylinder walls that were over-run and opened by the piston near the end of the expansion stroke, through which the major part of the spent gases escaped during release; and (2) he provided a pump set about half a stroke ahead of the engine piston, which compressed the new charge to about ten pounds above the atmosphere; as soon as the exhaust had sufficiently reduced the pressure in the cylinder, this new charge opened the inlet-valve and entered the cylinder, blowing the remainder of the spent gases out through the ports in the cylinder walls. The piston closed these ports and compressed the charge on the return stroke, so that only two strokes were required to complete the cycle, and the engine approximated the condition of a single-acting steam-engine in its

regularity of rotative velocity. The engine could also develop twice as much power for its size as a four-cycle engine, and in certain tests by Mr. Clerk, showed a slightly better economy than the older type of engine. But the operation of replacing the remnants of the spent charge by the fresh charge in engines of this type is rather delicate, there being a chance that some of the spent charge will remain, or that some of the fresh charge will be wasted; it is likely that the charges mingle and that the engine experiences both defects. Eventually the Clerk engine was withdrawn from the market, but the principles are used for two types of engines: (1) small gasoline engines for launches and other small craft, and (2) large engines built for burning blast-furnace gas.

Gasoline-engines of small power and moderate rotative speed have been made on the two-cycle principle by enclosing the crank- and connecting-rod in a casing, so that the piston may act as the compressing-pump. On the up-stroke a charge of air and gasoline is drawn into the crank-case, and it is slightly compressed on the down-stroke. There are two sets of ports cut through the cylinder walls near the end of the down-stroke and are opened by the piston; these are on opposite sides of the cylinder; one set, which is opened slightly earlier than the other, forms the exhaust-ports and the other the inlet-ports which are in communication with the crank-case, and therefore supply air and gasoline to replace the spent charge. A barrier is cast on the cylinder-head which prevents the fresh charge from flowing directly across from the inlet to the exhaust, but nevertheless the action is probably much inferior to that of Clerk's engine, which had the charge supplied at the cylinder-head. These engines are nearly valveless and can run in either direction, and on account of the simplicity and small cost have found favor for propelling small craft at moderate speeds.

If any attempt is made to run two-cycle engines at a high rotative speed there is difficulty in obtaining proper exhaust and supply, since both operations are performed under gaseous pressure that cannot well be increased. Recently two-cycle

engines have been introduced on automobiles to a limited extent. Two German engineering firms have developed two-cycle engines especially for burning blast-furnace gas on a large scale, as much as 1500 horse-power in a single cylinder.

The Körting engine (built by the de la Verne Machine Company) is a double-acting engine which has a piston nearly as long as the stroke of the engine. At the middle of the length of the cylinder is a ring of exhaust-ports that are uncovered at the end of each stroke, and discharge burnt gases from first one end of the cylinder and then the other. By the side of the engine-cylinder, and arranged in tandem so that they can be driven by one crank (which has a lead of 110°), are two pumps, one for compressing air, and the other gas. The capacities of the two pumps are designed for the kind of gas to be burned.

The air-pump compresses to eight pounds above the atmosphere and delivers air to the admission valves, which are lifted by cams at the time when the release is completed. The governor controls a bypass-valve which puts the two ends of the pump in communication for about half of the discharge stroke of that pump, which accomplishes two purposes. In the first place the compression of the gas begins only when the bypass-valve is closed, and consequently is to a less pressure than that of the air; consequently the air backs up in the gas-supply pipe, and when the engine admission valve is opened it supplies only air which clears the cylinder of spent gases; afterward the cylinder receives its charge of mixed gas and air. By careful design and adjustment it is attempted to fill the cylinder without wasting gas at the exhaust-ports, but tests show an appreciable percentage of unburned gas in the exhaust. And in the second place the governor can regulate the closure of the bypass-valve so as to adjust the amount of gas to the work. Since the range of explosive mixture of blast-furnace gas is not wide, this method of regulation appears to be adapted only to fairly uniform loads.

The Oechelhaüser gas-engine has two single-acting pistons or

plungers in a long open-ended cylinder; these plungers are connected to cranks at 180° so that they approach or recede from the middle of the cylinder simultaneously. The engine has a cross-head at each end of the cylinder to take the cross-thrust of the connecting-rod, so that the engine extends to a great length on a horizontal foundation. Toward the crank-end of the cylinder there is a ring of exhaust-ports uncovered by the inner (or crank-end) piston, and toward the outer end of the cylinder there is another row uncovered by the outer piston; a part of these outer ports supply air, and a part gas. These air- and gas-ports may be controlled by annular valves that are set by hand when the engine uses blast-furnace gas. Under these conditions the engine is regulated by a governor, which controls the pumps that supply air and gas. These pumps, which are driven from the outer cross-head, have bypass-valves which connect the two ends and begin to deliver only when the bypass-valves are shut by the governor, so that the charge is adjusted in amount to the load. When the engine uses a rich gas that has a wide explosive range, the governor controls the annular valves at the gas-ports and varies the mixture.

The Diesel Motor. — A new form of internal-combustion engine was described by Rudolf Diesel in 1893, which does away with many of the difficulties of gas- and oil-engines, and which at the same time gives a much higher efficiency. The essential feature of his engine consists in the adiabatic compression of atmospheric air to a sufficient temperature to ignite the fuel which is injected at a determined rate during part of the expansion or working stroke.

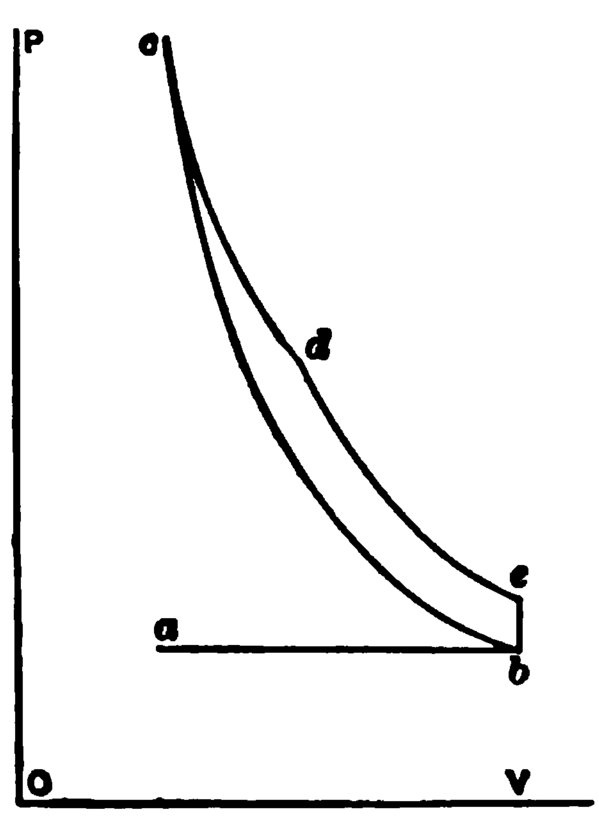

FIG 79

The theoretical cycle is shown by Fig. 79, which represents four strokes of a single-acting piston or plun-

ger. Atmospheric air is drawn in from *a* to *b* and is compressed from *b* to *c* to a pressure of 500 pounds to the square inch and a temperature of 1000° F. From *c* to *d* fuel is injected in a finely divided form, and as there is air in excess it burns completely at a rate that can be controlled by the injection mechanism. Thus far the only fuel used is petroleum or some other oil. At *d* the supply of fuel is interrupted, and the remainder of the working stroke, *de*, is an adiabatic expansion. The cycle is completed by a release at *e* and a rejection of the products of combustion from *b* to *a*.

The cycle has a resemblance to that of the Otto engine, but differs in that the air only is compressed in the cylinder and the combustion is accompanied by an expansion. Diesel, in his theoretic discussion of his engine, stipulates that the rate of combustion shall be so regulated that the temperature shall not rise during the injection of fuel, and that the line *cd* shall therefore be very nearly an isothermal for a perfect gas. Since the fuel is added during the operation represented by the line *cd*, the weight of the material in the cylinder increases and its physical properties change, so that the line will not be a true isothermal. The fact that there is air in excess makes it prob-

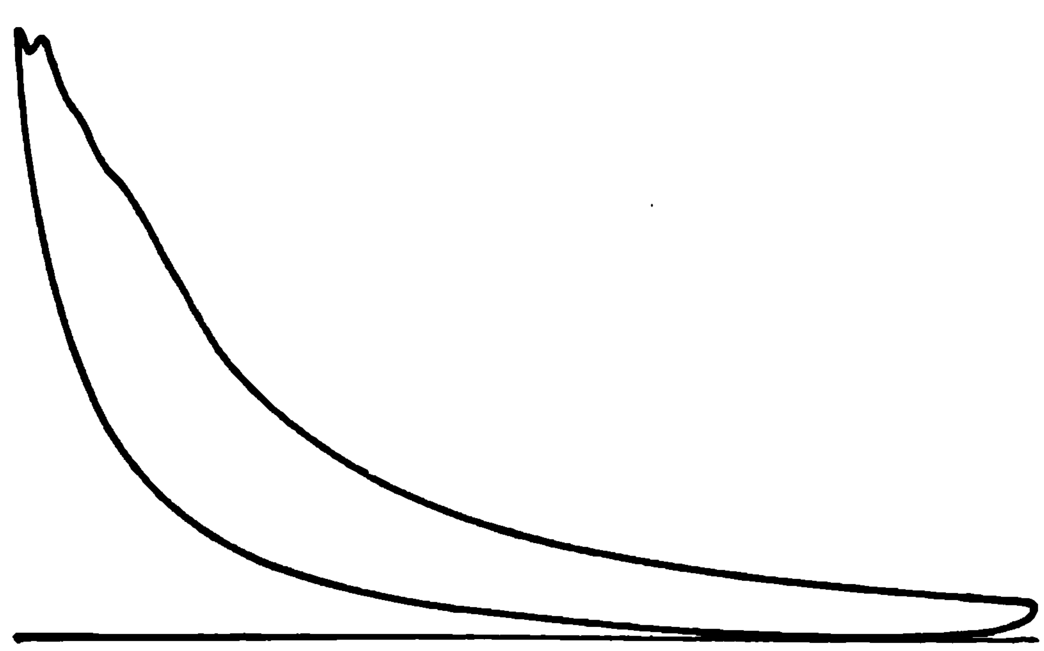

FIG. 80

able that these changes of weight and properties will be insignificant. On the other hand, it is not probable that in practice the rate of injection of fuel will be regulated so as to give no

rise of temperature, or that there is any great advantage in such a regulation if the temperature is not allowed to rise too high.

The diagram from an engine of this type is shown by Fig. 80, which appears to show an introduction of fuel for one-eighth or one-seventh of the working stroke. It is probable that the compression and the expansion (after the cessation of the fuel supply) are not really adiabatic, though as there is nothing but dry gas in the cylinder during those operations the deviation may not be large. The sides and heads of the cylinders of all the engines thus far constructed are water-jacketed, though the use of such a water-jacket and the consequent waste of heat was one of the difficulties in the use of internal-combustion engines that Diesel sought to avoid by controlling the rate of combustion. The statement on page 39 that the maximum efficiency is attained by adding heat only at the highest temperature has no application in this case. The real conditions are that heat cannot at first be added at a temperature higher than that due to compression (about 1000° F.), but as combustion proceeds heat can be added at higher temperature and with greater efficiency. The fuel may be regulated so as to avoid temperatures at which dissociation has an influence and after-burning can be avoided.

The oil used as fuel is injected in form of a spray by air that is compressed separately in a small pump to 30 or 40 pounds pressure above that in the main cylinder; of course it is necessary to cool this portion of the air after compression to avoid premature ignition. The engines that have been used are described as giving a clear and nearly dry exhaust. In damp weather the exhaust shows a little moisture, probably from the combustion of hydrogen in the oil. The cylinder when opened shows a slight deposit of soot on the head. It appears therefore that Diesel has succeeded in constructing an engine for burning heavy oils with good economy and without the annoyances of an igniting device. The engines have the further advantage in that the work can be regulated by the amount of fuel supplied, which amount is not controlled, as in explosive

engines, by the necessity to form an explosive mixture. The discussion of the theoretical efficiency of the cycle shows that the efficiency increases as the time of injection of fuel is shortened. In practice the engine shows a slight decrease in economy for light loads, due probably to the losses by radiation and to the water-jacket, which are nearly constant for all loads.

In the exposition of the theory of his motor, Diesel * claims that all kinds of fuel, solid, liquid, and gaseous, can be burned in his motor. As yet oil only has been used; the choice of petroleum or other heavy oil has probably been due to the low cost of such oils. It is evident that gas may be used in this type of engine; the gas can be compressed separately to a pressure somewhat higher than that in the main cylinder, much as the air is which is used for injecting oil. It does not appear necessary to cool the gas after compression, as it will burn only when supplied with air.

There appears to be no insurmountable difficulty in supplying powdered solid fuel to this engine. The presence of the ash from such fuel in the cylinder may, however, be expected to give trouble. Diesel claims that with a large excess of air (for example, a hundred pounds of air for one pound of coal) the ash will be swept out of the cylinder with the spent gases and will not give trouble; but that claim has not as yet been substantiated.

Diesel's original discussion of his motor contemplated a compound compressing-pump, one stage to give isothermal compression, and the second stage to give adiabatic compression; also a compound motor, the first cylinder having isothermal expansion with a supply of fuel, and the second cylinder an adiabatic expansion. He gives with that discussion a theoretical diagram approaching Carnot's cycle in appearance and efficiency. If this variety of the motor were mechanically practicable it would have the defects of Carnot's cycle for gas, namely, the diagram would be very long and attenuated, and even with the very high pressures contemplated would give a relatively small power.

* *Rational Heat Motor;* Rudolf Diesel, trans. Bryan Donkin.

A theoretical discussion of the efficiency of the cycle for the simple engine as represented by Fig. 79 may be obtained by considering that heat is added at constant temperature from c to d and that heat is rejected at constant volume from e to b, bearing in mind that bc and dc represent adiabatic changes.

From equation (75), page 63, the expression for the heat supplied from c to d is, for one pound of working substance,

$$Q_1 = Ap_c v_c \log_e \frac{v_d}{v_c} = ART_c \log_e \frac{v_d}{v_c}.$$

The heat rejected at constant volume is

$$Q_2 = c_v (T_e - T_b) = \frac{c_p}{\kappa} (T_e - T_b).$$

Since the expansion de is adiabatic,

$$T_e = T_d \left(\frac{v_d}{v_e}\right)^{\kappa - 1} = T_c \left(\frac{v_d}{v_e}\right)^{\kappa - 1};$$

but since the compression bc is also adiabatic,

$$T_c = T_b \left(\frac{v_b}{v_c}\right)^{\kappa - 1},$$

and consequently

$$T_e = T_b \left(\frac{v_b}{v_c}\right)^{\kappa - 1} \left(\frac{v_d}{v}\right)^{\kappa - 1} = T_b \left(\frac{v_d}{v_c}\right)^{\kappa - 1}$$

for $v_e = v_b$. Replacing T_e by its value in the expression for Q_2, we have

$$Q_2 = \frac{c_p}{\kappa} T_b \left\{ \left(\frac{v_d}{v_c}\right)^{\kappa - 1} - 1 \right\}$$

Finally, the efficiency appears to be

$$e = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{c_p T_b \left\{ \left(\frac{v_d}{v_c}\right)^{\kappa - 1} - 1 \right\}}{A\kappa RT_c \log_e \frac{v_d}{v_c}}. \quad (188)$$

Inspection of the equation shows that the efficiency may be increased by raising the temperature T_c or by reducing the

temperature T_b. The latter is practically the temperature of the atmosphere, but T_c may be made any desired temperature by reducing the clearance of the cylinder and thus raising the pressure at the end of compression. Again, the efficiency may be increased by reducing the time during which fuel is injected, that is, by reducing the ratio $v_d : v_c$, as may be proved by a series of calculations with different values for that ratio. This is a very important conclusion, as it shows that the engine will have in practice little if any falling off in efficiency at reduced loads.

It is reported that a clearance of something less than 7 per cent is associated with a compression to 500 pounds and a temperature of 1000° F., or more. Taking the pressure of the atmosphere at 14.7 pounds per square inch, adiabatic compression to 500 pounds above the atmosphere or to 514.7 pounds absolute requires a clearance of

$$v_a = v_b\left(\frac{p_b}{p_c}\right)^{\frac{1}{\kappa}} = v_b\left(\frac{14.7}{514.7}\right)^{\frac{1}{1.405}} = 0.0796\, v_b,$$

so that the clearance is

$$0.0796 \div \{1 - 0.079\} = 0.0865$$

of the piston displacement.

If the temperature of the atmosphere be taken at 70° F. or 530 absolute, the temperature after adiabatic compression becomes

$$T_c = T_b\left(\frac{p_c}{p_b}\right)^{\frac{\kappa - 1}{\kappa}} = 530\left(\frac{514.7}{14.7}\right)^{\frac{1.405-1}{1.405}} = 1480$$

absolute, or 1020° F.

If it be further assumed that fuel is supplied for one-tenth of the working stroke, then

$$v_d = 0.1\,(v_b - v_a) + v_a = [0.1\,(1 - 0.0796) + 0.0796]\, v_b$$
$$= 0.1716\, v_b.$$

The equation for efficiency gives in this case

$$e = 1 - \frac{778 \times 0.2375 \times 530 \left\{ \left(\frac{0.1716}{0.0796}\right)^{0.405} - 1 \right\}}{1.405 \times 53.22 \times 1480 \log_e \frac{0.1716}{0.0796}} = 0.58.$$

Engines for Special Purposes. — Small engines can be made to give any required degree of regularity for electrical or other purposes, by giving a sufficient weight to the fly-wheel; for large power the same object can be attained by using a number of cylinders, by making the engine double acting, by the construction of two-cycle engines, or by the combination of two or more of these devices.

The four-cycle engine has not as yet been made reversible, and even if the complexity of valve-gear for running in both directions could be accepted, it appears likely that a special starting device would be required for every reversal. Reversing launches and automobiles is done by aid of a mechanical reversing gear, except that for some small boats a reversing propeller is used. Such gear for large ships appears to be dangerous as well as impracticable.

Two-cycle engines would not require much complication of valve-gear to make them reversible, and would have some advantage on account of the greater frequency of working strokes; they also might require the use of a starting gear for every reversal. Small launches with two-cycle engines are readily reversed by hand, but such small craft can be fended off, and a failure to reverse need not be serious.

The engine with separate compressing-pump discussed on page 305, appears to show greater promise for marine or other purposes where ready reversal is essential. Even with the pump geared directly to the engine, it was found possible to reverse a two-cylinder engine promptly with a valve-gear but little more complicated than that for a steam-engine. But for marine purposes the engines could be placed in two groups; one

group could be connected to the propeller shaft (or shafts) and worked without compressor-pumps, and the other group at any convenient place could drive the compressor-pumps for the whole system. Such an arrangement should give practically the same certainty of maneuvering as steam-engines.

The application of gas-engines to large ships cannot be considered to be accomplished till producers have been made that can use all grades of bituminous coal, including inferior qualities.

Automobiles are commonly driven by four-cycle gasoline engines, and have a rather formidable array of mechanical devices, including clutches to release the engine for starting, or when the carriage is standing still, several change-speed gears for running slowly and climbing hills, and a reversing mechanism. All of this entails weight, cost and depreciation, and while gasoline vehicles can be handled efficiently by skilled drivers they have not the facility of control that is readily given to steam-carriages. The speed and power can be controlled by throttling the charge and by delaying the ignition; the mixture may be included in the methods of control, but probably it is better left alone when well adjusted.

Economy of Gas-Engines. — It will be convenient to consider the economy of gas-engines before discussing the economy of engines using special fuel like gasoline or oil, because it is only this class of engines that can, by association with the gas-producer, make use of all kinds of fuel, and especially of coal.

It will be convenient also to make such inquiry as may be possible concerning the influence of various conditions on the economy of gas-engines before trying to determine what economy may properly be attributed to them.

There are five conditions that can be enumerated which have an effect on the efficiency of gas-engines:

(1) Compression.

(2) Mixture.

(3) Size.

(4) Quality of gas.

(5) Time of ignition.

(1) The influence of compression is indicated theoretically by equation (187), page 312, which shows that the efficiency may be expected to increase progressively with increasing compression. To exhibit this feature and to compare it with the results obtained in practice, the following table has been computed for tests 2, 5, and 7 of Table XXXV, page 350. The composition of the illuminating-gas used was similar to that on page 315; the original detailed report of these tests shows little variation in composition.

Number of tests	2	5	7
Ratio of compression	4.98	4.59	3.84
Theoretical efficiency	0.479	0.461	0.420
Thermal efficiency	0.270	0.264	0.252
Ratio	0.564	0.573	0.600

Such a comparison is commonly considered to show that the actual efficiency follows the theoretical efficiency, the former being based on the indicated horse-power, and being obtained by dividing 42.42 (the equivalent of one horse-power in thermal units per minute) by the thermal units per indicated horse-power per minute. But if the brake horse-power is taken as the basis of comparison, as has already been shown to be proper, there appears to be practically no advantage in the higher compression for the illuminating-gas; for the power-gas there is no advantage in a compression beyond four and a half. There is, however, an advantage in that a higher compression gives a larger mean effective pressure and greater power.

(2) A stronger mixture of gas and air may in general be expected to yield more work than a weaker one, as is shown by comparing the trios of tests with the same compression both for illuminating-gas and for power-gas; but there is usually some mixture that will give the best economy. This mixture should be selected from a proper series of engine-tests rather than by some other method, but as this involves a large amount of experimental work, a satisfactory discussion of this feature is not always possible. The tests in Table XXXV show that for both

kinds of gas the richest mixture used is the most economical, basing the comparison on brake horse-power as should be done. The first trio of tests shows a distinct minimum for a ratio of ten

Table XXXV.

GAS-ENGINE WITH ILLUMINATING- AND WITH POWER-GAS.

DIAMETER 8.6 INCHES; STROKE 13 INCHES.

Professor Meyer, *Mitteilungen über Forschungsarbeiten Heft* 8, 1903.

		Compression.	Revolutions per minute.	Brake horse-power.	Indicated horse-power.	Mechanical efficiency.	Gas per brake horse-power per hour, cubic feet.	Ratio of air to gas.	Pressure after compression, pounds per square inch.	Ratio of charge to piston displacement.	Thermal units per brake horse-power per minute.	Thermal units per indicated horse-power per minute.
1	Illuminating-Gas	4.98	202	10.2	14.1	0.73	23.2	8.3	116	84	[illegible]	161
2		4.98	204	[redacted].3	12.5	0.66	23.8	10.5	117	87	[illegible]	157
3		4.98	204	6.2	10 0	0.62	29.7	11.4	118	87	[illegible]	174
4		4.59	200	9.7	13.1	0.74	23.2	8.6	102	87	[illegible]	166
5		4.59	200	[redacted] 1	11.6	0.70	24.0	10.5	105	88	[illegible]	161
6		4.59	202	6 2	10 7	0.58	28.4	11.2	108	89	[illegible]	162
7		3.84	207	10.5	13 3	0.79	22.9	10.9	82	86	213	168
[redacted]		3.84	208	8.5	12 5	0.68	24.5	11.0	89	87	252	170
9		3 84	207	6 3	10.1	0.62	28.0	11.1	89	88	275	170
10	Power-Gas.	4 98	202	9.2	12.6	0.73	107 5	1 05	107	80	251	181
11		4.98	204	8.3	12.4	0.67	115.5	1.18	107	80	247	164
12		4.98	207	6.2	9.3	0 65	121.5	1.78	111	81	282	178
13		4.59	201	[redacted] 8	12.1	0.73	108.0	1.13	98	81	250	184
14		4.59	203	8.2	11.7	0.70	121.5	1.12	101	82	267	187
15		4.59	202	6.2	10 0	0.62	145.0	1.40	101	83	302	186
16		3.84	204	8.3	11.4	0.73	124.0	1.20	81	86	263	195
17		3 84	205	7.3	10.4	0.70	128.5	1.39	82	85	275	194
18		3.84	205	6.3	9.5	0.66	138 0	1.64	81	85	292	190

to one; the minimum per brake horse-power will be found for a richer mixture, on account of the better mechanical efficiency which accompanies the larger power which such a mixture will develop; it cannot be far wrong to assume that the mixture of

eight to one will give the minimum per brake horse-power. The remainder of the table is less conclusive, but it appears likely that a ratio of one volume of illuminating-gas to eight volumes of air is proper, and that for power-gas the ratio should be somewhat larger than unity.

(3) A committee of the Institution of Civil Engineers * tested three gas-engines of varying size, but all having the same ratio of compression, and tested under the same conditions. The results that bear on the question of size are as follows:

Brake horse-power	5.2	20.9	52.7
Thermal units per horse-power per minute	159	150	143

It is to be remarked that the results just quoted are remarkably low, but that the composition of the committee and the precautions taken, place them beyond cavil. It is somewhat difficult to account for the difference between the results just quoted, and those given in Table XXXV, though part of it is due to the better mechanical efficiency of the former. This was estimated to be about 0.87, while that of the engine tested by Professor Meyer was about 0.72; allowance for this difference may be estimated to reduce the results of the first test in Table XXXV to 184 thermal units per brake horse-power per minute. This illustrates an inconvenience of using the brake horse-power as the basis of comparison of tests on different engines, since it makes the results depend on the mechanical condition of the engine; however, this condition is one of the elements of practical economy.

(4) It is likely that an engine will show a better heat economy when using a richer gas, as is indicated by comparing the results in Table XXXV with illuminating-gas and with power-gas; but there is not sufficient information to make this feature decisive.

(5) It is customary to time the ignition so that the maximum pressure shall come early in the stroke, and that is probably conducive to good economy; delaying ignition, as is done on automobiles to reduce the power, is known to be very wasteful.

* *Proc. Inst. Civ. Engrs.*, vol. clxii, p. 241.

Professor Meyer made some subsidiary tests to determine the influence of the time of ignition on illuminating-gas with the results following:

Lead of ignition,	1.2	5.6	9.7	11.0	10.9	14.2	20.7
Thermal units per indicated horse-power per minute	216	217	223	216	221	226	260

This appears to show that any lead up to 15° would give about the same result for this engine, but that a greater lead was undesirable.

The question as to the economy to be expected from gas-engines has been considered incidentally in our review of the influence of various conditions on the economy of gas-engines. The best result that is quoted is for an engine tested by the committee of the Institution of Civil Engineers, which used 143 thermal units per horse-power per minute, when developing 52.7 brake horse-power. The gas used had the composition by volume:

Hydrocarbons . . .	4.74	Carbon dioxide . .	2.62
Methane CH_4 . . .	33.73	Oxygen	0.27
Hydrogen	41.29	Nitrogen	10.22
Carbon monoxide . .	7.13	Total	100

Its heat of combustion determined by aid of a Junker calorimeter was 561 B.T.U.

The test of a producer gas-power plant at St. Louis given on page 354 used 198 thermal units per brake horse-power per minute.

An engine developing 728 metric horse-power at Seraing at 93 revolutions per minute, used 163 thermal units per brake horse-power per minute; the mechanical efficiency being 0.82. when tested by Hubert.*

A Producer-Gas Plant. — At the Louisiana Purchase Exposition at St. Louis in 1904, an extensive investigation was made of various fuels from all parts of the United States, including the

* *Bul. Soc. de l'Industrie Mineral*, 3d series, vol. xiv, p. 1461.

development of power by the combination of a Taylor gas-producer with necessary adjuncts, and a three-cylinder Westinghouse gas-engine; a detailed report of the tests is given by Messrs. Parker, Holmes, and Campbell,* the committee in charge.

The gas-producer had a diameter of 7 feet inside the brick lining, and at the bottom was a revolving ash table 5 feet in diameter; the blast was furnished by a steam-blower supplied from a battery of boilers used for other purposes; tests were made to determine the probable amount of steam taken by the blower, but the variation of steam-pressure acting at the blower during tests made this determination somewhat unsatisfactory. The cost of the steam in coal of the kind used for any test could be estimated closely from boiler-tests made with the same coal.

The gas from the producer passed through a coke-scrubber, and then through a centrifugal tar-extractor using a liberal amount of water. From the extractor the gas passed through a purifier filled with iron shavings to extract sulphur. On the way to the engine the gas was measured in a meter.

The engine-cylinders were 19 inches in diameter and had 22 inches stroke. At 200 revolutions the engine was rated at 235 brake horse-power. The engine was belted to a direct-current generator, and the energy was absorbed by a water-rheostat.

The results of a test on a bituminous coal from West Virginia have been selected for presentation. The composition of the coal by weight and the gas by volume are:

Coal.		Gas.	
Moisture	2.22	Carbon dioxide	8.90
Volatile matter	31.05	Carbon monoxide	14.77
Fixed carbon	59.83	Oxygen	.33
Ash	6.90	Hydrogen	9.52
		Methane	6.65
Thermal units per pound coal	14224	Nitrogen	59.83
		Thermal units per cu. ft. (62° F., 14.7 pounds)	160.5

* *U. S. Geological Survey, Professional Paper* No. 48.

TEST ON PRODUCER AND ENGINE.

Duration, hours	24
Total coal fired in producer, pounds	6,000
Coal equivalent of steam used by blower, pounds	835
Coal equivalent of power to drive auxiliary machinery	299
Total equivalent coal	7,134
Thermal value of total, equivalent coal, B.T.U.	101,500,000
Total gas (at 62° F. and 14.7 pounds), cu. ft.	415,660
Thermal value of total gas	66,700,000
Efficiency of producer	0.657
Electrical horse-power	199.3
Mechanical efficiency, estimated	0.85
Brake horse-power	234
Gas per horse-power per hour, cubic feet	74.1
Thermal units per horse-power per minute	198
Thermal efficiency of brake-power	0.214
Coal per brake horse-power per hour	1.27
Combined thermal efficiency of producer and engine	0.14

It is interesting to compare these results of a test on a producer-plant with the tests at the pumping-station at Chestnut Hill from which the results quoted on page 239 were taken.

TEST AT CHESTNUT HILL PUMPING STATION.

Duration hours,	24
Coal required by plant, corrected	16,269
Thermal value of George's Creek coal, estimated	14,500
Heat abstracted from one pound of coal by boiler	10,690
Efficiency of boiler	0.74
Indicated horse-power, engine	576
Indicated horse-power, pump	530
Mechanical efficiency	0.920
Thermal units per pump horse-power per minute	222
Thermal efficiency pump-power	0.191
Combined thermal efficiency pump and boiler	0.14
Coal per pump horse-power per hour	1.21

If allowance is made for the higher thermal value of George's Creek coal, the coal consumptions are very nearly equivalent.

A test on a Dowson suction producer by Mr. M. A. Adam * gave an efficiency of 0.80 to 0.84 after the producer was well started. If the thermal efficiency of an engine using the gas may be estimated from 0.20 to 0.24, the combined efficiency may be estimated from 0.16 to 0.20, which for anthracite coal would

* *Proc. Inst. Civ. Engrs.*, vol. clviii, p. 320.

correspond to one pound per brake horse-power per hour, or 0.9 of a pound per indicated horse-power; the makers of producer power-plants are now ready to guarantee a consumption of one pound of anthracite per brake horse-power per hour.

Economy of Oil-Engine. — An engine of the type described on page 335 was tested by Messrs. A. E. Russell and G. S. Tower * of the Massachusetts Institute of Technology. The engine had a diameter of 11.22 inches and a stroke of 15 inches, and at 220 revolutions per minute developed ten brake horse-power; the mechanical efficiency was about 0.72, so that the indicated power was about 14; the clearance or charging space was about 0.44 of the piston displacement.

With kerosene the best economy was 1.5 pounds per brake horse-power per hour; this kerosene weighed 6.52 pounds per gallon, flashed at 104° F., and had a calorific power of 17,222 thermal units per pound.

The engine was also tested with a crude distillate which comes from petroleum after the kerosene, weighing 6.66 pounds per gallon, with a flash-point at 148° F., and having a calorific power of 19,410 thermal units per pound; of this oil the engine used 1.3 pounds per brake horse-power per hour.

The thermal units per horse-power per minute were 430 for kerosene and 420 for the distillate; the thermal efficiencies corresponding are 0.099 and 0.11 on the basis of brake horse-power.

Economy of a Diesel Motor. — A 70 horse-power Diesel motor using Russian petroleum, which had a calorific power of 18,450 thermal units per pound, was tested by Professor Meyer † in 1904. The diameter of the cylinder was 15.75 inches, the stroke was 23.7 inches, and the ratio of compression was 15.4. The air-pump had a diameter of 2.2 inches and a stroke of 5.5 inches. At the normal load of 69.63 metric horse-power by the brake (68.6 English horse-power) the oil-consumption was 0.429 pound per horse-power per hour, or 132 thermal units per brake horse-power per minute. The thermal efficiency was conse-

* Thesis, M. I. T. 1905.

† *Mitteilungen über Forschungsarbeiten Heft* 17, p. 35.

quently 0.32. At an overload amounting to 85.7 brake horse-power, the oil-consumption was 0.42 pound, and at half load (34.4 horse-power) the consumption was 0.50 of a pound.

Since oil for lubrication of the cylinder is liable to be burned together with the fuel, it is specially necessary in tests of engines of this type that error from the effect of excessive use of lubricating-oil is to be guarded against.

Distribution of Heat. — A very interesting and instructive matter in the discussion of tests on gas-engines is the distribution of the heat, and especially of the heat that is not changed into work. It cannot be considered that all of this lost heat is wasted, because any heat-engine must reject heat, and that for the theoretical cycles, which are the limits for practical engines, the major part of the heat is unavoidably rejected.

The following table is taken from a lecture by Mr. Dugald Clerk.*

Dimension of Engine.	Distribution of Heat.		
	Work.	Jacket.	Exhaust.
6.75 × 13.7	0.16	0.51	0.31
9.5 × 18.0	0.22	0.43	0.35
26 × 36 } 2 cyls.	0.28	0.24	0.39
51.2 × 55.13	0.28	0.52	0.20

The first three show, together with a notable gain in efficiency, a strong tendency to shift the waste heat from the water-jacket to the exhaust, as the engine increases in size; the last test is from an engine using blast-furnace gas, and which is liberally cooled with water. The whole table, and especially the last two examples, show that to a large extent an engineer may decide in the design of an engine, whether he will withdraw heat by thorough cooling, or allow the heat to be suppressed by dissociation and thrown out in the exhaust.

Mean Effective Pressure. — In the design of a gas-engine the

* Forest Lecture. Inst. Civ. Eng. cxliii. p. 21.

first question to be determined is the mean effective pressure that is desired or can be obtained. This must depend on the fuel and its mixture with air, and on the degree of compression. There does not at the present time appear to be information that will serve as the basis of a working theory for determining the mean effective pressure even when these features are determined.

It is desirable, in order that the engine shall be powerful and compact, that the mean effective pressure shall be high; English engineers commonly make use of 90 to 100 pounds mean effective pressure; but German engineers who have had experience with very large engines for which pre-ignition is dangerous, have been content with 60 pounds or less.

Waste-heat Engines. — On page 180 attention was called to the fact that the exhaust-steam from a steam-engine could be used for vaporizing some fluid like sulphur dioxide, and that thereby the temperature range could be extended. The only tests quoted failed to show the advantage that might be expected when this method is used with steam-engines. But the exhaust from a gas-engine is very hot, probably 1000° F., or over, and there appears to be no reason why the heat should be wasted, as it could readily be used to form steam in a boiler or for other purposes.

CHAPTER XV.

COMPRESSED AIR.

COMPRESSED air is used for transmitting power, for storing energy, and for producing refrigeration. Air at moderate pressure, produced by blowing-engines, is used in the production of iron and steel; and currents of air at slightly higher pressure than that of the atmosphere (produced by centrifugal fan-blowers) are used to ventilate mines, buildings, and ships, and for producing forced draught for steam-boilers. Attention will be given mainly to the transmission and storage of energy. The production and use of ventilating currents require and are susceptible of but little theoretical treatment. Refrigeration will be reserved for another chapter.

A treatment of the transmission of power by compressed air involves the discussion of air-compressors, of the flow of air through pipes, and of compressed-air engines or motors. The storage of energy differs from the transmission of power in that the compressed air, which is forced into a reservoir at high pressure, is used at a much lower pressure at the air-motor.

Air-Compressors. — There are three types of machines used for compressing or moving air: (1) piston air-compressors, (2) rotary blowers, (3) centrifugal blowers or fans.

The piston air-compressor is always used for producing high pressures. It consists of a piston moving in a cylinder with inlet- and exit-valves at each end. Commonly the valves are actuated by the air itself, but some compressors have their valves moved mechanically. Blowing-engines are usually piston-compressors, though the pressures produced are only ten or twenty pounds per square inch.

Rotary blowers have one or more rotating parts, so arranged that as they rotate, chambers of varying capacity are formed,

which receive air at atmospheric pressure, compress it, and deliver it against a higher pressure. They are simple and compact, but are wasteful of power on account of friction and leakage, and are used only for moderate pressures.

Fan-blowers consist of a number of radial plates or vanes, fixed to a horizontal axis and enclosed in a case. When the axis and the vanes attached to it are rotated at a high speed, air is drawn in through openings near the axis and is driven by centrifugal force into the case, from which it flows into the delivery-main or duct. Only low pressures, suitable for ventilation and forced draught, can be produced in this way. But little has been done in the development of the theory or the determination of the practical efficiency of fan-blowers. Some ventilating-fans have their axes parallel to the direction of the air-current, and the vanes have a more or less helicoidal form, so that they may force the air by direct pressure; they are in effect the converse of a windmill, producing instead of being driven by the current of air. They are useful rather for moving air than for producing a pressure.

Fluid Piston-Compressors. — It will be shown that the effect of clearance is to diminish the capacity of the compressor; consequently the clearance should be made as small as possible. With this in view the valves of compressors and blowers are commonly set in the cylinder-heads. Single-acting compressors with vertical cylinders have been made with a layer of water or some other fluid on top of the piston, which entirely fills the clearance-space when the piston is at the end of the stroke. An extension of this principle gives what are known as fluid piston-compressors. Such a compressor commonly has a double-acting piston in a horizontal cylinder much longer than the stroke of the piston, thus giving a large clearance at each end. The clearance-spaces extend upward to a considerable height, and the admission- and exhaust-valves are placed at or near the top, and the entire clearance-space is filled with water. The spaces and heights must be so arranged that when the piston is at one end of its stroke the water at that end shall fill the clearance

and cover the valves, and at the other end the water shall not fall to the level of the top of the cylinder. There are consequently two vertical fluid pistons actuated by a double-acting horizontal piston. It is essential that the spaces in which the fluid pistons act shall give no places in which air may be caught as in a pocket, and that there are no projecting ribs or other irregularities to break the surface of the water; and, further, the compressor must be run at a moderate speed. The water forming the fluid pistons becomes heated and saturated with air by continuous use, and should be renewed.

Air-pumps used with condensing-engines or for other purposes may be made with fluid pistons which are renewed by the water coming with the air or vapor. In case the water thus supplied is insufficient, water from without may be admitted, or water from the delivery may be allowed to flow back to the admission side of the pump.

Displacement Compressors. — When a supply of water under sufficient head is available, air may be compressed in suitably arranged cylinders or compressors by direct action of the water on air, compressing it and expelling it by displacement. Such compressors are very wasteful of power, and in general it is better to use water-power for driving piston-compressors, properly geared to turbine-wheels or other motors.

Cooling during Compression. — There is always a considerable rise of temperature due to compressing air in a piston air-compressor, which is liable to give trouble by heating the cylinder and interfering with lubrication. Blowing-engines which produce only moderate pressures usually have their cylinders lubricated with graphite, and no attempt is made to cool them. All compressors which produce high pressures have their cylinders cooled either by a water-jacket or by injecting water, or by both methods.

Since the air after compression is cooled either purposely or unavoidably, there would be a great advantage in cooling the air during compression, and thereby reducing the work of compression. Attempts have been made to cool the air by spray-

ing water into the cylinder, but experience has shown that the work of compression is not much affected by so doing. The only effective way of reducing the work of compression is to use a compound compressor, and to cool the air on the way from the first to the second cylinder. Three-stage compressors are used for very high pressures. It is, however, found that air which has been compressed to a high pressure and great density is more readily cooled during compression.

Moisture in the Cylinder. — If water is not injected into the cylinder of an air-compressor the moisture in the air will depend on the hygroscopic condition of the atmosphere. But even if the air were saturated with moisture the absolute and the relative weight of water in the cylinder would be insignificant. Thus at 60° F. the pressure of saturated steam is about one-fourth of a pound per square inch, and the weight of one cubic foot is about 0.0008 of a pound, while the weight of one cubic foot of air is about 0.08 of a pound. It is probable that the only effect of moisture in the atmosphere is to slightly reduce the exponent of the equation (77), page 64. This conclusion probably holds when the cylinder is cooled by a water-jacket.

When water is sprayed into the cylinder of a compressor the temperature of the air and the amount of vapor mixed with it vary, and there is no ready way of determining its condition. But, as has been stated, the spraying of water into the cylinder does not much reduce the work of compression, and consequently it is probable we can assume that the compression always follows the law expressed by an exponential equation; such as

$$pv^n = p_1 v_1^n.$$

The value to be given to n is not well known; it may be as small as 1.2 for a fluid piston-compressor, and it may approach 1.4 when the cooling of the air is ineffective, as is usually the case.

Power Expended. — The indicator-diagram of an air-compressor with no clearance-space is represented by Fig. 81. Air is drawn in at atmospheric pressure in the part of the cycle

of operations represented by *dc*; in the part represented by *cb* the air is compressed, and in the part represented by *ba* it is expelled against the higher pressure.

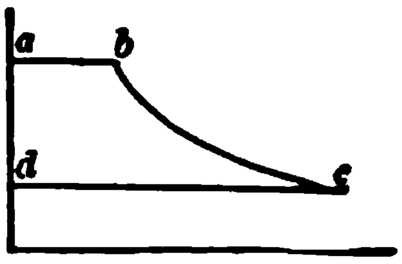

FIG. 81.

If p_1 is the specific pressure and v_1 the specific volume of one pound of air at atmospheric pressure, and p_2 and v_2 corresponding quantities at the higher pressure, then the work done by the atmosphere on the piston of the compressor while air is drawn in is p_1v_1. Assuming that the compression curve *cb* may be represented by an exponential curve having the form

$$pv^n = p_1v_1^n = \text{const.},$$

then the work of compression is

$$\int pdv = p_1v_1^n\int_{v_2}^{v_1}\frac{dv}{v^n} = \frac{p_1v_1}{n-1}\left\{\left(\frac{v_1}{v_2}\right)^{n-1} - 1\right\}$$

$$= \frac{p_1v_1}{n-1}\left\{\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right\}.$$

The work of expulsion from *b* to *a* is

$$p_2v_2 = p_2v\left(\frac{p_1}{p_2}\right)^{\frac{1}{n}} = p_1v_1\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$$

The effective work of the cycle is therefore

$$W = \frac{p_1v_1}{n-1}\left\{\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right\} + p_1v_1\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - p_1v_1\,;$$

$$\therefore\quad W = p_1v_1\frac{n}{n-1}\left\{\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right\} \quad . \; . \; . \; . \; . \; . \; (189)$$

Equation (189) gives the work done to compress one pound of air, p_1 and p_2 being specific pressures (in pounds per square foot), and v_1 the specific volume, which may be calculated by aid of the equation

$$\frac{pv}{T} = \frac{p_0v_0}{T_0},$$

in which the subscripts refer to the normal properties of air at freezing-point and at atmospheric pressure.

If, instead of the specific volume v_1, we use the volume V_1 of air drawn into the compressor we may readily transform equation (189) to give the horse-power directly, obtaining

$$\text{H. P.} = \frac{144\, p_1 V_1 n}{33000(n-1)} \left\{ \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1 \right\} \quad . \quad (190)$$

where p_1 is the pressure of the atmosphere in pounds per square inch, and n is the exponent of the equation representing the compression curve, which may vary from 1.4 for dry-air compressors to 1.2 for fluid piston-compressors.

Effect of Clearance. — The indicator-diagram of an air-compressor with clearance may be represented by Fig. 82. The end of the stroke expelling air is at a, and the air remaining in the cylinder expands from a to d, till the pressure becomes equal to the pressure of the atmosphere before the next supply of air is drawn in. The expansion curve ad may commonly be represented by an exponential equation having the same exponent as the compression curve cb, in which case the air in the clearance acts as a cushion which stores and restores energy, but does not affect the work done on the air passing through the cylinder. The work of compressing one unit of weight of air in such a compressor may be calculated by aid of equation (189), but the equation (190) for the horse-power cannot be used directly.

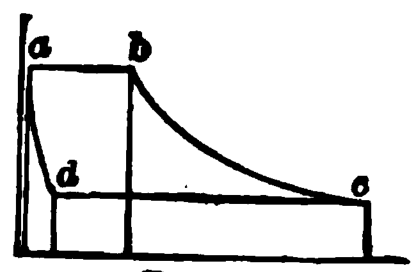

FIG. 82.

The principal effect of clearance is to increase the size of the cylinder required for a certain duty in the ratio of the entire length of the diagram in Fig. 82 to the length of the line dc.

Let the clearance be $\frac{1}{m}$ part of the piston displacement. At the beginning of the filling stroke, represented by the point a, that volume will be filled with air at the pressure p_2. After the expansion represented by ad the air in the clearance will have

the pressure p_1, and, assuming that the expansion follows the law expressed by the exponential equation

$$pv^n = p_1 v_1^n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (190a)$$

its volume will be

$$\frac{1}{m}\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}}$$

part of the piston displacement. The ratio of the line dc to the length of the diagram will consequently be

$$\frac{dc}{ac} = 1 - \frac{1}{m}\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}} + \frac{1}{m} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (191)$$

and this is the factor by which the piston displacement calculated without clearance must be divided to find the actual piston displacement.

Temperature at the End of Compression. — When the air in the compressor-cylinder is dry or contains only the moisture brought in with it, it may be assumed that the mixture of air and vapor follows the law of perfect gases,

$$\frac{pv}{T} = \frac{p_1 v_1}{T_1},$$

which, combined with the exponential equation

$$pv^n = p_1 v_1^n,$$

gives

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (192)$$

from which the final temperature T_2 at the end of compression may be determined when T_1 is known. When water is used freely in the cylinder of a compressor the final temperature cannot be determined by calculation, but must be determined from tests on compressors.

Contraction after Compression. — Ordinarily compressed air loses both pressure and temperature on the way from the com-

pressor to the place where it is to be used. The loss of pressure will be discussed under the head of the flow of air in long pipes; it should not be large, unless the air is carried a long distance. The loss of temperature causes a contraction of volume in two ways: first, the volume of the air at a given pressure is directly as the absolute temperature; second, the moisture in the air (whether brought in by the air or supplied in the condenser) in excess of that which will saturate the air at the lowest temperature in the conduit, is condensed. Provision must be made for draining off the condensed water. The method of estimating the contraction of volume due to the condensation of moisture will be exhibited later in the calculation of a special problem.

Interchange of Heat. — The interchanges of heat between the air in the cylinder of an air-compressor and the walls of the cylinder are the converse of those taking place between the steam and the walls of the cylinder of a steam-engine, and are much less in amount. The walls of the cylinder are never so cool as the incoming air, nor so warm as the air expelled; consequently the air receives heat during admission and the beginning of compression, and yields heat during the latter part of compression and during expulsion. The presence of moisture in the air increases this effect.

Volume of the Compressor Cylinder. — Let a compressor making n revolutions per minute be required to deliver V_3 cubic feet of air at the temperature t_3° F., or T_3° absolute, and at the absolute pressure p_3 pounds per square inch, at the place where the air is to be used. Assuming that the air is dry when it is delivered and that the atmosphere is dry when it is taken into the compressor, then the volume drawn into the compressor per minute at the temperature T_1 and the pressure p_1 will be

$$V_1 = V_3 \frac{p_3 T_1}{p_1 T_3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (193)$$

cubic feet; and this expression will be correct whatever may be the intermediate temperatures, pressures, or condition of saturation of the air.

If the compressor has no clearance the piston displacement will be

$$\frac{V_1}{2n} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (194)$$

if the clearance is $\frac{1}{m}$ part of the piston displacement, dividing by the factor (191) gives for the piston displacement

$$\frac{V_1}{2n} \div \left\{ 1 - \frac{1}{m}\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}} + \frac{1}{m} \right\} \quad . \quad . \quad . \quad . \quad . \quad (195)$$

expressed in cubic feet.

The pressure in the compressor-cylinder when air is drawn in, is always less than the pressure of the atmosphere, and when the air is expelled it is greater than the pressure against which it is delivered. From these causes and from other imperfections the compressor will not deliver the quantity of air calculated from its dimensions, and consequently the volume of the cylinder as calculated, whether with o: without clearance, must be increased by an amount to be determined by experiment.

Compound Compressors. — When air is to be compressed from the pressure p_1 to the pressure p_2, but is to be delivered at the initial temperature t_1, the work of compression may be reduced by dividing it between two cylinders, one of which takes the air at atmospheric pressure and delivers it at an intermediate pressure p' to a reservoir, from which the other cylinder takes it and delivers it at the required pressure p_2, provided that the air be cooled, at the pressure p', between the two cylinders.

The proper method of dividing the pressures and of proportioning the volumes of the cylinders so that the work of compression may be reduced to a minimum may be deduced from equation (189) when there is no clearance or when the clearance is neglected.

The work of compressing one pound of air from the pressure p_1 to the pressure p' is

$$W_1 = p_1 v_1 \frac{n}{n-1} \left\{ \left(\frac{p'}{p_1}\right)^{\frac{n-1}{n}} - 1 \right\}. \quad . \quad . \quad . \quad . \quad (196)$$

The work of compressing one pound from the pressure p' to p_2 is

$$W_2 = p'v' \frac{n}{n-1} \left\{ \left(\frac{p_2}{p'}\right)^{\frac{n-1}{n}} - 1 \right\} = p_1 v_1 \frac{n}{n-1} \left\{ \left(\frac{p_2}{p'}\right)^{\frac{n-1}{n}} - 1 \right\}, (197)$$

because the air after compression in the first cylinder is cooled to the temperature t_1 before it is supplied to the second cylinder, and consequently $p'v' = p_1 v_1$. The total work of compression is

$$W = W_1 + W_2 = p_1 v_1 \frac{n}{n-1} \left\{ \left(\frac{p'}{p_1}\right)^{\frac{n-1}{n}} + \left(\frac{p_2}{p'}\right)^{\frac{n-1}{n}} - 2 \right\}, \quad (198)$$

and this becomes a minimum when

$$\left(\frac{p'}{p_1}\right)^{\frac{n-1}{n}} + \left(\frac{p_2}{p'}\right)^{\frac{n-1}{n}}$$

becomes a minimum. Differentiating with regard to p', and equating the first differential coefficient to zero, gives

$$p' = \sqrt{p_1 p_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (199)$$

Since the air is supplied to each cylinder at the temperature t_1, their volumes should be inversely as the absolute pressures p_1 and p'. This method also leads to an equal distribution of work between the two cylinders, for if the value of p' from equation (189) is introduced into equations (197) and (198) we shall obtain

$$W_1 = W_2 = p_1 v_1 \frac{n}{n-1} \left\{ \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{2n}} - 1 \right\} . \quad . \quad . \quad (200)$$

and the total work of compression is

$$W = 2 p_1 v_1 \frac{n}{n-1} \left\{ \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{2n}} - 1 \right\} . \quad . \quad . \quad . \quad (201)$$

Three-stage Compressors. — When very high pressures are required, as where air is used for storing energy, it is customary to use a compressor with a series of three cylinders, through which the air is passed in succession, and to cool the air on the way from one cylinder to the next. If the initial and final pressures are p_1 and p_2, and if p' and p'' are the pressures in the intermediate receivers in which the air is cooled, the conditions for most economical compression may be deduced in the following way:

The work of compressing one pound of air in the several cylinders will be

$$W_1 = p_1 v_1 \frac{n}{n-1}\left\{\left(\frac{p'}{p_1}\right)^{\frac{n-1}{n}} - 1\right\} \quad . \quad . \quad . \quad (202)$$

$$W_2 = p'v' \frac{n}{n-1}\left\{\left(\frac{p''}{p'}\right)^{\frac{n-1}{n}} - 1\right\} \quad . \quad . \quad . \quad (203)$$

$$W_3 = p''v'' \frac{n}{n-1}\left\{\left(\frac{p_2}{p''}\right)^{\frac{n-1}{n}} - 1\right\} \quad . \quad . \quad . \quad . \quad (204)$$

But since the air is cooled to the initial temperature on its way from one cylinder to the other so that

$$p_1 v_1 = p'v' = p''v'';$$

the total work of compressing one pound of air will be

$$W = W_1 + W_2 + W_3$$

$$= p_1 v_1 \frac{n}{n-1}\left\{\left(\frac{p'}{p_1}\right)^{\frac{n-1}{n}} + \left(\frac{p''}{p'}\right)^{\frac{n-1}{n}} + \left(\frac{p_2}{p''}\right)^{\frac{n-1}{n}} - 3\right\} \quad . \quad . \quad (205)$$

This expression will be a minimum when

$$\left(\frac{p'}{p_1}\right)^{\frac{n-1}{n}} + \left(\frac{p''}{p'}\right)^{\frac{n-1}{n}} + \left(\frac{p_2}{p''}\right)^{\frac{n-1}{n}} = k$$

becomes a minimum; that is, when

$$\frac{\delta k}{\delta p'} = \frac{n-1}{n}\frac{p'^{-\frac{1}{n}}}{p_1^{\frac{n-1}{n}}} - \frac{n-1}{n}\frac{p''^{\frac{n-1}{n}}}{p'^{\frac{2n-1}{n}}} = 0 \quad . \quad . \quad (206)$$

and

$$\frac{\delta k}{\delta p''} = \frac{n-1}{n}\frac{p''^{-\frac{1}{n}}}{p'^{\frac{n-1}{n}}} - \frac{n-1}{n}\frac{p_2^{\frac{n-1}{n}}}{p''^{\frac{2n-1}{n}}} = 0 \quad . \quad . \quad . \quad (207)$$

Equations (206) and (207) lead to

$$p'^2 = p_1 p'' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (208)$$

$$p''^2 = p' p_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (209)$$

from which by elimination we have

$$p' = \sqrt[3]{p_1^2 p_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (210)$$

and

$$p'' = \sqrt[3]{p_1 p_2^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (211)$$

Since the temperature is the same at the admission to each of the three cylinders, the volumes of the cylinders should be inversely proportional to the absolute pressures p_1, p', and p''. As with the compound compressors, this method of arranging a three-stage compressor leads to an equal distribution of work between the cylinders. For, if the values of p' and p'' from equations (210) and (211) are introduced into equations (202) to (204), taking account also of the equation (190a) we shall have

$$W_1 = W_2 = W_3 = p_1 v_1 \frac{n}{n-1}\left\{\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{3n}} - 1\right\} \quad . \quad (212)$$

and consequently the total work of compression is

$$W = 3p_1 v_1 \frac{n}{n-1}\left\{\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{3n}} - 1\right\} \quad . \quad . \quad . \quad . \quad . \quad (213)$$

Friction and Imperfections. — The discussion has thus far taken no account of friction of the compressor nor of imperfections due to delay in the action of the valves and to heating the air as it enters the cylinder of the compressor.

From comparisons of indicator-diagrams taken from the steam- and the air-cylinders of certain combined steam-engines and air-compressors at Paris, Professor Kennedy found a mechanical efficiency of 0.845. Professor Gutermuth found an efficiency of 0.87 for a new Riedler compressor. It will be fair to assume an efficiency of 0.85 for compressors which are driven by steam-

engines; compressors driven by turbines will probably be affected to a like extent by friction.

The following table given by Professor Unwin * shows the effect of imperfect valve-action and of heating the entering air as deduced from tests on a Dubois-François compressor which had a diameter of 18 inches and a stroke of 48 inches.

RATIO OF ACTUAL AND APPARENT CAPACITIES OF AN AIR-COMPRESSOR.

Piston speed, feet per minute.	Revolutions per minute.	Ratio of air delivered at atmospheric pressure and temperature to volume displaced by piston.
80	10	0.94
160	20	0.92
200	25	0.90
240	30	0.86
280	35	0.78

This table does not take account of the effect of clearance, nor is the clearance for the compressor stated. It is probable that five or ten per cent will be enough to allow for imperfect valve-action after the effect of clearance is properly calculated. The effect of clearance is to require a larger volume of cylinder than would be needed without clearance. The effect of imperfect valve-action and of heating of the entering air is to require an additional increase in the size of the cylinder of the air-compressor and also to increase the work of compression.

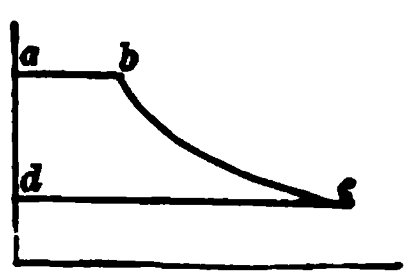

FIG. 83.

Efficiency of Compression. — If air could be so cooled during compression that the temperature should not rise, the compression line *cb*, Fig. 83, would be an isothermal line, and the work of compressing one pound of air

* *Development and Transmission of Power*, p. 182.

would be

$$W = p_2v_2 + p_1v_1 \log_e \frac{v_1}{v_2} - p_1v_1;$$

but $p_1v_1 = p_2v_2$ for an isothermal change, and consequently

$$W = p_1v_1 \log_e \frac{p_2}{p_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (214)$$

Some investigators have taken the work of isothermal compression, represented by equation (214), as a basis of comparison for compressors, and have considered its ratio to the actual work of compression as the efficiency of compression. This throws together into one factor the effect of heating during compression and the effect of imperfect valve-action.

Professor Riedler* obtained indicator-diagrams from the cylinders of a number of air-compressors and drew upon them the diagrams which would represent the work of isothermal compression, without clearance or valve losses. A comparison of the areas of the isothermal and the actual diagrams gave the arbitrary efficiency of compression just described. The following table gives his results:

ARBITRARY EFFICIENCY OF COMPRESSION.

Type of compressor.	Pressures in main, atmospheres.	Lost work in per cent of useful work.	Arbitrary efficiency.
Colladon, St. Gothard	6	105.0	0.488
do.	6	92.0	0.521
Sturgeon	3	94.3	0.515
Colladon	4	38.15	0.772
Slide-valve	5	49.3	0.670
Paxman	6	42.7	0.701
Cockerill	6	40.2	0.713
Riedler two-stage	6	12.07	0.892

A similar comparison for a fluid piston-compressor showed an efficiency of 0.84.

* *Development and Distribution of Power*, Unwin.

There are three notable conclusions that may be drawn from this table: (1) there is much difference between compressors working at the same pressures, (2) a simple compressor loses efficiency rapidly as the pressure rises, and (3) the compound or two-stage compressor shows a great advantage over a simple compresson.

Test of a Blowing-Engine. — Pernolet * gives the following test of a blowing-engine used to produce the blast for Bessemer converters at Creusot. The engine was a two-cylinder horizontal engine, with the cranks at right angles. The piston-rod for each cylinder extended through the cylinder-head and actuated a double-acting compressor. The dimensions were:

Diameter, steam-pistons	47½ inches
" air-pistons	59 "
Stroke	70.9 "
Diameter of fly-wheel	26¼ feet

At 28 revolutions per minute the following results were obtained:

Indicated horse-power of steam-cylinders	1078
" " " air-cylinders	986
Efficiency	0.92
Temperature of air admitted	50° F.
" " " delivered	140° F.
Pressure of air delivered, pounds per square inch gauge	23.4
Pressure of air in supply-pipe, pounds per square inch gauge	0.44

At 25 revolutions there was no sensible depression of pressure in the supply-pipe.

The air from such a blowing-engine probably suffers little loss of temperature after compression.

Hydraulic Air-Compressor. — The Taylor hydraulic air-compressor makes use of water-power for compressing air at constant

* *L'Air Comprimé*, 1876.

temperature. The essential features are an aspirator for charging the water with air, a column of water to give the required pressure, and a separator to gather the air from the water after compression. The water is brought to the compressor in a penstock, as it would be to a water-wheel, and below the dam it flows away in a tailrace; the power available is determined from the weight of water flowing and the head in the penstock above the tailrace, in the usual manner. Below the dam a shaft is excavated to a depth proper to give the required pressure (about 2.3 feet depth per pound pressure), and then a chamber is excavated to provide space for the separator. In the shaft is a plate-iron pipe or cylinder, down which the water flows; after, passing the separator the water ascends in the shaft and flows away at the tailrace.

The head of the pipe is surrounded by a vertical plate-iron drum into which the penstock leads, so that water is supplied to the head all round the periphery. The head itself is formed of two inverted conical iron-castings, so formed that the space into which the water flows at first contracts and then expands; the changes of velocity being gradual, no appreciable loss of energy ensues. At the throat of the inlet, where the velocity is highest, there is a partial vacuum, and air is admitted through numerous small pipes so that the water is charged with bubbles of air. The upper conical casting can be set by hand to control the supply of water and air.

As the mingled column of water and air-bubbles goes down the pipe, the air is compressed at appreciably the temperature of the water. At the lower end, the pipe expands to reduce the velocity, and delivers the air and water into a plate-iron bell; the air gathers in the top of the bell, from which it is led by a pipe, and the water escapes under the edge of the bell. Air in solution is unavoidably lost, and forms the chief source of loss of power in the device. The air is, of course, saturated with moisture at the temperature of the water, but that is probably the condition of compressed air however produced. The efficiency of the compressor may be taken as about 0.60 to

0.70; making allowance for loss in transmission and for the efficiency of the compressed-air motors, the system appears to be inferior to the ordinary turbine water-wheel.

Air-Pumps. — The feed-water supplied to a steam-boiler usually contains air in solution, which passes from the boiler with the steam to the engine and thence to the condenser. In like manner the injection-water supplied to a jet-condenser brings in air in solution. Also there is more or less leakage of air into the cylinder communicating with the condenser and into the exhaust-pipe or the condenser itself. An air-pump must therefore be provided to remove this air and to maintain the vacuum. The air-pump also removes the condensed steam from a surface-condenser, and the mingled condensed steam and injection-water from a jet-condenser. If no air were brought into the condenser the vacuum would be maintained by the condensation of the steam by the injection, or the cooling water, and it would be sufficient to remove the water by a common pump, which, with a surface-condenser, might be the feed-pump.

The weight of injection-water per pound of steam, calculated by the method on page 149, will usually be less than 20 pounds, but it is customary to provide 30 pounds of injection-water per pound of steam, with some method of regulating the quantity delivered.

It may be assumed that the injection-water will bring in with it one-twentieth of its volume of air at atmospheric pressure, and that this air will expand in the condenser to a volume inversely proportional to the absolute pressure in the condenser. The capacity of the air-pump must be sufficient to remove this air and the condensed steam and injection-water.

An air-pump for use with a surface-condenser may be smaller than one used with a jet-condenser. In marine work it is common to provide a method of changing a surface- into a jet-condenser, and to make the air-pump large enough to give a fair vacuum in case such a change should become advisable in an emergency.

Seaton * states that the efficiency of a vertical single-acting air-pump varies from 0.4 to 0.6, and that of a double-acting horizontal air-pump from 0.3 to 0.5, depending on the design and condition; that is, the volume of air and water actually discharged will bear such ratios to the displacement of the pump.

He also gives the following table of ratios of capacity of air-pump cylinders to the volume of the engine cylinder or cylinders discharging steam into the condenser:

RATIO OF ENGINE AND AIR-PUMP CYLINDERS.

Description of Pump.	Description of Engine.	Ratio.
Single-acting vertical	Jet-condensing, expansion 1½ to 2	6 to 8
" "	Surface- " " 1½ to 2	8 to 10
" "	Jet- " " 3 to 5	10 to 12
" "	Surface- " " 3 to 5	12 to 15
" "	" compound . . .	15 to 18
Double-acting horizontal . . .	Jet-condensing, expansion 1½ to 2	10 to 13
" " . . .	Surface- " " 1½ to 2	13 to 16
" " . . .	Jet- " " 3 to 5	16 to 19
" " . . .	Surface- " " 3 to 5	19 to 24
" " . . .	" " compound . . .	24 to 28

Dry-air Pump. — In the recent development of steam-engineering, especially for steam-turbines, great emphasis is given to obtaining a high vacuum. For this purpose the old form of air-pump which withdraws air and water from the condenser has been replaced by a feed-pump which takes water only from the condenser, and a dry-air pump which removes the air. The air is necessarily saturated with moisture at the temperature in the condenser, and allowance must be made for this moisture or steam, in the design of the pump. For this purpose Dalton's law is used, which says that the total pressure in any receptacle containing air and vapor is equal to the sum of the pressures due to the air and to the vapor.

* *Manual of Marine Engineering.*

If the amount of air brought by the water to a jet-condenser can be determined or assumed, a calculation for a dry-air pump can readily be made. The leakage to a surface-condenser cannot be estimated, and consequently the only way of proportioning the air-pump for a surface-condenser is that already given on page 375.

To illustrate the method of calculation for a dry-air pump use will be made of the data from the test of the Chestnut Hill Pumping Station already quoted on page 239.

The vacuum in the condenser was 27.25 inches of mercury, and the barometer stood at 30.25 inches reduced to 32° F., so that the absolute pressure was 1.47 of a pound. The condensing water entered the surface-condenser at 51°.9 F. and left at 85°.2 F.; had there been a jet-condenser this would have been the temperature in the condenser and will be used for our calculation. Making use of the equation for the quantity of condensing water on page 150, we have,

$$\frac{H - q_k}{q_k - q_i} = \frac{1111.2 - 53.3}{53.3 - 20} = 32.$$

Since the engine used 11.22 pounds of steam per horse-power per hour and developed 575.7 horse-power, the total condensing water per hour would be

$$\frac{32 \times 11.22 \times 575.7}{62.4} = 3310;$$

the denominator being the weight of a cubic foot of water. If the water brings one-twentieth of its volume of atmospheric air, the volume of air will be 166 cubic feet per hour.

Steam at 85°.2 F. has the pressure of 0.598 of a pound absolute; consequently the pressure 1.47 of a pound in the condenser is made up of 0.598 steam-pressure and 0.872 air-pressure. The atmospheric pressure is 30.25 inches of mercury or 14.85 pounds, so that taking account of the influence of the pressures and absolute temperatures the volume of air (saturated with moisture) to be removed from the condenser per hour is

$$166 \times \frac{459.5 + 85.2}{459.5 + 51.9} \times \frac{14.85}{0.872} = 3010 \text{ cubic feet.}$$

Assuming the air-pump to be single-acting and to be connected directly to the engine which made about 50 revolutions per minute, the effective displacement of the air-pump bucket should be

$$3010 \div (50 \times 60) = 1.0 \text{ cubic foot.}$$

To allow for the effect of the air-pump clearance, imperfection of valve-action, and for variation in the temperature of condensing water, this quantity may be increased by 50 to 100 per cent.

The engine had $3\frac{1}{2}$ feet for the diameter and 6 feet for the stroke of the low-pressure piston, so that its displacement was nearly 50 cubic feet; the air-pump had a diameter of 2 feet and a stroke of one foot, so that its displacement was 3.14 cubic feet; the ratio of displacements was about sixteen. This discrepancy shows that the conventional method of designing air-pumps provides liberal capacity.

Calculation for an Air Compressor. — Let it be required to find the dimensions of an air-compressor to deliver 300 cubic feet of air per minute at 100 pounds per square inch by the gauge, and also the horse-power required to drive it.

If it is assumed that the air is forced into the delivery-pipe at the temperature of the atmosphere, and, further, that there is no loss of pressure between the compressor and the delivery-pipe, equation (193) for finding the volume drawn into the compressor will be reduced to

$$V_1 = V_3 \frac{p_3}{p_1} = 300 \times \frac{114.7}{14.7} = 2341 \text{ cubic feet.}$$

If now we allow five per cent for imperfect valve-action and for heating the air as it is drawn into the compressor the apparent capacity of the compressor will be

$$2341 \div 0.95 = 2464 \text{ cubic feet.}$$

This is the volume on which the *power* for the compressor must be calculated.

If the clearance of the compressor is 0.02 of the piston displacement, then the factor for allowing for clearance will be

$$1 - \frac{1}{m}\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}} + \frac{1}{m} = 1 - \frac{2}{100}\left(\frac{114.7}{14.7}\right)^{\frac{1}{1.4}} + \frac{2}{100} = 0.9332$$

if the exponent of the equation representing the expansion of the air in the clearance is 1.4. Consequently the volume on which the dimensions of the compressor must be based is

$$2464 \div 0.9332 = 2640 \text{ cubic feet.}$$

At 80 revolutions per minute the mean piston displacement will be

$$2640 \div (2 \times 80) = 16.5 \text{ cubic feet.}$$

Assuming a stroke of 3 feet, the mean area of the piston must be

$$(144 \times 16.5) \div 3 = 792 \text{ square inches.}$$

Allowing 16 square inches for a piston-rod 4½ inches in diameter gives a mean area of 800 square inches for the piston, which corresponds very nearly to 32 inches for the diameter of the piston.

The power expended in the compressor-cylinder may be calculated by equation (190), using for V_1 the apparent capacity of the compressor, giving

$$\text{H.P.} = \frac{144 \times 14.7 \times 2464 \times 1.4}{33000 \times (1.4 - 1)}\left\{\left(\frac{114.7}{14.7}\right)^{\frac{1.4-1}{1.4}} - 1\right\} = 442.$$

If the friction of the combined steam-engine and compressor is assumed to be 15 per cent the horse-power of the steam-cylinder must be

$$442 \div 0.85 = 520.$$

If the temperature of the atmosphere drawn into the compressor is 70° F., then by an equation like (80), page 65, the delivery temperature will be

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = (460 + 70)\left(\frac{114.7}{14.7}\right)^{\frac{1.4-1}{1.4}} = 953°$$

absolute, or about 493° F.

The calculation has been carried on for a simple compressor, but there will be a decided advantage in using a compound compressor for such work. Such a compressor should have for the pressure in the intermediate reservoir

$$p' = \sqrt{p_1 p_2} = \sqrt{114.7 \times 14.7} = 41.06 \text{ pounds.}$$

The factor for allowing for clearance of the low-pressure cylinder will now be

$$1 - \frac{1}{m}\left(\frac{p'}{p_1}\right)^{\frac{1}{n}} + \frac{1}{m} = 1 - \frac{2}{100}\left(\frac{41.06}{14.7}\right)^{\frac{1}{1.4}} + \frac{2}{100} = 0.9784.$$

The loss from imperfect action of the valves and for heating of the air as it enters the compressor will be less for a compound than for a simple compressor, but we will here retain the value 2464 cubic feet, previously found for the apparent capacity of the compressor. The volume from which the dimensions of the compressor will be found will now be

$$2464 \div 0.9784 = 2518 \text{ cubic feet,}$$

which with 80 revolutions per minute will give 15.74 cubic feet for the piston displacement, and 755.5 square inches for the effective piston area, if the stroke is made 3 feet, as before. Adding 16 inches for the piston-rod, which will be assumed to pass entirely through the cylinder, will give for the diameter of the low-pressure cylinder 31⅜ inches.

Since the pressure p' is a mean proportional between p_1 and p_2, the clearance factor for the high-pressure cylinder will be the same as that for the low-pressure cylinder, and, as the volumes are inversely proportional to the pressures p_1 and p', the high-pressure piston displacement will be

$$(15.74 \times 14.7) \div 41.06 = 5.64 \text{ cubic feet.}$$

If we allow 8 inches for a rod 4½ inches in diameter at one side of the piston, then the mean area of the piston will be 278.7 square inches, which corresponds to a diameter of 18⅞ inches for the high-pressure cylinder. In reality the piston-rod for the compound compressor may have a less diameter than the rod for

a simple compressor, because the maximum pressure on both pistons will be less than that for the piston of the simple compressor. Again, the rod which extends from the large to the small piston may be reduced in size. But details like these which depend on the calculation of strength cannot properly receive much attention at this place.

The power required to drive the compressor may be derived from equation (190), replacing v_1, the specific volume, by V_1, the apparent capacity of the low-pressure cylinder. Using the apparent capacity already obtained, 2464 cubic feet, we have for the power expended in the air-cylinders

$$\text{H.P.} = \frac{2 \times 144 \times 14.7 \times 2464 \times 1.4}{33000 \times (1.4 - 1)} \left\{ \left(\frac{114.7}{14.7}\right)^{\frac{1.4-1}{2 \times 1.4}} - 1 \right\} = 377;$$

and, as before, allowing 15 per cent for friction of the engine and compressor, we have for the indicated horse-power of the steam-engine

$$377 \div 0.85 = 444.$$

The temperature at the delivery from the low-pressure cylinder will be for 70° F. atmospheric temperature

$$(460 + 70)\left(\frac{41.06}{14.7}\right)^{\frac{1.4-1}{1.4}} = 711^\circ$$

absolute, or 251° F. Since p' is a mean proportional between p_1 and p_2, this will also be the temperature of the air delivered by the high-pressure cylinder.

Friction of Air in Pipes. — The resistance to the flow of a liquid through a pipe is represented in works on hydraulics by an expression having the form

$$\zeta \frac{u^2}{2g} \frac{l}{m} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (215)$$

in which ζ is an experimental coefficient, u is the velocity in feet per second, g is the acceleration due to gravity, l is the length of the pipe in feet, and m is the hydraulic mean depth,

which last term is obtained by dividing the area of the pipe by its perimeter. For a cylindrical pipe we have consequently

$$m = \tfrac{1}{4}\pi d^2 \div \pi d = \tfrac{1}{4}d \quad . \quad . \quad . \quad . \quad . \quad . \quad (216)$$

The expression (215) represents the head of liquid required to overcome the resistance of friction in the pipe when the velocity of flow is u feet per second. Such an expression cannot properly be applied to flow of air through a pipe when there is an appreciable loss of pressure, for the accompanying increase in volume necessitates an increase of velocity, whereas the expression treats the velocity as a constant. If, however, we consider the flow through an infinitesimal length of pipe, for which the velocity may be treated as constant, we may write for the loss of head due to friction

$$\zeta \frac{u^2}{2g} \frac{dl}{m} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (217)$$

This loss of head is the vertical distance through which the air must fall to produce the work expended in overcoming friction, and the total work thus expended may be found by multiplying the loss of head by the weight of air flowing through the pipe. It is convenient to deal with one pound of air, so that the expression for the loss of head also represents the work expended.

The air flowing through a long pipe soon attains the temperature of the pipe and thereafter remains at a constant temperature, so that our discussion for the resistance of friction may be made under the assumption of constant temperature, which much simplifies our work, because the intrinsic energy of the air remains constant. Again, the work done by the air on entering a given length dl will be equal to the work done by the air when it leaves that section, because the product of the pressure by the volume is constant.

Since there is a continual increase of volume corresponding to the loss of pressure to overcome friction, and consequently a continual increase of velocity from the entrance to the exit end of the pipe, there is also a continual gain of kinetic energy.

But the velocity of air in long pipes is small, and the changes of kinetic energy can be neglected.

The air expands by the amount dv as it passes through the length dl of pipe, and each pound does the work pdv. This work must be supplied by the loss of head, and, since there is no other expenditure of energy, the work expended in the loss of head is equal to the work done by expansion; consequently

$$pdv = \zeta \frac{u^2}{2g} \frac{dl}{m} \quad \ldots \ldots \quad (218)$$

But from the characteristic equation

$$pv = RT \quad \ldots \ldots \quad (219)$$

we have

$$dv = - \frac{RT}{p^2} dp,$$

which substituted in equation (217) gives

$$\zeta \frac{u^2}{2g} \frac{dl}{m} = - \frac{RT}{p} dp \quad \ldots \quad (220)$$

If the area of the pipe is A square feet, and if W pounds of air flow through it per second, then

$$u = \frac{Wv}{A} = \frac{WRT}{Ap} \quad \ldots \ldots \quad (221)$$

in which v is the specific volume, for which a value may be derived from equation (219). Replacing u in equation (220) by the value just derived, we have

$$\zeta \frac{W^2 T^2 R^2 dl}{2gA^2 p^2 m} = - \frac{RT}{p} dp;$$

$$\therefore \quad \zeta \frac{W^2 dl}{2gA^2 m} = - \frac{p}{RT} dp \quad \ldots \quad (222)$$

Integrating between the limits L and o, and p_2 and p_1, we have

$$\zeta \frac{W^2 L}{gA^2 m} = \frac{p_1^2 - p_2^2}{RT}. \quad \ldots \ldots \quad (223)$$

But from equation (221) the velocity at the entrance to the pipe where the pressure is p_1 will be

$$u_1 = \frac{WRT}{Ap_1} \quad \text{and} \quad W = \frac{Ap_1u_1}{RT},$$

so that equation (223) may be reduced to

$$\zeta \frac{A^2p_1^2u_1^2L}{gA^2mR^2T^2} = \frac{p_1^2 - p_2^2}{RT};$$

$$\therefore \quad \zeta \frac{u_1^2L}{gRTm} = \frac{p_1^2 - p_2^2}{p_1^2} \quad . \quad . \quad . \quad . \quad . \quad (224)$$

Equation (224) may be solved as follows:

$$u_1 = \left\{ \frac{gRTm}{\zeta L} \frac{p_1^2 - p_2^2}{p_1^2} \right\}^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad . \quad (225)$$

$$p_2 = p_1 \left\{ 1 - \frac{\zeta u_1^2 L}{gRTm} \right\}^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (226)$$

$$\zeta = \frac{gRTm}{u_1^2L} \frac{p_1^2 - p_2^2}{p_1^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (227)$$

The first two forms allow us to calculate either the velocity or the loss of pressure; the last form may be used to calculate values of ζ from experiments on the flow through pipes.

From experiments made by Riedler and Gutermuth* Professor Unwin † deduces the following values for ζ:

Diameter of pipe, feet.	ζ
0.492	0.00435
0.656	0.00393
0.980	0.00351

For pipes over one foot in diameter he recommends for use

$$\zeta = 0.003.$$

* *Neue Erfahrungen über die Kraftversorgung von Paris durch Druckluft*, 1891.
† *Development and Distribution of Power.*

Replacing the hydraulic mean depth m by $\frac{1}{4}d$, its value for round pipes, and using $R = 53.22$ and $g = 32.16$, we have in place of equation (226)

$$p_2 = p_1 \left\{ 1 - \frac{\zeta u_1^2 L}{430 T d} \right\}^{\frac{1}{2}} . \quad . \quad . \quad . \qquad (228)$$

All of the dimensions are given in feet, but from the form of the equation it is evident that the pressures may be in any convenient units, for example, in pounds per square inch absolute.

For example, let us find the loss of pressure of 300 cubic feet per minute if delivered through a six-inch pipe a mile long, the initial pressure being 100 pounds by the gauge.

The velocity of the air will be

$$(300 \div 60) \div \frac{\pi d^2}{4} = 5 \div \frac{\pi \left(\frac{1}{2}\right)^2}{4} = 25.5 \text{ feet.}$$

The terminal pressure will consequently be

$$p_2 = p_1 \left\{ 1 - \frac{\zeta u_1^2 L}{430 T d} \right\} = 114.7 \left\{ 1 - \frac{0.0044 \times \overline{25.5}^2 \times 5280}{430\,(460 + 70)\frac{1}{2}} \right\}^{\frac{1}{2}}$$

$$= 107 \text{ pounds,}$$

with 70° F. for the temperature of the atmosphere and with $\zeta = 0.0044$. Consequently the loss of pressure is about eight pounds.

Compressed-air Engines. — Engines for using compressed air differ from steam-engines only in details that depend on the nature of the working fluid. In some instances compressed air has been used in steam-engines without any change; for example, in Fig. 84 the dotted diagram was taken from the cylinder of an engine using compressed air, and the dot-and-dash diagram was taken from the same end of the cylinder when

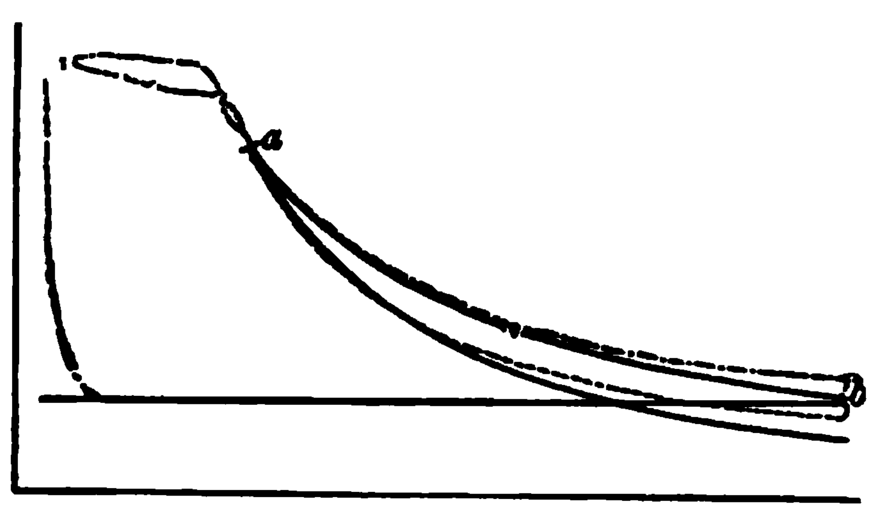

FIG. 84.

steam was used in it. The full line *ab* is a hyperbola, and the line *ac* is the adiabatic line for a gas; both lines are drawn through the intersection of the expansion lines of the two diagrams.

Power of Compressed-air Engines. — The probable mean effective pressure attained in the cylinder of a compressed-air engine, or to be expected in a projected engine, may be found in the same manner as is used in designing a steam-engine. In Fig. 85 the expansion curve 1 2 and the compression curve 3 0 may be assumed to be adiabatic lines for a gas represented by the equation

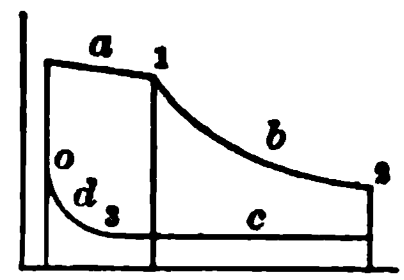

FIG. 85.

$$pv^{\kappa} = p_1 v_1^{\kappa},$$

and the area of the diagram may be found in the usual way, and therefrom the mean effective pressure can be determined. Having the mean effective pressure, the power of a given engine or the size required for a given power may be determined directly. The method will be illustrated later by an example.

Air-Consumption. — The air consumed by a given compressed-air engine may be calculated from the volume, pressure, and temperature at cut-off or release, and the volume, temperature, and pressure at compression, in the same way that the indicated consumption of a steam-engine is calculated; but in this case the indicated and actual consumption should be the same, since there is no change of state of the working fluid. Since the intrinsic energy of a gas is a function of the temperature only, the temperature will not be changed by loss of pressure in the valves and passages, and the air at cut-off will be cooler than in the supply-pipe, only on account of the chilling action of the walls of the cylinder during admission, which action cannot be energetic when the air is dry, and probably is not very important when the air is saturated.

Final Temperature. — If the expansion in a compressed-air engine is complete, i.e., if it is carried down to the pressure in the exhaust-pipe, then, assuming that there are no losses of

pressure in valves and passages, the final temperature may be found by the equation

$$T_4 = T_3 \left(\frac{p_4}{p_3}\right)^{\frac{\kappa - 1}{\kappa}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (229)$$

If the expansion is not complete, then the temperature at the end of expansion may be found by the equation

$$T_r = T_3 \left(\frac{V_c}{V_r}\right)^{\kappa - 1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (230)$$

in which V_c is the volume in the cylinder at cut-off and V_r at release, T_r is the absolute temperature at the end of expansion, and T_3 is the temperature at cut-off, assumed to be the same as in the supply-pipe. T_r is not the temperature during back-pressure nor in the exhaust-pipe. When the exhaust-valve is opened at release the air will expand suddenly, and part of the air will be expelled at the expense of the energy in the air remaining — much as though that air expanded behind a piston, and the temperature in the cylinder during exhaust and at the beginning of compression may be calculated by equation (229). The temperature in the exhaust-pipe will not be so low, for the temperature of the escaping air will vary during the expulsion produced by sudden expansion, and will only at the end of that operation have the temperature T_4, while the energy expended on that air to give it velocity will be restored when the velocity is reduced to that in the exhaust-pipe.

Volume of the Cylinder. — The determination of the volume of the cylinder of a compressed-air engine which uses a stated volume of air per minute is the converse of the determination of the air consumed by a given engine, and can be found by a similar process. We may calculate the volume of air, at the pressure in the supply-pipe, consumed per stroke by an engine having one unit of volume for its piston displacement, and therefrom find the number of units of volume of the piston displacement for the required engine.

Interchange of Heat. — The interchanges of heat between

the walls of the cylinder of a compressed-air engine and the air working therein are of the same sort as those taking place between the steam and the walls of the cylinder of a steam-engine; that is to say, the walls absorb heat during admission and compression if the latter is carried to a considerable degree, and yield heat during expansion and exhaust. Since the walls of the cylinder are never so warm as the entering air nor so cold as the air exhausted, the walls may absorb heat during the beginning of expansion and yield heat during the beginning of compression.

The amount of interchange of heat is much less in a compressed-air engine than in a steam-engine. With a moderate expansion the interchanges of heat between dry air and the walls of the cylinder are insignificant. Moisture in the air increases the interchanges in a marked degree, but does not make them so large that they need be considered in ordinary calculations.

Moisture in the Cylinder. — The chief disadvantage in the use of moist compressed air — and it is fair to assume that compressed air is nearly if not quite saturated when it comes to the engine — is that the low temperature experienced when the range of pressures is considerable causes the moisture to freeze in the cylinder and clog the exhaust-valves. The difficulty may be overcome in part by making the valves and passages of large size. Freezing of the moisture may be prevented by injecting steam or hot water into the supply-pipe or the cylinder, or the air may be heated by passing it through externally heated pipes or by some similar device. In the application of compressed air to driving street-cars the air from the reservoir has been passed through hot water, and thereby made to take up enough hot moisture to prevent freezing. The study of gas-engines suggests a method of heating compressed air which it is believed has never been tried. The air supplied to a compressed-air engine, or a part of the air, could be caused to pass through a lamp of proper construction to give complete combustion, and the products of combustion passed to the engine with the air. Should such a device be used it would be advisable that the tem-

perature of the air should be raised only to a moderate degree to avoid destruction of the lubricants in the cylinder, and the combustion at all hazards must be complete, or the cylinder would be fouled by unburned carbon.

Compound Air-Engines. — When air is expanded to a considerable degree in a compressed-air engine a gain may be realized by dividing the expansion into two or more stages in as many cylinders, provided that the air can be economically reheated between the cylinders. The heat of the atmosphere or of water at the same temperature may sometimes be used for this purpose. It is not known that machines of this construction have been used. If they were to be constructed the practical advantages of equal distribution of work and pressure would probably control the ratio of the volumes of the cylinders.

Calculation for a Compressed-air Engine. — Let it be required to find the dimensions for a compressed-air engine to develop 100 indicated horse-power at the pressure of 92 pounds by the gauge and at 70° F. Assume the clearance to be five per cent of the piston displacement, and assume the cut-off to be at quarter stroke, the release to be at the end of the stroke, and the compression at one-tenth of the stroke.

If the piston displacement is represented by D, then the volume in the cylinder at cut-off will be 0.30 D, that at release will be 1.05 D, and that at compression will be 0.15 D. The absolute pressures during supply and exhaust may be assumed to be 106.7 and 14.7 pounds per square inch. The work for one stroke of the piston will be

$$W = 144 \times 106.7 \times 0.25D + \frac{144 \times 106.7 \times 0.30D}{\kappa - 1}\left\{1 - \left(\frac{0.30}{1.05}\right)^{1.4-1}\right\}$$

$$-144 \times 14.7 \times 0.9D - \frac{144 \times 14.7 \times 0.15D}{\kappa - 1}\left\{1 - \left(\frac{0.05}{0.15}\right)^{14-1}\right\}$$

$$= 144D\,(26.68 + 31.530 - 13.23 - 1.96) = 144 \times 43.02D.$$

The corresponding mean effective pressure is 43.02 pounds per square inch. If the engine is furnished with large ports and

automatic valve-gear the actual mean effective pressure may be 0.9 of that just calculated, or 38.7 pounds per square inch.

For a piston displacement D the engine will develop at 150 revolutions per minute

$$\frac{144 \times 38.7D \times 2 \times 150}{33000} \text{ horse-power;}$$

and conversely to develop 100 horse-power the piston displacement must be

$$D = \frac{100 \times 33000}{144 \times 38.7 \times 2 \times 150} = 1.974 \text{ cubic feet,}$$

and with a stroke of 2 feet the effective area of the piston will be

$$1.974 \times 144 \div 2 = 142.1 \text{ square inches.}$$

If the piston-rod is 2 inches in diameter it will have an area of 3.14 square inches, so that the mean area of the piston will be 143.7 square inches, corresponding to a diameter of $13\frac{1}{2}$ inches.

We find, consequently, that an engine developing 100 horse-power under the given conditions will have a diameter of $13\frac{1}{2}$ inches and a stroke of 2 feet, provided that it runs at 150 revolutions per minute.

In order to determine the amount of air used by the engine we must consider that the air caught at compression is compressed to the full admission-pressure of 106.7 pounds absolute. Part of this compression is done by the piston and part by the entering air, but for our present purpose it is immaterial how it is done. The volume filled by air at atmospheric pressure when the exhaust-valve closes (including clearance) is 0.15 of the piston displacement. When the pressure is increased to 106.7 pounds the volume will be reduced to

$$0.15 \left(\frac{14.7}{106.7}\right)^{\frac{1}{1.4}} = 0.017$$

of the piston displacement. The volume drawn in from the supply-pipe will consequently be

$$0.25 + 0.05 - 0.017 = 0.283$$

of the piston displacement. If the compression occurred sufficiently early to raise the pressure to that in the supply-pipe before the admission-valve opened, then only 0.25 of the piston displacement would be used per stroke and a saving of about 13 per cent would be attained; in such case the mean effective pressure would be smaller and the size of the cylinder would be larger.

The air-consumption for the engine appears to be

$2 \times 150 \times 0.283 \times$ pist. displ. $= 2 \times 150 \times 0.283 \times 1.974 = 167.6$ cubic feet per minute. The actual air-consumption will be somewhat less on account of loss of pressure in the valves and passages; it may be fair to assume 160 cubic feet per minute for the actual consumption.

In order to make one complete calculation for the use of compressed air for transmitting power, the data for the compressed-air engine have been made to correspond with the results of calculations for an air-compressor on page 377 and for the loss of pressure in a pipe on page 384. Since there is a loss of pressure in flowing through the pipe at constant temperature, there is a corresponding increase of volume, so that the pipe delivers

$$300 \times 114.7 \div 106.7 = 322.6$$

cubic feet per minute. Our calculation for the air-consumption of an engine to deliver 100 horse-power gives about 160 cubic feet, from which it appears that the system of compressor, conducting-pipe, and compressed-air engine should deliver

$$100 \times 322.6 \div 160 = 200 + \text{horse-power.}$$

If the friction of the compressed-air engine is assumed to be ten per cent, the power delivered by it to the main shaft (or to the machine driven directly from it) will be

$$200 \times .9 = 180 \text{ horse-power.}$$

The steam-power required to drive a simple compressor was found to be 520 horse-power; it consequently appears that

$$180 \div 520 = 0.34$$

of the indicated steam-power is actually obtained for doing work

from the entire system of transmitting power. If, however, a compound compressor is used, then the indicated steam-power is 444, and of this

$$180 \div 444 = 0.40$$

will be obtained for doing work.

If, however, we consider that the power would in any case be developed in a steam-engine, and that the transmission system should properly include only the compressor-cylinder, the pipe, and the compressed-air engine, then our basis of comparison will be the indicated power of the compressor-cylinder. For the simple compressor we found the horse-power to be 442, which gives for the efficiency of transmission

$$180 \div 442 = 0.41,$$

while the compound compressor demanded only 377 horse-power, giving an efficiency of

$$180 \div 377 = 0.48.$$

It appeared that the failure to obtain complete compression involved a loss of about 13 per cent in the air-consumption. It may then be assumed that with complete compression our engine could deliver 200 horse-power to the main shaft. In that case the efficiency of transmission when a compound compressor is used may be 0.53.

Efficiency of Compressed-air Transmission. — The preceding calculation exhibits the defect of compressed air as a means of transmitting power. It is possible that somewhat better results may be obtained by a better choice of pressures or proportions. Professor Unwin estimates that when used on a large scale from 0.44 to 0.51 of the indicated steam-power may be realized on the main shaft of the compressed-air engine. On the other hand, when compressed air is used in small motors, and especially in rock-drills and other mining-machinery, much less efficiency may be expected.

Experiments made by M. Graillot * of the Blanzy mines showed an efficiency of from 22 to 32 per cent. Experiments

* Pernolet, *L'Air Comprimé*, pp. 549, 550.

made by Mr. Daniel at Leeds gave an efficiency varying from 0.255 to 0.455, with pressures varying from 2.75 atmospheres to 1.33 atmospheres. An experiment made by Mr. Kraft * gave an efficiency of 0.137 for a small machine, using air at a pressure of five atmospheres without expansion.

Compressed air has been used for transmitting power either where power for compression is cheap and abundant, or where there are reasons why it is specially desirable, as in mining and tunnelling. It is now used to a considerable extent for driving hand-tools, such as drills, chipping-chisels, and calking-tools, in machine- and boiler-shops, and in shipyards. It is also used for operating cranes and other machines where power is used only at intervals, so that the condensation of steam (when used directly) is excessive, and where hydraulic power is liable to give trouble from freezing.

Compressed air has been used to a very considerable extent for transmitting power in Paris. The system appears to be expensive and to be used mainly on account of its convenience for delivering small powers or in places where the cold exhaust can be used for refrigeration. The trouble from freezing of moisture in the cylinder has been avoided by allowing the air to flow through a coil of pipe which is heated externally by a charcoal fire. Professor Unwin estimates that an efficiency of transmission of 0.75 may be attained under favorable conditions when the air is heated near the compressor, but he does not include the cost of fuel for reheating in this estimate.

Storage of Power by Compressed Air. — Reservoirs or cylinders charged with compressed air have been used to store power for driving street-cars. A system developed by Mekarski uses air at 350 to 450 pounds per square inch in reservoirs having a capacity of 75 cubic feet. The car also carries a tank of hot water at a temperature of about 350° F., through which the air passes on the way to the motor and by which it is heated and charged with steam. This use of hot water gives a secondary method of storing power, and also avoids trouble from freezing

* *Revue universelle des Mines*, 2 série, tome vi.

in the motor-cylinders. Air at much higher pressures has been used for driving street-cars in New York City, but the particulars have not been given to the public.

The calculation for storage of power may be made in much the same way as that for the transmission of power; the chief difference is due to the fact that the air is reduced in pressure by passing it through a reducing-valve on the way from the reservoir to the motor. By the theory of perfect gases such a reduction of pressure should not cause any change of temperature, but the experiments of Joule and Thomson (page 69) show that there will be an appreciable, though not an important, loss of temperature when there is a large reduction of pressure. Thus at 70° F. or 21°.1 C. the loss of temperature for each 100 inches of mercury will be

$$0°.92 \times \left(\frac{273}{294}\right)^2 = 0°.79 \text{ C.} = 1\tfrac{3}{4}° \text{ F.}$$

Now 100 inches of mercury are equivalent to about 49 pounds to the square inch, so that 100 pounds difference of pressure will give about $3\frac{1}{2}$° F. reduction of temperature, and 1000 pounds difference of pressure will give about 35° F. reduction of temperature. The last figures are far beyond the limits of the experiments, and the results are therefore crude. Again, the air in passing through the reducing-valve and the piping beyond will gain heat and consequently show a smaller reduction of temperature. The whole subject of loss of temperature due to throttling is uncertain, and need not be considered in practical calculations for air-compressors.

For an example of the calculation for storage of power let us find the work required to store air at 450 pounds per square inch in a reservoir containing 75 cubic feet. Replacing the specific volume v_1 in equation (213) by the actual volume, we have for the work of compression (not allowing for losses and imperfections)

$$W = 3 \times 464.7 \times 144 \times 75 \frac{1.4}{1.4 - 1} \left\{ \left(\frac{464.7}{14.7}\right)^{\frac{1.4-1}{3 \times 1.4}} - 1 \right\}$$
$$= 20520000 \text{ foot-pounds.}$$

If the pressure is reduced to 50 pounds by the gauge before it is used, the volume of air will be

$$75 \times 464.7 \div 64.7 = 539 \text{ cubic feet.}$$

The work for complete expansion of one pound to the pressure of the atmosphere will be

$$W = p_3 v_3 + \frac{p_3 v_3}{\kappa - 1}\left\{1 - \left(\frac{v_3}{v_4}\right)^{\kappa - 1}\right\} - p_4 v_4$$

$$= p_3 v_3 \frac{\kappa}{\kappa - 1}\left\{1 - \left(\frac{p_4}{p_3}\right)^{\frac{\kappa - 1}{\kappa}}\right\}$$

and the work for 539 cubic feet will be

$$144 \times 64.7 \times 539 \frac{1.4}{1.4 - 1}\left\{1 - \left(\frac{14.7}{64.7}\right)^{\frac{1.4-1}{1.4}}\right\} = 5976000$$

foot-pounds, without allowing for losses or imperfections. The maximum efficiency of storing and restoring energy by the use of compressed air in this case is therefore

$$5976000 \div 20520000 = 0.29.$$

In practice the efficiency cannot be more than 0.25, if indeed it is so high.

Sudden Compression. — It may not be out of place to call attention to a danger that may arise if air at high pressure is suddenly let into a pipe which has oil mingled with the air in it or even adhering to the side of the pipe. The air in the pipe will be compressed, and its temperature may become high enough to ignite the oil and cause an explosion. That this danger is not imaginary is shown by an explosion which occurred under such conditions in a pipe which was strong enough to withstand the air-pressure.

Liquid Air. — The most practical way of liquefying air on a large scale is that devised by Linde depending on the reduction of the temperature by throttling. On page 69, is given the empirical expression deduced by Joule and Kelvin for the reduction in temperature of air flowing through a porous plug with a difference of pressure measured by 100 inches of mercury,

$$0.92\left(\frac{273.7}{T}\right)^2,$$

in which $273°.7$ C. is taken to be the absolute temperature of freezing, and T is the absolute temperature of the air.

A modern three-stage air-compressor can readily give a pressure of 2000 pounds per square inch, and if the above expression is assumed to hold approximately for such a reduction in pressure, it indicates a cooling of

$$0.92 \times \frac{2000}{100 \times 0.491} = 37°.5 \text{ C.}$$

or about 67° F. By a cumulative effect to be described, the air may be cooled progressively till it reaches the boiling-point of its liquid and then liquefied. Linde's liquefying apparatus consists essentially of an air-compressor, a throttling-orifice, and a heat interchange apparatus.

The air-compressor should be a good three-stage machine giving a high pressure. The throttling-orifice may be a small hole in a metallic plate. The heat interchange apparatus may be made up of a double tube about 400 feet long, the inner tube having a diameter of 0.16 and the outer tube a diameter of 0.4 of an inch; these tubes for convenience are coiled and are then thoroughly insulated from heat. The air from the compressor is passed through the inner tube to the throttle-orifice and then from the reservoir below the orifice, through the space between the inner and outer tubes back to the compressor. The cumulative effect of this action brings the air to the critical temperature in a comparatively short period, and then liquid air begins to accumulate in the reservoir below the orifice, whence it may be drawn off.

The atmospheric air before it is supplied to the condenser should be freed from carbon dioxide and moisture, which would interfere with the action, and should be cooled by passing it through pipes cooled with water and by a freezing mixture. The portion of air liquefied must be made up by drawing air from the atmosphere, which is, of course, purified and cooled.

The principal use of liquid air is the commercial production of oxygen by fractional distillation; several plants have been installed for this purpose.

CHAPTER XVI.

REFRIGERATING-MACHINES.

A REFRIGERATING-MACHINE is a device for producing low temperatures or for cooling some substance or space. It may be used for making ice or for maintaining a low temperature in a cellar or storehouse.

Refrigeration on a small scale may be obtained by the solution of certain salts; a familiar illustration is the solution of common salt with ice, another is the solution of sal ammoniac in water. Certain refrigerating-machines depend on the rapid absorption of some volatile liquid, for example, of ammonia by water; if the machine is to work continuously there must be some arrangement for redistilling the liquid from the absorbent. The most recent and powerful refrigerating-machines are reversed heat-engines. They withdraw the working substance (air or ammonia) from the cold-room or cooling-coil, compress it, and deliver it to a cooler or condenser. Thus they take heat from a cold substance, do work and add heat, and finally reject the sum of the heat drawn in and the heat equivalent of the work done. These reversed heat-engines, however, are very far from being reversible engines, not only on account of imperfections and losses but because they work on a non-reversible cycle.

Two forms of refrigerating-machines are in common use, air refrigerating-machines and ammonia refrigerating-machines. Sometimes sulphur dioxide or some other volatile fluid is used instead of ammonia. Carbon dioxide has been used, but there are difficulties due to high pressure and the fact that the critical temperature is 88° F.

Air Refrigerating-Machine. — The general arrangement of an air refrigerating-machine is shown by Fig. 86. It consists

of a compression-cylinder *A*, an expansion-cylinder *B* of smaller size, and a cooler *C*. It is commonly used to keep the atmosphere in a cold-storage room at a low temperature, and has certain advantages for this purpose, especially on shipboard. The air from the storage-room comes to the compressor at or about freezing-point, is compressed to two or three atmospheres and delivered to the cooler, which has the same form as a surface-condenser, with cooling water entering at *e* and leaving at *f*. The diaphragm *mn* is intended to improve the circulation of the cooling water. From the cooler the air, usually somewhat warmer than the atmosphere, goes to the expansion-cylinder *B*,

FIG. 86.

in which it is expanded nearly to the pressure of the air and cooled to a low temperature, and then delivered to the storage-room. The inlet-valves *a*, *a* and the delivery-valves *b*, *b* of the compressor are moved by the air itself; the admission-valves *c*, *c* and the exhaust-valves *d*, *d* of the expansion-cylinder are like those of a steam-engine and must be moved by the machine. The difference between the work done on the air in the compressor and that done by the air in the expansion-cylinder, together with the friction work of the whole machine, must be supplied by a steam-engine or other motor.

It is customary to provide the compression-cylinder with a water-jacket to prevent overheating, and frequently a spray of water is thrown into the cylinder to reduce the heating and the work of compression. Sometimes the cooler *C*, Fig. 86,

is replaced by an apparatus resembling a steam-engine jet-condenser, in which the air is cooled by a spray of water. In any case it is essential that the moisture in the air, as well as the water injected, should be efficiently removed before the air is delivered to the expansion-cylinder; otherwise snow will form in that cylinder and interfere with the action of the machine. Various mechanical devices have been used to collect and remove water from the air, but air may be saturated with moisture after it has passed such a device. The Bell-Coleman Company use a jet-cooler with provision for collecting and withdrawing water, and then pass the air through pipes in the cold-room on the way to the expansion-cylinder. The cold-room is maintained at a temperature a little above freezing-point, so that the moisture in the air is condensed upon the sides of the pipes and drains back into the cooler.

When an air refrigerating-machine is used as described, the pressure in the cold-room is necessarily that of the atmosphere, and the size of the machine is large as compared with its performance. The performance may be increased by running the machine on a closed cycle with higher pressures; for example, the cold air may be delivered to a coil of pipe in a non-freezing salt solution, from which the air abstracts heat through the walls of the pipe and then passes to the compressor to be used over again. The machine may then be used to produce ice, or the brine may be used for cooling spaces or liquids. A machine has been used for producing ice on a small scale, without cooling water, on the reverse of this principle; that is, atmospheric air is first expanded and chilled and delivered to a coil of pipe in a salt solution, then the air is drawn from this coil, after absorbing heat from the brine, compressed to atmospheric pressure, and expelled.

Proportions of Air Refrigerating-Machines. — The performance of a refrigerating-machine may be stated in terms of the number of thermal units withdrawn in a unit of time, or in terms of the weight of ice produced. The latent heat of fusion of ice may be taken to be 80 calories or 144 B.T.U.

Let the pressure at which the air enters the compression-cylinder be p_1, that at which it leaves be p_2; let the pressure at cut-off in the expanding-cylinder be p_3 and that of the back-pressure in the same be p_4; let the temperatures corresponding to these pressures be t_1, t_2, t_3, and t_4, or, reckoned from the absolute zero, T_1, T_2, T_3, and T_4. With proper valve-gear and large, short pipes communicating with the cold-chamber p_4 may be assumed to be equal to p_1 and equal to the pressure in that chamber. Also t_1 may be assumed to be the temperature maintained in the cold-chamber, and t_3 may be taken to be the temperature of the air leaving the cooler. With a good cut-off mechanism and large passages p_3 may be assumed to be nearly the same as that of the air supplied to the expanding-cylinder. Owing to the resistance to the passage of the air through the cooler and the connecting pipes and passages, p_3 is considerably less than p_2.

It is essential for best action of the machine that the expansion and compression of the expanding-cylinder shall be complete. The compression may be made complete by setting the exhaust-valve so that the compression shall raise the pressure in the clearance-space to the admission-pressure p_3 at the instant when the admission-valve opens. The expansion can be made complete only by giving correct proportions to the expanding- and compression-cylinders.

The expansion in the expanding-cylinder may be assumed to be adiabatic, so that

$$\frac{T_4}{T_3} = \left(\frac{p_4}{p_3}\right)^{\frac{\kappa - 1}{\kappa}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (231)$$

Were the compression also adiabatic the temperature t_2 could be determined in a similar manner; but the air is usually cooled during compression, and contains more or less vapor, so that the temperature at the end of compression cannot be determined from the pressure alone, even though the equation of the compression curve be known.

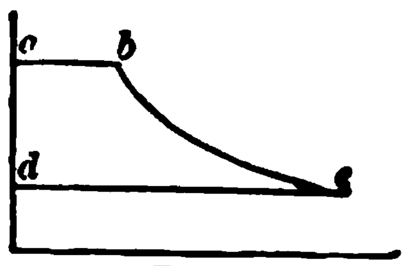

FIG. 87.

Let the air passing through the refrigerating-machine per minute be M; then the heat withdrawn from the cold-room is

$$Q_1 = Mc_p\,(t_1 - t_4). \quad . \quad . \quad . \quad . \quad . \quad (232)$$

The work of compressing M pounds of air from the pressure p_1 to the pressure p_2 in a compressor without clearance is (Fig. 87)

$$W_c = M\left\{p_2v_2 + \int_{v_2}^{v_1} p\,dv - p_1v_1\right\};$$

$$\therefore\quad W_c = M\left\{p_2v_2 + \frac{p_1v_1}{n-1}\left[\left(\frac{v_1}{v_2}\right)^{n-1} - 1\right] - p_1v_1\right\},$$

$$\therefore\quad W_c = M\left\{p_1v_1\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} + \frac{p_1v_1}{n-1}\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right] - p_1v_1\right\},$$

$$\therefore\quad W_c = Mp_1v_1\,\frac{n}{n-1}\left\{\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right\} \quad . \quad . \quad . \quad . \quad . \quad (233)$$

provided that the compression curve can be represented by an exponential equation. If the compression can be assumed to be adiabatic,

$$W_c = Mp_1v_1\,\frac{\kappa}{\kappa-1}\left\{\left(\frac{p_2}{p_1}\right)^{\frac{\kappa-1}{\kappa}} - 1\right\} = \frac{Mc_p}{A}(t_2 - t_1); \quad (234)$$

for in such case we have the equations

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\kappa-1}{\kappa}} \qquad AR = c_p - c_v = c_p\,\frac{\kappa-1}{\kappa}.$$

If the expansion is complete in the expanding-cylinder, as should always be the case, then the equation for the work done by the air will have the same form as equation (233) or (234), replacing t_1 and p by t_4 and p_4, and t_2 and p_2 by t_3 and p_3; so that

$$W_e = Mp_4v_4\,\frac{n}{n-1}\left\{\left(\frac{p_3}{p_4}\right)^{\frac{n-1}{n}} - 1\right\} \quad . \quad . \quad . \quad (235)$$

and for adiabatic expansion

$$W_e = \frac{Mc_p}{A}\,(t_3 - t_4) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (236)$$

The difference between the works of compression and expansion is the net work required for producing refrigeration; consequently

$$W = W_c - W_e = \frac{Mc_p}{A}\{t_2 - t_1 - t_3 + t_4\} \quad . \quad (237)$$

or, replacing M by its value from equation (232),

$$W = \frac{Q_1}{A}\frac{t_2 + t_4 - t_1 - t_3}{t_1 - t_4} \quad . \; . \; . \; . \; . \; . \; . \quad (238)$$

The net horse-power required to abstract Q_1 thermal units per minute is consequently

$$P_n = \frac{778Q_1}{33000}\frac{t_2 + t_4 - t_1 - t_3}{t_1 - t_4} \quad . \; . \; . \; . \; . \; . \quad (239)$$

where t_1 is the temperature of the air drawn into the compressor, and t_2 is the temperature of the air forced by the compressor into the cooler, and t_3 is the temperature of the air supplied to the expanding-cylinder, and t_4 is the temperature of the cold air leaving the expanding-cylinder. The gross horse-power developed in the steam-engine which drives the refrigerating-machine is likely to be half again as much as the net horse-power or even larger. The relation of the gross and the net horse-powers for any air refrigerating-machine may readily be obtained by indicating the steam- and air-cylinders and may serve as a basis for calculating other machines.

The heat carried away by the cooling water is

$$Q_2 = Q_1 + AW \quad . \; . \; . \; . \; . \; . \; . \; . \quad (240)$$

If compression and expansion are adiabatic, then

$$Q_2 = Mc_p(t_1 - t_4 + t_2 + t_4 - t_1 - t_3) = Mc_p(t_2 - t_3) \quad . \quad (241)$$

or, replacing M by its value from equation (232),

$$Q_2 = Q_1\frac{t_2 - t_3}{t_1 - t_4} \quad . \; . \; . \; . \; . \; . \; . \quad (242)$$

If the initial and final temperatures of the cooling water are

t_i and t_k, and if q_i and q_k are the corresponding heats of the liquid, then the weight of cooling water per minute is

$$G = \frac{Q_2}{q_k - q_i} = Q_1 \frac{t_2 - t_3}{(t_1 - t_4)\,(q_k - q_i)} \quad . \quad . \quad . \quad (243)$$

The compressor-cylinder must draw in M pounds of air per minute at the pressure p_1 and the temperature t_1, that is, with the specific volume v_1; consequently its apparent piston displacement without clearance will be at N revolutions per minute,

$$D_c = \frac{Mv_1}{2N} = \frac{MRT_1}{2Np_1} \quad . \quad . \quad . \quad . \quad . \quad (244)$$

for the characteristic equation gives

$$p_1 v_1 = RT_1.$$

Replacing M by its value from equation (232), we have

$$D_c = \frac{Q_1 RT_1}{2Nc_p p_1\,(t_1 - t_4)} \quad . \quad . \quad . \quad . \quad (245)$$

Since all the air delivered by the compressor must pass through the expanding-cylinder, its apparent piston displacement will be

$$D_e = D_c \frac{p_1 T_4}{p_4 T_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (246)$$

If p_1, the pressure of the air entering the compression-cylinder is equal to p_4, that of the air leaving the expanding-cylinder (as may be nearly true with large and direct pipes for carrying the air to and from the cold-room), equation (246), will reduce to

$$D_e = D_c \frac{T_4}{T_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (247)$$

Both the compressor- and the expanding-cylinder will have a clearance, that of the expanding-cylinder being the larger. As is shown on page 363, the piston displacement for an air-compressor with a clearance may be obtained by dividing the apparent piston displacement by the factor

$$1 - \frac{1}{m}\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}} + \frac{1}{m}.$$

If the expansion and compression of the expanding-cylinder are complete, the same factor may be applied to it. For a refrigerating-machine n may be replaced by κ for both cylinders. To allow for losses of pressure and for imperfect valve action the piston displacements for both compressor- and expanding-cylinders must be increased by an amount which must be determined by practice; five or ten per cent increase in volume will probably suffice. In practice the expansion in the expanding-cylinder is seldom complete. A little deficiency at this part of the diagram will not have a large effect on the capacity of the machine, and will prevent the formation of a loop in the indicator-diagram; but a large drop at the release of the expanding-cylinder will diminish both the capacity and the efficiency of the machine.

The temperature t_4 and the capacity of the machine may be controlled by varying the cut-off of the expanding-cylinder. If the cut-off is shortened the pressure p_2 will be increased, and consequently T_4 will be diminished. This will make D_e, the piston displacement of the expanding-cylinder, smaller. A machine should be designed with the proper proportions for its full capacity, and then, when running at reduced capacity, the expansion in the expanding-cylinder will not be quite complete.

Calculation for an Air-refrigerating Machine. — Required the dimensions and power for an air refrigerating-machine to produce an effect equal to the melting of 200 pounds of ice per hour. Let the pressure in the cold-chamber be 14.7 pounds per square inch and the temperature 32° F. Let the pressure of the air delivered by the compressor-cylinder be 39.4 pounds by the gauge or 54.1 pounds absolute, and let there be ten pounds loss of pressure due to the resistance of the cooler and pipes and passages between the compressor- and the expanding-cylinder. Let the initial and final temperatures of the cooling water be 60° F. and 80° F., and let the temperature of the air coming from the cooler be 90° F. Let the machine make 60 revolutions per minute.

With adiabatic expansion and compression the temperatures

of the air coming from the compressor- and discharged from the expanding-cylinder will be

$$T_2 = 492 \left(\frac{54.1}{14.7}\right)^{\frac{0.4}{1.4}} = 714; \qquad \therefore \; t_2 = 254° \text{ F.}$$

$$T_4 = (460 + 90) \left(\frac{14.7}{44.1}\right)^{\frac{0.4}{1.4}} = 402; \qquad \therefore \; t_4 = -58° \text{ F.}$$

The melting of 200 pounds of ice is equivalent to

$$200 \times 144 \div 60 = 480 \text{ B.T.U.}$$

per minute; consequently the net horse-power of the machine is by equation (239)

$$P_n = \frac{778\,Q_1}{33000} \frac{t_2 + t_4 - t_1 - t_3}{t_1 - t_4}$$

$$= \frac{778 \times 480}{33000} \times \frac{254 - 58 - 32 - 90}{32 + 58}$$

$$= \frac{778 \times 480 \times 74}{33000 \times 90} = 9.2 \text{ H. P.},$$

and the indicated power of the steam-engine may be assumed to be 14 horse-power.

By equation (245) the apparent piston displacement of the compressor without clearance will be

$$D_c = \frac{Q_1 R T_1}{2 N c_p p_1 (t_1 - t_4)}$$

$$= \frac{480 \times 53.35 \times 492}{2 \times 60 \times 0.2375 \times 144 \times 14.7\,(32 + 58)} = 2.33 \text{ cu. ft.}$$

By equation (247) the apparent piston displacement of the expanding-cylinder without clearance will be

$$D_e = D_c \frac{T_4}{T_1} = 2.33 \times \frac{402}{492} = 1.90 \text{ cubic feet.}$$

If the clearance of the compressor-cylinder is 0.02 of its piston displacement, then the factor for clearance by equation (191) is

$$1 - \frac{1}{m}\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}} + \frac{1}{m} = 1 - \frac{2}{100}\left(\frac{54.1}{14.7}\right)^{\frac{1}{1.4}} + \frac{2}{100} = 0.979,$$

so that the piston displacement becomes

$$2.33 \div 0.979 = 2.38 \text{ cubic feet.}$$

If, further, the clearance of the expander-cylinder is 0.05 of its piston displacement, the factor for clearance becomes

$$1 - \frac{5}{100}\left(\frac{44.1}{14.7}\right)^{\frac{1}{1.4}} + \frac{5}{100} = 0.963,$$

which makes the piston displacement

$$1.90 \div 0.963 = 1.97 \text{ cubic feet.}$$

If now we allow ten per cent for imperfections, we will get for the dimensions: stroke 2 feet, diameter of the compressor-cylinder $15\frac{1}{2}$ inches, and diameter of the expanding-cylinder 14 inches.

Compression Refrigerating-Machine. — The arrangement of a refrigerating-machine using a volatile liquid and its vapor is

FIG. 88.

shown by Fig. 88. The essential parts are the compressor *A*, the condenser *B*, the valve *D*, and the vaporizer *C*. The compressor draws in vapor at a low pressure and temperature, compresses it, and delivers it to the condenser, which consists of coils of pipe surrounded by cooling water that enters at *e* and leaves at *f*. The vapor is condensed, and the resulting liquid

gathers in a reservoir in the bottom, from whence it is led by a small pipe having a regulating-valve D to the vaporizer or refrigerator. The refrigerator is also made up of coils of pipe, in which the volatile liquid vaporizes. The coils may be used directly for cooling spaces, or they may be immersed in a tank of brine, which may be used for cooling spaces or for making ice. Fig. 88 shows the compressor with one single-acting vertical cylinder which has head-valves, foot-valves, and valves in the piston. Small machines usually have one double-acting compressor cylinder. Large machines have vertical compressors which may be single-acting or double acting.

The cycle which has been stated for the compression refrigerating-machine is incomplete, because the working fluid is allowed to flow through the expansion-cock into the expanding-coils without doing work. To make the cycle complete, there should be a small expanding-cylinder in which the liquid could do work on the way from the condenser to the vaporizing-coils; but the work gained in such a cylinder would be insignificant, and it would lead to complications and difficulties.

Proportions of Compression Refrigerating-Machines. — The liquid condensed in the coils of the condenser flows to the expansion-cock with the temperature t_1 and has in it the heat q_1. In passing through the expansion-cock there is a partial vaporization, but no heat is gained or lost. The vapor flowing from the expansion-coils at the temperature t_2 and the pressure p_2 is usually dry and saturated, or perhaps slightly superheated, as it approaches the compressor. Each pound consequently carries from the expanding-coils the total heat H_2. Consequently the heat withdrawn from the expanding-coil by a machine using M pounds of fluid per minute is

$$Q_1 = M\,(H_2 - q_1) \quad . \quad . \quad . \quad . \quad . \quad . \quad (246)$$

The compressor-cylinder is always cooled by a water-jacket, but it is not probable that such a jacket has much effect on the working substance, which enters the cylinder dry and is superheated by compression. We may consequently calculate the

temperature of the vapor delivered by the compressor by aid of equation (80), page 65, giving

$$T_s = T_2 \left(\frac{p_1}{p_2}\right)^{\frac{k-1}{k}} = T_2 \left(\frac{p_1}{p_2}\right)^a \quad . \quad . \quad . \quad (249)$$

This equation may be used because it is equivalent to the assumption with regard to entropy on page 121. The value of a is $\frac{1}{4}$ for ammonia and 0.22 for sulphur dioxide as given on pages 119 and 124.

As has already been pointed out, the vapor approaching the compressor may be treated as though it were dry and saturated, each pound having the total heat H_2. The vapor discharged by the compressor at the temperature t_s and the pressure p_1 will have the heat

$$c_p (t_s - t_1) + H_1.$$

The heat added to each pound of fluid by the compressor is consequently

$$c_p (t_s - t_1) + H_1 - H_2,$$

and an approximate calculation of the horse-power of the compressor may be made by the equation

$$P_c = \frac{778M \{c_p (t_s - t_1) + H_1 - H_2\}}{33000} \quad . \quad . \quad (250)$$

or, substituting for M from equation (249),

$$P_c = \frac{778Q_1 \{c_p (t_s - t_1) + H_1 - H_2\}}{33000 (H_2 - q_1)} \quad . \quad . \quad (251)$$

The power thus calculated must be multiplied by a factor to be found by experiment in order to find the actual power of the compressor. Allowance must be made for friction to find the indicated power of the steam-engine which drives the motor; for this purpose it will be sufficient to add ten or fifteen per cent of the power of the compressor.

The heat in the fluid discharged by compressor is equal to the sum of the heat brought from the vaporizing-coils and the heat-equivalent of the work of the compressor. The heat that

must be carried away by the cooling water per minute is consequently

$$Q_2 = M(H_2 - q_1) + M\{c_p(t_s - t_1) + H_1 - H_2\};$$

$$\therefore \quad Q_2 = M\{c_p(t_s - t_1) + r_1\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (252)$$

where r_1 is the heat of vaporization at the pressure p_1.

If the cooling water has the initial temperature t_w and the final temperature t'_w, and if q_w and q'_w are the corresponding heats of the liquid for water, then the weight of cooling water used per minute will be

$$G = \frac{M[c_p(t_s - t_1) + r_1]}{q_w - q'_w} \quad . \quad . \quad . \quad . \quad . \quad (253)$$

If the vapor at the beginning of compression can be assumed to be dry and saturated, then the volume of the piston displacement of a compressor without clearance, and making N strokes per minute, is

$$D = \frac{Mv_2}{N} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (254)$$

To allow for clearance, the volume thus found may be divided by the factor

$$1 - \frac{1}{m}\left(\frac{p_1}{p_2}\right)^{n} + \frac{1}{m},$$

as is explained on page 363. The volume thus found is further to be multiplied by a factor to allow for inaccuracies and imperfections.

The vapors used in compression-machines are liable to be mingled with air or moisture, and in such case the performance of the machine is impaired. To allow for such action the size and power of the machine must be increased in practice above those given by calculation. Proper precautions ought to be taken to prevent such action from becoming of importance.

Calculation for a Compression Refrigerating-Machine. — Let it be required to find the dimensions and power for an ammonia refrigerating-machine to produce 2000 pounds of ice per hour from water at 80° F. Let the temperature of the brine in the

freezing-tank be 15° F., and the temperature in the condenser be 85° F. Assume that the machine will have one double-acting compressor, and that it will make 80 revolutions per minute.

The heat of the liquid for water at 80° F. is 48 B.T.U., and the heat of liquefaction of ice is 144, so that the heat which must be withdrawn to cool and freeze one pound of water will be

$$48 + 144 = 192 \text{ B.T.U.}$$

If we allow 50 per cent loss for radiation, conduction, and melting the ice from the freezing-cans, the heat which the machine must withdraw for each pound of ice will be about 300 B.T.U.; consequently the capacity of the machine will be

$$Q_1 = 2000 \times 300 \div 60 = 10000 \text{ B.T.U. per minute.}$$

The pressures for ammonia corresponding to 15° and 85° F., are 42.43 and 165.47 pounds absolute per square inch, so that by equation (249)

$$T_s = T_2 \left(\frac{p_1}{p_2}\right)^a = (15 + 460) \left(\frac{165.47}{42.43}\right)^{\frac{1}{4}} = 668.$$

$$\therefore \quad t_s = 668 - 460 = 208° \text{ F.}$$

The horse-power of the compressor is

$$P_c = \frac{778 Q_1 \{c_p (t_s - t_1) + H_1 - H_2\}}{33000 (H_2 - q_1)}$$

$$= \frac{778 \times 10000 \{0.50836 (208 - 85) + 556 - 535\}}{33000 (535 - 58)} = 41.$$

If we allow 10 per cent for imperfections, the compressor will require 45 horse-power. If, further, 15 per cent is allowed for friction, the steam-engine must develop 53 horse-power.

From equation (248) the weight of ammonia used per minute is

$$M = Q_1 \div (H_2 - Q_1) = 10000 \div (535 - 58) = 21 \text{ pounds;}$$

and by equation (254) the piston displacement for the compressor will be

$$D = \frac{M v_2}{N} = \frac{21 \times 6.93}{2 \times 80} = 0.91 \text{ cubic feet.}$$

If 10 per cent is allowed for clearance and imperfect valve action, the piston displacement will be one cubic foot, and the diameter may be made 10½ inches and the stroke 20 inches.

Fluids Available. — The fluids that have been used in compression refrigerating-machines are ether, sulphur dioxide, ammonia, and a mixture of sulphur dioxide and carbon dioxide, known as Pictet's fluid. The pressures of the vapors of these fluids at several temperatures, and also the pressure of the vapors of methylic ether and carbon dioxide, are given in the following table:

PRESSURES OF VAPORS, MM. OF MERCURY.

Temperatures Degrees Centigrade.	Ether.	Sulphur Dioxide.	Methyl-Ether.	Ammonia.	Carbon Dioxide.	Pictet's Fluid.
− 30	...	287.5	576.5	866.1	...	585
− 20	68.9	479.5	882.0	1392.1	15142	745
− 10	114.7	762.5	1306.6	2144.6	20340	1018
0	184.4	1165.1	1879.0	3183.3	26907	1391
10	286.8	1719.6	2629.0	4574.0	34999	1938
20	432.8	2462.1	3586.0	6387.8	44717	2584
30	634.8	3431.8	4778.0	8701.0	56119	3382
40	907.0	4670.2	...	11595.3	69184	4347

Ether was used in the early compression-machines, but at the temperatures maintained in the refrigerator the pressure is small and the specific volume large, so that the machines, like air refrigerating-machines, were either feeble or bulky. Moreover, air was liable to leak into the machine and unduly heat the compressor-cylinder. Sulphur dioxide has been used successfully, but it has the disadvantage that sulphuric acid may be formed by the leakage of moisture into the machine, in which case rapid corrosion occurs. Ammonia has been extensively used in the more recent machines with good results. When distilled from an aqueous solution it is liable to contain considerable moisture. As is shown by the table, Pictet's fluid has a pressure at low temperature intermediate between the pressures of sulphur dioxide and ammonia, and the pressure increases slowly with the temperature. It has been used by the inventor

only, and does not appear in practice to have any advantage over ammonia.

Absorption Refrigerating Apparatus. — Fig. 89 gives an ideal diagram of a continuous absorption refrigerating apparatus. It consists of the following essential parts: (1) the generator *B*, containing a concentrated solution of ammonia in water, from which the ammonia is driven by heat; (2) the condenser *C*, consisting of a coil of pipe in a tank, through which cold water is circulated; (3) the valve *V*, for regulating the pressures in *C* and in *I*; (4) the refrigerator *I*, consisting of a coil of pipe in a tank containing a non-freezing salt solution; (5) the absorber *A*, containing a dilute solution of ammonia, in which the vapor of ammonia is absorbed; and (6) the pump *P* for transferring the solution from the bottom of *A* to the top of *B*; there is also a pipe connecting the bottom of *B* with the top of *A*. It is apparent that the condenser and refrigerator or vaporizer correspond to the parts *B* and *C* of Fig. 88, and that the absorber and generator take the place of the compressor. The pipes connecting *A* and *B* are arranged to take the most

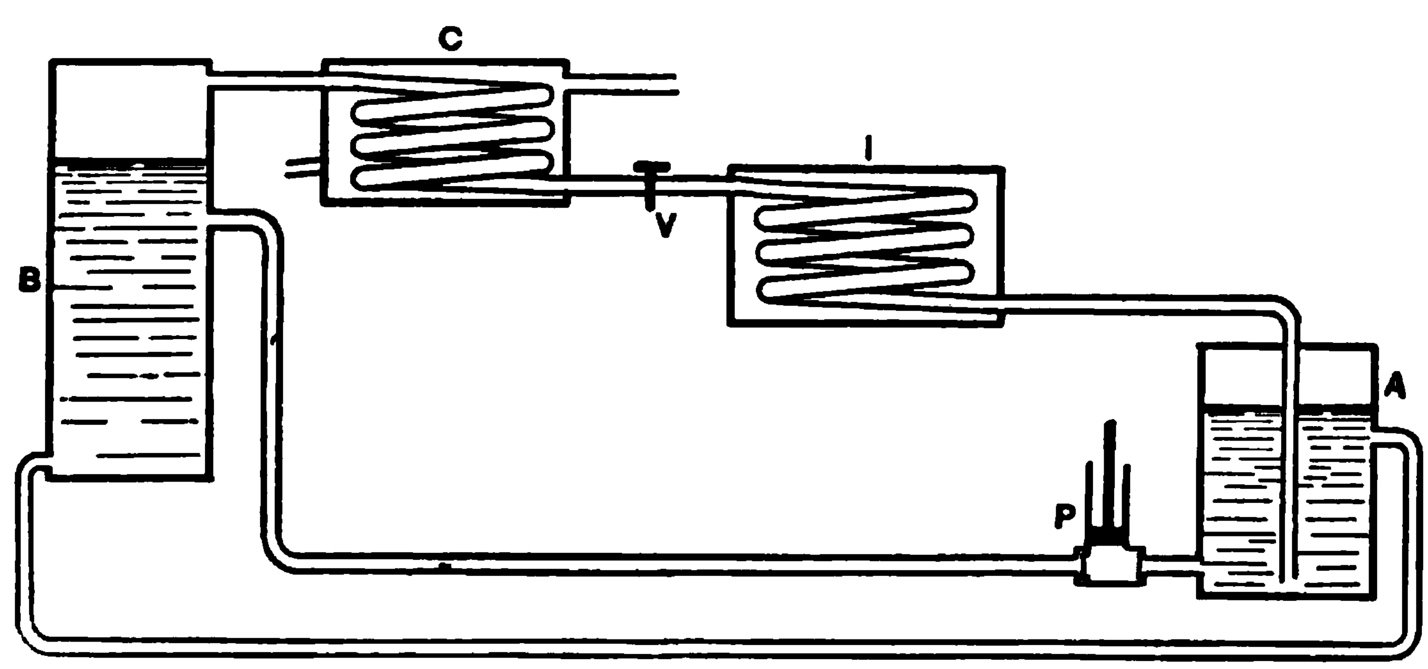

FIG. 89.

concentrated solution from *A* to *B*, and to return the solution from which the ammonia has been driven, from *B* to *A*. In practice the generator *B* is placed over a furnace, or is heated by a coil of steam-pipe, to drive off the ammonia. Also, arrangements are made for transferring heat from the hot liquid flowing from *B* to *A* to the cold liquid flowing from *A* to *B*. As

the ammonia is distilled from water in B the vapor driven off contains some moisture, which causes an unavoidable loss of efficiency.

Tests of an Air Refrigerating-Machine. — An air refrigerating-machine, constructed under the Bell-Coleman patent, was tested by Professor Schröter * at an abattoir in Hamburg, where it was used to maintain a low temperature in a storage-room. The machine is horizontal, and has the pistons for the expansion- and compression-cylinders on one piston-rod, the expansion-cylinder being nearer the crank. Power is furnished by a steam-engine acting on a crank at the other end of the main shaft and at right angles to the crank driving the air-pistons. Both the steam-cylinder and the expansion-cylinder have distribution slide-valves, with independent cut-off valves. The main dimensions are given in the following table:

DIMENSIONS BELL-COLEMAN MACHINE.

	Steam-Cylinder.		Compression-Cylinder.		Expansion-Cylinder.	
	Head End.	Crank End.	Head End.	Crank End.	Head End.	Crank End.
Diameter of piston, cm.	53	53	71	71	53	53
Diameter of piston-rod, cm.	8.1	6.9	9.0	6.8	9.0	9.0
Stroke, m.	0.605	0.605	0.605	0.605	0.605	0.605
Clearance, per cent of piston displacement.	5.9	5.8	1.4	1.4	8 9	8.9

Water is sprayed into the compression-cylinder, and the air is further cooled by passing through an apparatus resembling a steam-engine jet-condenser, after which it is dried by passing it through a system of pipes in the cold-room before it passes to the expansion-cylinder.

In the tests, indicators were attached to each end of the several cylinders, and the temperature of the air was taken at entrance to and exit from each of the air-cylinders. Specimens of the indicator-diagrams from the air-cylinders show for the compressor a slight reduction of pressure during admission and some irregularity during expulsion, and for the expansion-

* *Untersuchungen an Kältemachinen*, 1887.

cylinder a little wire-drawing at cut-off, and a good expansion and compression, though neither is complete. No attempt was made to measure the amount and temperatures of the cooling water.

The data and results of the tests and the calculations are given in Table XXXVI.

Table XXXVI.

TESTS ON BELL-COLEMAN MACHINE.

Number of test	I.	II.	III.
Duration in hours	6	1.63	2.92
Revolutions per minute	65.05	61.2	63.5
Temperatures of air, degrees Centigrade:			
At entrance to compression-cylinder	19.3	17.5	19.1
At exit from compression-cylinder	27.3	26.8	27.2
At entrance to expansion-cylinder	19.00	16.6	19.1
At exit from expansion-cylinder	−47.0	−47.0	−47.0
Mean effective pressure, kgs. per sq. cm.:			
Steam-cylinder: head end	2.263	2.336	2.343
crank end	2.239	2.294	2.301
Compression-cylinder: head end	1.900	1.861	1.870
crank end	1.869	1.825	1.906
Expansion-cylinder: head end	1.592	1.589	1.626
crank end	1.615	1.594	1.624
Indicated horse-power:			
Steam-cylinder	85.12	82.35	85.71
Compression-cylinder	128.85	118.55	126.01
Expansion-cylinder	60.10	56.12	59.46
Mean pressure during expulsion from compression-cylinder, kgs.	3.35	3.25	3.40
Mean pressure during admission to expansion-cylinder, kgs.	2.82	2.83	2.84
Difference	0.53	0.42	0.56
Calculation from compression diagram:			
Absolute pressure at end of stroke, kgs.	1.04	1.04	1.04
Absolute pressure at opening of admission-valve, kg.:			
Head end	0.783	0.788	0.764
Crank end	0.765	0.749	0.765
Volume at admission, per cent of piston displacement:			
Head end	6.15	5.95	6.03
Crank end	8.50	8.41	7.91
Weight of air discharged per stroke, kg.:			
Head end	0.2744	0.2764	0.2750
Crank end	0.2716	0.2742	0.2730
Weight of air discharged per revolution, kg.	0.546	0.551	0.548
Calculation from expansion diagram:			
Absolute pressure at release, kgs.:			
Head end	1.32	1.31	1.33
Crank end	1.45	1.44	1.46
Absolute pressure at compression, kgs.:			
Head end	1.14	1.14	1.17
Crank end	1.20	1.19	1.22
Volume at release, per cent of piston displacement:			
Head end	104.65	104.7	104.8
Crank end	106.1	106.3	106.4
Volume at compression, per cent of piston displacement:			
Head end	16.5	16.0	16.6
Crank end	19.8	19.6	20.6
Air used per stroke, kg.:			
Head end	0.234	0.233	0.238
Crank end	0.254	0.254	0.255
Air used per revolution	0.488	0.487	0.493
Difference of weights, calculated by compression and expansion diagrams, kg.	0.058	0.064	0.055
In per cent of the former	10.6	11.6	10.0
Mean weight of air per revolution, kg.	0.514	0.519	0.520
Elevation of temperature at constant pressure, degrees Centigrade.	66.3	64.5	66.1
Heat withdrawn per H. P. per hour, calories	371	354	363

TABLE XXXVII.

TESTS ON REFRIGERATING MACHINES.

By PROFESSOR SCHRÖTER.

Number.	System of the machine.	Dimensions of the steam cylinder. Diameter of piston, mm.	Diameter of piston-rod, mm.	Stroke, mm.	Dimensions of the compression cylinder. Diameter of piston, mm.	Diameter of piston-rod, mm.	Stroke, mm.	Duration of test.
1	Linde.	371.25	55.5	800	325	63	540	36 min.
2	"	"	"	"	"	"	"	34 "
3	"	400	....	602	250	55	420	106 "
4	"	"	....	"	"	"	"	50 "
5	"	"	....	"	"	"	"	46 "
6	"	"	....	"	"	"	"	35 "
7	"	330	52	740	"	"	"	3 hrs.
8	"	"	"	"	"	"	"	3 "
0	Pictet.	450	68	900	430	65	900	8.5 "
19	"	"	"	"	"	"	"	11.08 "
11	"	"	"	"	"	"	"	9.83
12	"	"	"	"	"	"	"	4.00 "

Number.	Revolutions per minute compressor.	Indicated horse-power of steam cylinder.	Indicated horse-power of compressor.	Absolute pressures of vapor, kilos. per sq. centimeter. In compressor during expulsion.	In condenser.	In compressor during admission.	In vaporizer.	Cooling water. Initial temperature.	Final temperature.
1	64.8	53.6	....	....	6.99	....	2.76	....	....
2	59.8	66.1	45.9	9.58	9.31	2.50	2.64	....	....
3	54.7	....	26.27	....	13.66	....	{4.85 4.55}	11.19	22.56
4	55.1	....	27.30	....	14.06	....	{4.90 4.53}	11.2	23.58
5	59.1	....	29.23	....	14.11	....	{4.91 4.55}	11.2	23.04
6	49.6	....	24.49	....	13.78	....	{4.27 4.83}	11.1	26.10
7	65.15	26.1	18.1	8.13	7.87	2.36	2.63	8.77	12.41
8	65.8	34.5	25.8	10.68	10.41	2.97	3.24	8.82	20.45
9	64.2	91.2	52.01	3.77	3.22	0.45	0.82	10.15	14.0
10	64.7	94.5	61.70	4.11	3.50	0.63	1.03	10.1	15.95
11	64.5	99.2	66.42	4.23	3.62	0.73	1.15	10.15	17.20
12	64.0	....	75 02	5.81	5.11	0.67	1.06	10.3	31.1

Number	Ice formed. Temperature of water supplied, degrees C.	Per compressor horse-power, per hour, gross, kilos.	Per compressor horse-power, per hour, net, kilos.	Temperature of water or brine cooled. At entrance.	At exit.	Heat withdrawn per comp. h. p. per hour calories.
1	9.0	....	....	−4.4	−4.4	4444
2	8.3	34.8	31.7	−5.9	−5.9	3120
3	....	....	....	11.19	2.95	3249
4	....	....	....	11.2	2.38	3367
5	....	....	....	11.2	2.24	3072
6	....	....	....	11.1	4.71	3263
7	....	....	....	−9.50	−9.97	3684
8	....	....	....	−3.1	−4.1	3086
9	11.3	16.8	15.2	−18.2	−18.2	1674
10	11.3	25.0	22.6	−10.0	−10.0	2385
11	11.3	28.2	25.9	−9.7	−9.7	2638
12	11.3	20.6	18.5	−6.05	−6.05	1958

Tests of Compression-Machines.—In Table XXXVII are given the data and results of tests on three refrigerating-machines on the Linde system using ammonia, and of a machine on Pictet's system using Pictet's fluid, all by Professor Schröter. The tests on machines used for making ice were necessarily of considerable length, but the tests on machines used for cooling liquids were of shorter duration.

The cooling water when measured was gauged on a weir or through an orifice. In the tests 3 to 6 on a machine used for cooling fresh water the heat withdrawn was determined by taking the temperatures of the water cooled, and by gauging the flow through an orifice, for which the coefficient of flow was determined by direct experiment. The heat withdrawn in the tests 7 and 8 was estimated by comparison with the tests 3 to 6. The net production of ice in the tests 1 and 2 was determined directly; and in the test 2 the loss from melting during the removal from the moulds was found by direct experiment to be 8.45 per cent. By comparison with this the loss by melting in the first test was estimated to be 7.7 per cent. The gross production of ice in the refrigerator was calculated from the net production by aid of these figures. In the tests 9 to 12 on the Pictet machine the gross production was determined from the weight of water supplied, and the net production from the weight of ice withdrawn.

A separate experiment on the machine used for cooling brine gave the following results for the distribution of power:

Total horse-power	57.1
Power expended on compressor	19.5
" " " centrifugal pump	9.8
" " " water-pump	3.6

The centrifugal pump was used for circulating the brine through a system of pipes used for cooling a cellar of a brewery. The water-pump supplied cooling water to the condenser and for other purposes.

A similar test on the Pictet machine gave:

Power of engine alone	7.9 H. P.
" " " and intermediate gear	16.6 "
" " " gear, and pump	20.0 "

In 1888 comparative tests were made by Professor Schröter, on a Linde and on a Pictet refrigerating-machine, in a special building provided by the Linde Company which had every convenience and facility for exact work. The following table gives the principal dimensions of the machines:

PRINCIPAL DIMENSIONS OF LINDE AND PICTET REFRIGERATING-MACHINES.

	Linde.	Pictet.
Diameter of steam-cylinder, cm.	30.55	31.63
compressor-cylinder, cm.	25.03	28.6
steam piston-rod, cm.	4.85	5
compressor-rod, cm.	5.5	5
Stroke of steam-piston, cm.	70	62
compressor, cm.	42	62
Diameter of pipe in vaporizers, external, mm.	40.5	44
internal, mm.	32	36
Length of pipe in first vaporizer, m.	556.5	538.2
second vaporizer, m.	558.5	538.2
Diameter of pipe in condenser, external, mm.	38.5	44
internal, mm.	30	36
Length of pipe in condenser, m.	556.2	483.1

The Linde machine used ammonia and was allowed to draw a mixture of liquid and vapor into the compressor, so that no water-jacket was required. The Pictet machine used Pictet's fluid, which is a mixture of sulphur dioxide and carbon dioxide and had the compressor cooled by a water-jacket.

The data and results of the tests are given in Table XXXVIII. Five tests were made on each machine. The temperature of the salt solution or brine, from which heat was withdrawn by the vaporizers, was allowed to vary about three degrees centigrade from entrance to exit. The entrance temperatures were intended

TABLE XXXVIII.

TESTS ON REFRIGERATING-MACHINES.

By Professor M. SCHRÖTER, *Vergleichende Versuche an Kältemaschinen.*

Pictet machine.	One vaporizer. I	II	III	IV	V
Steam-engine:					
Revolutions per minute	57.0	56.8	57.1	57.6	59.3
Indicated horse-power	21 81	20.88	18.75	15.93	27.56
Compressor:					
Horse-power	16.82	16.10	14.26	11.83	22.91
Mechanical efficiency	0.771	0.771	0.761	0.743	0.831
Pressure in condenser, kilograms per square centimetre	3.99	3.91	3 84	4.25	6.39
Pressure in vaporizer, kilograms per square centimetre	1.47	1.05	0.68	0.17	1.05
Vaporizer:					
Mean temperature of brine, entrance	6.10	—1.96	— 9.92	—17.93	—2.04
Mean temperature of brine, exit	3.08	—4.98	—12.91	—20.96	—5.01
Specific heat per litre	0.850	0.847	0.845	0.841	0.846
Initial temperature of brine, entrance	6.09	—2.02	— 9.91	—18.00	—1.99
Initial temperature of brine, exit	3.03	4.99	—12.91	—21.00	—5.02
Final temperature of brine, entrance	6.11	—2.04	— 9.94	—18.00	—2.05
Final temperature of brine, exit	3.05	—4.98	—12.88	—21.00	—4.96
Condenser:					
Mean temperature of cooling-water, entrance	9.65	9.60	9.61	9.68	9.68
Mean temperature of cooling-water from condenser	19 72	19.70	19.59	19.51	35.18
Mean temperature of cooling-water from jacket	15.5	15.6	16.8	16.7	18.6
Initial temperature of condensing-water, entrance	9.57	9.64	9.58	9.68	9.73
Initial temperature of condensing-water, exit	19.71	19.72	19.37	19.52	35.08
Final temperature of condensing-water, entrance	9.67	9.57	9.61	9.72	9.72
Final temperature of condensing-water, exit	19.71	19.64	19.35	19.59	35.01
Error in heat account, per cent	+0.6	+0.6	+0.4	−1.3	+8.9
Refrigerative effect, calories per horse-power per hour	3507	2556	1852	1075	1702
Linde machine.					
Steam-engine:					
Revolutions per minute	44.9	45.1	45.1	44.8	45.0
Horse-power	18.14	18.26	17.03	15.70	24.41
Compressor:					
Horse-power	15.53	15.20	14.31	12.63	21.86
Mechanical efficiency	0.856	0.833	0.840	0.805	0.895
Pressure in condenser, kilograms per square centimetre	9.52	9.24	9.00	8.89	14.03
Pressure in vaporizer, kilograms per square centimetre	3.89	2.95	2.13	1.56	2.95
Vaporizer:					
Mean temperature of brine, entrance	6.00	—2.02	— 9.99	—17.92	— 2.03
Mean temperature of brine, exit	2 89	—5.02	−12.91	—20.82	— 5.01
Specific heat per litre	0.850	0.846	0.843	0.840	0.845
Initial temperature of brine, entrance	5 98	—2.05	— 9.95	—17.97	— 2.03
Initial temperature of brine, exit	2.89	—5.02	—12.94	—20.83	— 5.00
Final temperature of brine, entrance	5.97	2.04	9.97	17.96	2.03
Final temperature of brine, exit	2.94	5.04	12.89	20.83	5.01
Condenser:					
Mean temperature of cooling-water, entrance	9.56	9.54	9.61	9.61	9.68
Mean temperature of cooling-water, exit	19.76	19.63	19.84	19.72	35.33
Initial temperature of water, entrance	9.56	9.55	9.61	9.64	9.68
Initial temperature of water, exit	19.74	19.42	19.82	19.79	35.45
Final temperature of water, entrance	9.57	9.54	9.60	9.56	9.65
Final temperature of water, exit	19.74	19.45	19.89	19.88	35.44
Error in heat account, per cent	—1.8	—1.8	— 1.9	— 2.1	+1
Refrigerative effect, calories per horse-power per hour	4308	3182	2336	1711	2022

to be 6° C., − 2° C., − 10° C., and − 18° C. The cooling water was supplied to the condenser at about 9°.5 C., for all tests, and for all but one it left the condenser with a temperature of nearly 20° C.; the fifth test on each machine was made with the exit temperature of the cooling water at about 35° C.

The pressure in the compressor depended, of course, on the temperatures of the brine and the cooling-water. For all the tests except the fifth on each machine, the maximum pressure of the working substance was nearly constant: about 9 kilograms per square centimetre for ammonia and about 4 kilograms for Pictet's fluid. The fifth test had considerably higher pressure, corresponding to the higher temperature in the condenser. The minimum pressure of the working substance of course diminished as the brine temperature fell.

The heat yielded per hour to the ammonia in the vaporizer was calculated by multiplying together the amount of brine used in an hour, the specific heat of the brine, and its increase of temperature. But the initial and final temperatures were not quite constant, and so a correction was applied. The heat abstracted from the ammonia in the condenser was calculated from the water used per hour and its increase of temperature. The calculation for Pictet's machine involves also the jacket-water and its increase of temperature. A correction is applied for the variations of initial and final temperatures of the cooling-water. If the heat equivalent of the work of the compressor is added to the heat yielded by the vaporizer the sum should be equal to the heat abstracted by the cooling-water. The per cent of difference between these two calculations of the heat abstracted by the cooling-water is a measure of the accuracy of the tests.

The refrigerative effect is obtained by dividing the heat yielded by the vaporizer by the horse-power of the steam-cylinder. The first four tests with constant temperature in the condenser show a regular decrease in the refrigerative effect for each machine as the temperature of the brine and the minimum pressure of the working substance is reduced. The fifth test, with a

higher temperature in the condenser, shows a less refrigerative effect than the second test, which has nearly the same brine temperatures. These results are in concordance with the idea that a refrigerating-machine is a reversed heat-engine; for a heat-engine will have a higher efficiency and will use less heat per horse-power when the range of temperatures is increased, and *per contra*, a refrigerating-machine will be able to transfer less heat per horse-power as the range of temperatures is increased.

TABLE XXXIX.

TESTS ON AMMONIA REFRIGERATING-MACHINE.

By Professor J. E. DENTON, *Trans. Am. Soc. Mech. Engr.*, vol. xii, p. 326.

	I	II	III	IV
Pressure above atmosphere, pounds per square inch:				
Ammonia from compressor	151	152	147	161
Ammonia back-pressure	28	8.2	13	27.5
Barometer, inches of mercury	30.07	29.59	29.87	30.01
Temperature, degrees Fahrenheit:				
Brine, inlet	36.76	6.27	14.29	......
outlet	28.86	2.03	2.29	28.45
Condensing-water inlet	44.65	56.65	46.9	54.00
outlet	83.66	85.4	85.46	82.86
Jacket-water, inlet	44.65	56.7	46.9	54.3
Ammonia-vapor, leaving brine-tank	34.2	14.7	3.0	29.2
entering compressor	39	25	10.13	34
leaving compressor	213	263	239	221
calculated	229	304	260	237
entering condenser	200	218	209	168
Brine, pounds per minute	2281	2173	942.8	2374
Specific gravity	1.163	1.174	1.174	1.174
Specific heat	0.82	0.78	0.78	0.78
Ammonia, lbs. per min. by metre	....	14.68	16.67	28.32
from compressor displacement	....	....	22.10	34.51
Heat account, B.T.U. per minute:				
Given to ammonia by brine.	14776	71876	8824	14647
compressor	27860	2320	2518	3020
atmosphere	140	147	167	141
Total received by ammonia	17702	9653	11409	17708
Taken from ammonia by condenser	17242	9056	9910	17359
jackets	608	712	656	406
atmosphere	182	338	250	252
Total taken from ammonia	18032	10106	10816	18017
Error, per cent	2	5	3.5	2
Power, etc.:				
Revolutions per minute	58.00	57.7	57.88	58.89
Horse-power steam-cylinder	85.0	71.7	73.6	88.6
compressor	65.7	54.7	59.4	71.2
Mechanical efficiency	0.81	0.83	0.86	0.83
Refrigerative effect:				
Tons of ice (melted) in 24 hours	74.8	36.43	44.64	74.56
B.T.U. abstracted from brine per horse-power minute	174	197	197	196
Pounds of ice (melted) per pound of coal	24.1	14.1	17.27	23.37

Table XXXIX gives the data and results of tests made by Professor Denton on an ammonia refrigerating-machine. The

only items requiring explanation are the refrigerative effect and the calculated temperature of the vapor leaving the condenser; the latter was calculated by the equation

$$T_s = T_2\left(\frac{p_1}{p_2}\right)^{0.24}$$

and shows both the cooling effect of the jacket and the error in assuming an adiabatic compression. The exponent used here is a trifle smaller than that of equation (249) page 407. The refrigerative effect was obtained by dividing the B.T.U. given to the ammonia in a minute by the horse-power of the steam-cylinder. The tons per horse-power in 24 hours was obtained by multiplying the refrigerative effect in thermal units per minute by the number of minutes in a day and then dividing the product by 2000 (the pounds in a short ton) and by 144 (the heat of melting a pound of ice). The pounds of ice per pound of coal was based on an assumed consumption of three pounds of coal per horse-power per hour, and was calculated by multiplying the B.T.U. per horse-power per minute by 60 and dividing by 3×144.

The main dimensions of the machine were:

Diameter of ammonia cylinder (single-acting)	12 inches
Stroke of ammonia cylinder	30 "
Diameter of steam-cylinder	18 "
Stroke of steam-cylinder	36 "
Diameter of pipe for vaporizer and condenser	1 "
Length of pipe in vaporizer	8000 feet
" condenser	5000 "

Test of an Absorption-machine. — The principal data and the results of a test made by Professor J. E. Denton * on an absorption ammonia refrigerating-machine are given in Table XL. The machine is applied to chill a room of about 400,000 cubic feet capacity at a pork-packing establishment at New Haven, Conn. In connection with this test the specific heat of the brine, which served as a carrier of heat from the cold room to the ammonia, was determined by direct experiment. The

* *Trans. Am. Soc. Mech. Eng.*, vol. x, May, 1889.

Table XL.

TEST OF AN ABSORPTION-MACHINE.

Seven Days' Continuous Test, Sept. 11–18, 1888.

Average pressures above atmosphere in lbs. per sq. in.	Generator	150.77
	Steam	47.70
	Cooler	23.69
	Absorber	23.4
Average temperatures in Fahrenheit degrees.	Atmosphere in vicinity of machine	80
	Generator	272
	Brine, Inlet	21.2
	Brine, Outlet	16.16
	Condenser, Inlet	54½
	Condenser, Outlet	80
	Absorber, Inlet	80
	Absorber, Outlet	111
	Heater, Upper outlet to generator	212
	Heater, Lower " " absorber	178
	Heater, Inlet from absorber	132
	Inlet from generator	272
	Water returned to main boilers from steam coil	260
Average range of temperatures Fahr. degrees.	Condenser	25½
	Absorber	31
	Brine	5.13
Brine circulated per hour.	Cubic feet	1,633.7
	Pounds	119,260
Specific heat of brine		0.800
Cooling capacity of machine in tons of ice per day of 24 hours		40.67
Steam consumption per hour, to volatilize ammonia, and to operate ammonia pump pounds		1,986
British thermal units:	Eliminated, Per pound of brine	4.1
	Eliminated, Total per hour	481,260
	Of refrigerating effect per pound of steam consumption	243
	Rejected, At condenser, per hour	918,000
	Rejected, At absorber "	1,116,000
	Per pound of steam, On entering generator coil	1,203
	Per pound of steam, On leaving generator coil	271
	Consumed by generator per lb. of steam condensed	932
Condensing water per hour, in pounds		36,000
Equivalent ice production per pound of coal, if one pound of coal evaporates ten pounds of steam at boiler		17.1
Calories, refrigerating effect per kilogram of steam consumed		135
Approximate coil surface in sq. ft.	Condensing coil	870
	Absorber "	350
	Steam "	200

brine chilled and the cooling water used were measured with meters, which were afterwards tested under the conditions of the experiment.

It is interesting to compare the refrigerative effects expressed in pounds of ice per pound of coal. On this basis the compression-machine tested by Professor Denton has an advantage of

$$\frac{24.1 - 17.1}{24.1} \times 100 = 19 \text{ per cent.}$$

But this comparison is really unfair to the compression-machine, for its steam-engine is assumed to require a consumption of three pounds of coal per horse-power per hour, while the calculation for the absorption-machine is based on the assumption that a pound of coal can evaporate ten pounds of water; but an automatic condensing-engine of the given power should be able to run on 20 or 22 pounds of steam per horse-power per hour.

CHAPTER XVII.

FLOW OF FLUIDS.

THUS far the working substance has been assumed to be at rest or else its velocity has been considered to be so small that its kinetic energy has been neglected; now we are to consider thermodynamic operations involving high velocities, so that the kinetic energy becomes one of the important elements of the problem. These operations are clearly irreversible and consequently the first law of thermodynamics only is available, and if any element of computation involves reference to equations that were deduced by aid of the second law, care must be taken that such computations are allowable. It is true that all practical thermal operations are irreversible for one reason or another; for example, the cycle for a steam engine is irreversible, both because steam is supplied and exhausted from the cylinder and because the cylinder is made of conducting material. But all adiabatic operations in cylinders (which serve as the basis of theoretical discussions) are properly treated as reversible and all the deductions from the second law may be applied to that part of the cycle. In particular the limitations of the discussion of entropy on page 32 have been observed.

Three cases of continuous thermal operations have been discussed (1) flow through a porous plug, (2) the throttling calorimeter, (3) friction of air in pipes; to which it may be well to return now. In all, the velocity of the fluid has been so small that its kinetic energy was neglected; in none of them was any reference made to equations deduced by the aid of the second law of thermodynamics. Rather curiously, all the operations were adiabatic, using the word to mean that no heat was taken from or lost to external objects; in the case of transmission of air in pipes, this comes from the natural conditions of the case

and in the other two operations there was careful insulation from heat. None of the operations are isoentropic; for instance, the entropy of steam supplied to the calorimeter on page 192 is about 1.60 and the entropy of the superheated steam in the calorimeter is about 1.72; but this does not enter into the solution of the problem and is more curious than useful.

The flow of fluids through orifices and nozzles has become even of more importance than formerly on account of the development of steam turbines. Thus far all computations have been based on adiabatic action, and when attempt is made to allow for friction it is done by the application of an experimental factor to results from adiabatic computations.

The following is the customary method of establishing the fundamental equation. Suppose that a fluid is flowing from the larger pipe A into the pipe B; there will clearly be an increase in velocity, with a reduction in pressure.

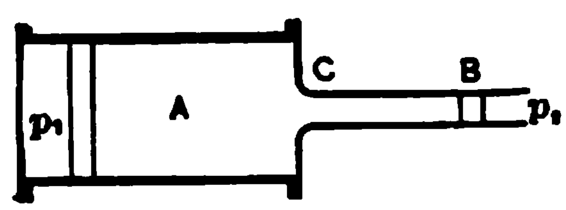

FIG. 90

The first law of thermodynamics as expressed by equation (16), page 14, needs the addition of a term to take account of the change in kinetic energy, and may be written

$$dQ = A\,(dE + dW + dK);$$

the last term in the parenthesis represents the increase of kinetic energy.

Let it be supposed that there is a frictionless piston in each cylinder; the piston in A exerts the pressure p_1 on the fluid in front of it, and the piston in B has on it the fluid pressure p_2. Each unit of weight of fluid passing from A through the orifice has the work p_1v_1 done on it, while each pound entering the cylinder B does the work p_2v_2. The assumption of pistons is merely a matter of convenience, and if they are suppressed the same conditions with regard to external work will hold.

If the velocity in A is V_1 the kinetic energy of one unit of weight in that cylinder is $\frac{V_1^2}{2g}$; the kinetic energy in B is $\frac{V_2^2}{2g}$ for a velocity V_2.

The intrinsic energies in A and B are E_1 and E_2. If there is no heat communicated to or from the fluid the sum of the intrinsic energy, external work, and kinetic energy must remain constant, so that

$$E_1 + p_1v_1 + \frac{V_1^2}{2g} = E_2 + p_2v_2 + \frac{V_2^2}{2g}; \quad . \quad . \quad (255)$$

this is the fundamental equation for the flow of a fluid.

If the walls of the pipes are well insulated there will not be much radiation or other external loss even if the pipes have considerable length, and in cases that arise in practice that loss may properly be neglected. There is likely to be a considerable frictional action even if the pipes are short, and the logical method appears to call for the introduction of frictional terms at this place. Such is not the custom, and a substitute will be discussed later.

Usually the velocity in the large cylinder A is small and the term depending on it may be neglected. Solving for the term depending on the velocity in B and dropping the subscript, we have

$$\frac{V^2}{2g} = E_1 - E_2 + p_1v_1 - p_2v_2 \quad . \quad . \quad . \quad . \quad (256)$$

Incompressible Fluids. — There is little if any change of volume or of intrinsic energy in a liquid in passing through an orifice under pressure, so that the equation of flow becomes in this case

$$\frac{V^2}{2g} = (p_1 - p_2)v_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad (257)$$

If the difference of pressure is due to a difference of level or head, h, we have

$$p_1 - p_2 = hd,$$

where d is the density, or weight of a unit of volume, and is the reciprocal of the specific volume; consequently equation (257) reduces to

$$\frac{V^2}{2g} = h,$$

which is the usual equation for the flow of a liquid through a small orifice.

Flow of Gases. — The intrinsic energy of a unit of weight of a gas is

$$E = \frac{pv}{\kappa - 1},$$

which depends only on the condition of the gas and not on any changes that have taken or may take place. The equation for the flow of a gas therefore becomes

$$\frac{V^2}{2g} = \frac{p_1 v_1}{\kappa - 1} - \frac{p_2 v_2}{\kappa - 1} + p_1 v_1 - p_2 v_2;$$

$$\therefore \quad \frac{V^2}{2g} = \frac{\kappa}{\kappa - 1}(p_1 v_1 - p_2 v_2) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (258)$$

At this place it is customary to use the equation

$$p_2 v_2^{\kappa} = p_1 v_1^{\kappa} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (259)$$

for the reduction of the equation (258) just as though we were dealing with an adiabatic expansion in a non-conducting closed cylinder. Now the fact that the isoenergic line and the isothermal line are practically identical (page 63) shows that a perfect gas has no disgregation energy and consequently for an adiabatic change all the change in intrinsic energy is available for doing outside work, which in this case is applied to increasing the kinetic energy of the gas, instead of being applied to the piston of a compressed air motor. If this analogy is allowed equation (259) may be used, and will yield

$$p_2 v_2 = p_1 v_1 \left(\frac{v_1}{v_2}\right)^{\kappa - 1} = p_1 v_1 \left(\frac{p_2}{p_1}\right)^{\frac{\kappa - 1}{\kappa}} \quad . \quad . \quad . \quad (260)$$

so that equation (258) may be reduced to

$$\frac{V^2}{2g} = p_1 v_1 \frac{\kappa}{\kappa - 1}\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{\kappa - 1}{\kappa}}\right] \quad . \quad . \quad . \quad . \quad (261)$$

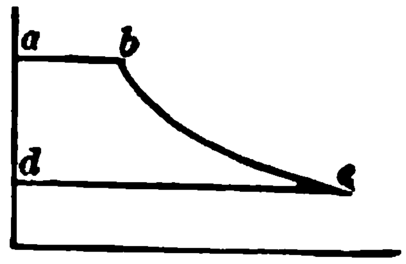

FIG. 91.

This equation may also be deduced for the work of air in the cylinder of a compressed air motor (Fig. 91). The work of admission is p_1v_1; the work of expansion is by equation (81), page 65.

$$\frac{p_1v_1}{\kappa - 1}\left\{1 - \left(\frac{v_1}{v_2}\right)^{\kappa - 1}\right\} = \frac{p_1v_1}{\kappa - 1}\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{\kappa - 1}{\kappa}}\right];$$

and the work of exhaust is

$$-p_2v_2 = -p_1v_1\left(\frac{p_2}{p_1}\right)^{\frac{\kappa - 1}{\kappa}};$$

so that the effective work is

$$p_1v_1 + \frac{p_1v_1}{\kappa - 1}\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{\kappa - 1}{\kappa}}\right] - p_1v_1\left(\frac{p_2}{p_1}\right)^{\frac{\kappa - 1}{\kappa}}$$

which is readily reduced to equation (261).

For the calculation of velocities it is convenient to replace the coefficient p_1v_1 in equation (261) by RT_1, since pressures and temperatures are readily determined and are usually given, thus

$$\frac{V^2}{2g} = RT_1\frac{\kappa}{\kappa - 1}\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{\kappa - 1}{\kappa}}\right] \quad . \quad . \quad (262)$$

If the area of the orifice is a, then the volume discharged per second is

$$aV,$$

and the weight discharged per second is

$$w = \frac{aV}{v_2},$$

when v_2 is the specific volume at the lower pressure and is equal to

$$v_2 = v_1\left(\frac{p_1}{p_2}\right)^{\frac{1}{\kappa}} = \frac{RT_1}{p_1}\left(\frac{p_1}{p_2}\right)^{\frac{1}{\kappa}} \quad . \quad . \quad . \quad . \quad . \quad (263)$$

Substituting V from equation (262) and v_2 from (263) and reducing

$$w = a \frac{p_1}{\sqrt{T_1}} \sqrt{\frac{2g}{R}} \left\{ \frac{\kappa}{\kappa - 1} \left[\left(\frac{p_2}{p_1}\right)^{\frac{2}{\kappa}} - \left(\frac{p_2}{p_1}\right)^{\frac{\kappa+1}{\kappa}} \right] \right\}^{\frac{1}{2}} . \quad (264)$$

The equations deduced for the flow of air apply to the flow from a large cylinder or reservoir into a small straight tube through a rounded orifice. The lower pressure is the pressure in the small tube and differs materially from the pressure of the space into which the tube may deliver. In order that the flow shall not be much affected by friction against the sides of the tube it should be short — not more than once or twice its diameter. The flow does not appear to be affected by making the tube very short, and the degree of rounding is not important; the equations for the flow of both air and steam may be applied with a fair degree of approximation to orifices in thin plates and to irregular orifices.

Professor Fliegner * made a large number of experiments on the flow of air from a reservoir into the atmosphere, with pressures in the reservoir varying from 808 mm. of mercury to 3366 mm. He used two different orifices, one 4.085 and the other 7.314 mm. in diameter, both well rounded at the entrance.

He found that the pressure in the orifice, taken by means of a small side orifice, was 0.5767 of the absolute pressure in the reservoir so long as that pressure was more than twice the atmospheric pressure; under such conditions the pressure in the orifice is independent of the pressure of the atmosphere.

If the ratio $\frac{p_2}{p_1}$ is replaced by the number 0.5767 and if κ is replaced by its value 1.405 in equation (264) we shall have for the equation foı the flow of a gas

$$w = 0.4822a \sqrt{\frac{2g}{R}} \frac{p_1}{\sqrt{T_1}} \quad . \quad . \quad . \quad . \quad . \quad (265)$$

* *Der Civilingenieur*, vol. xx, p. 14, 1874.

For the flow into the atmosphere from a reservoir having a pressure less than twice the atmospheric pressure Fliegner found the empirical equation

$$w = 0.9644a\sqrt{\frac{2g}{R}} \cdot \sqrt{\frac{p_a(p_1 - p_a)}{T_1}}, \quad . \quad . \quad . \quad (266)$$

where p_a is the pressure of the atmosphere.

These equations were found to be justified by a comparison with experiments on the flow of air, made by Fliegner himself, by Zeuner, and by Weisbach.

Although these equations were deduced from experiments made on the flow of air into the atmosphere, it is probable that they may be used for the flow of air from one reservoir into another reservoir having a pressure differing from the pressure of the atmosphere.

Fliegner's Equations for Flow of Air. — Introducing the values for g and R in the equations deduced by Fliegner, we have the following equations for the French and English systems of units:

French units.

$$p_1 > 2p_a, \quad w = 0.395a\frac{p_1}{\sqrt{T_1}};$$

$$p_1 < 2p_a, \quad w = 0.790a\sqrt{\frac{p_a(p_1 - p_a)}{T_1}}.$$

English units.

$$p_1 > 2p_a, \quad w = 0.530a\frac{p_1}{\sqrt{T_1}};$$

$$p_1 < 2p_a, \quad w = 1.060a\sqrt{\frac{p_a(p_1 - p_a)}{T_1}}.$$

p_1 = pressure in reservoir;

p_a = pressure of atmosphere;

T_1 = absolute temperature of air in reservoir (degrees centigrade, French units; degrees Fahrenheit, English units).

In the English system p_1 and p_a are pounds per square inch and a is the area of the orifice in square inches, while w is the flow of air through the orifice in pounds per second. If desired, the area may be given in square feet and the pressures in pounds on the square foot, as is the common convention in thermodynamics.

In the French system w is the flow in kilograms per second. The pressures may be given in kilograms per square metre and the area a in square metres; or the area may be given in square centimetres, and the pressures in kilograms on the same unit of area. If the pressures are in millimetres of mercury, multiply by 13.5959; if in atmospheres, multiply by 10333.

Theoretical Maxima. — From a discussion of the mean velocity of molecules of a gas Fliegner deduces for the maximum velocity through an orifice

$$V \max = \sqrt{gRT_1} = 16.9 \sqrt{T_1}$$

in metric units. His ratio of pressure 0.5767 inserted in equation (262) gives

$$V \max = 17.1 \sqrt{T_1}.$$

The algebraic maximum of equation (264) occurs for the ratio $p_2 \div p_1 = 0.5274$, but this figure probably has no physical significance.

Flow of Saturated Vapor. — For a mixture of a liquid and its vapor equation (110), page 95, gives

$$E = \frac{1}{A}(q + x\rho),$$

so that equation (256) gives for the adiabatic flow from a receptacle in which the initial velocity is zero

$$\frac{V^2}{2g} = \frac{1}{A}(q_1 - q_2 + x_1\rho_1 - x_2\rho_2) + p_1v_1 - p_2v_2. \quad (267)$$

Substituting for v_1 and v_2 from

$$v = xu + \sigma,$$

$$A\frac{V^2}{2g} = q_1 - q_2 + x_1\rho_1 - x_2\rho_2 + Ap_1x_1u_1 - Ap_2x_2u_2 + A\sigma(p_1 - p_2).$$

But

$$\rho + Apu = r;$$

$$\therefore\ A\frac{V^2}{2g} = x_1r_1 - x_2r_2 + q_1 - q_2 + A\sigma\,(p_1 - p_2).$$

The last term of the right-hand member is small, and frequently can be omitted, in which case the right-hand member is the same as the expression for the work done per pound of steam in a non-conducting engine, equation (143), page 136, except that as in that place the steam is assumed to be initially dry, x_1 is then unity. The intrinsic energy depends only on the condition of the steam, and consequently reference to the second law of thermodynamics first comes into this discussion with the proposal to compute the quality x_2 in the orifice by aid of the standard equation for entropy

$$\frac{x_1r_1}{T_1} + \theta_1 = \frac{x_2r_2}{T_2} + \theta_2;$$

the acceptance of this method infers that the flow of steam through a nozzle differs from its action in the cylinder of an engine in that the work done is applied to increasing the kinetic energy of the steam instead of driving the piston.

Values of the right-hand member of equation (268) may be found in the temperature-entropy table which was computed for solving problems of this nature.

The weight of fluid that will pass through an orifice having an area of a square metres or square feet may be calculated by the formula

$$w = \frac{aV}{x_2u_2 + \sigma} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (268)$$

The equations deduced are applicable to all possible mixtures of liquid and vapor, including dry saturated steam and hot water. In the first place steam will be condensed in the tube, and in the second water will be evaporated.

If steam blows out of an orifice into the air, or into a large receptacle, and comes to rest, the energy of motion will be turned into heat and will superheat the steam. Steam blowing into the air will be wet near the orifice, superheated at a little distance, and if the air is cool will show as a cloud of mist further from the orifice.

Rankine's Equations. — After an investigation of the experiments made by Mr. R. D. Napier on the flow of steam, Rankine concluded that the pressure in the orifice is never less than the pressure which gives the maximum weight of discharge, and that the discharge in pounds per second may be calculated by the following empirical equations:

$$p_1 = \text{or} > \frac{5}{3} p_a, \quad w = a\frac{p_1}{70};$$

$$p_1 < \frac{5}{3} p_a, \qquad w = 0.029\, a\, [p_a\, (p_1 - p_a)]^{\frac{1}{2}};$$

in which p_1 is the pressure in the reservoir, p_a is the pressure of the atmosphere, both in pounds on the square inch, and a is the area in square inches.

The error of these equations is liable to be about two per cent; but the flow through a given orifice may be known more closely if tests are made on it at or near the pressure during the flow, and a special constant is found for that orifice.

Grashoff's Formula. — For pressures exceeding five-thirds of the external or back pressure Grashoff gives the following formula for the discharge of steam through a converging orifice,

$$w = 15.26\, ap^{0.97}$$

the weight being in grams per second, the area in square centimetres and the pressure in kilograms per square centimetre. For English units the equation becomes

$$w = 0.0165\, ap^{0.97}$$

the discharge being in pounds per second, the area in square inches and the pressure in pounds absolute per square inch. Rateau shows that this formula is well verified by his experiments

on the flow of steam, and that when the pressure is less than that required by the formula the flow can be represented by a curve which has for coördinates the ratio of the back pressure to the internal pressure and the ratio of the actual discharge to that computed by the equation on the preceding page.

The following values were taken from his curves:

Ratio of back pressure to internal pressure.	Ratio of actual to computed discharge.	
	Converging orifice.	Orifice in thin plates.
0.95	0.45	0.30
0.90	0.62	0.42
0.85	0.73	0.51
0.80	0.82	0.58
0.75	0.89	0.64
0.70	0.94	0.69
0.65	0.97	0.73
0.60	0.99	0.77
0.55	...	0.80
0.45	...	0.82
0.40	...	0.83

He further gives a curve for the discharge from a sharp-edged orifice from which the third column was taken.

Flow of Superheated Steam. — Though there is no convenient expression for the intrinsic energy of superheated steam, and though the general equation (256) cannot be used directly, an equation for velocity can be obtained by the addition of a term to equation (268) to allow for the heat required to superheat one pound of steam, making it read

$$A\,\frac{V^2}{2g} = \int_{t_1}^{t} c\,dt + r_1 + q_1 - x_2 r_2 - q_2. \quad . \quad . \quad (269)$$

The accompanying equation for finding the quality of steam x_2 is

$$\int_{T_1}^{T} \frac{c\,dt}{T} + \frac{r_1}{T_1} + \theta_1 = \frac{x_2 r_2}{T_2} + \theta_2 \quad . \quad . \quad . \quad (270)$$

Here t and T are the thermometric and the absolute temperatures of the superheated steam, t_1 is the temperature of saturated steam at the initial pressure, and t_2 the temperature at the final

pressure, and the letters r_1 and r_2 and θ_1 and θ_2 represent the corresponding heats of vaporization and entropies of the liquid.

Both equations apply only if the steam becomes moist at the lower pressure, which is the usual case. They may obviously be modified to apply to steam that remains superheated, but such a form does not appear to have practical application.

The method of reduction of the integrals in equation (269) and (270) is given on page 114; attention is called to the fact that the temperature-entropy table affords ready solution of equation (269), also of the velocity flow during which the steam remains superheated.

Flow in Tubes and Nozzles. — The velocity of air or steam flowing through a tube or nozzle with a large difference in pressure is very high, reaching 3000 feet a second in some cases; and consequently the effect of friction is marked even in short tubes and nozzles. A test by Büchner * on a straight tube 3.52 inches long and 0.158 of an inch internal diameter, under an absolute pressure of 177 pounds to the square inch delivered only about 0.9 of the amount of steam calculated by the adiabatic method, and the pressure in the tube fell gradually from 131 pounds near the entrance to 14.5 pounds near the exit when delivering to a condenser at about atmospheric pressure. If there were any use for such a device in engineering the problem would appear to call for a method of dealing with friction resembling that on page 380 for flow of air in long pipes, but probably more difficulty would be found in getting a satisfactory treatment.

From the investigations that have been made on the flow of steam through nozzles it appears that they should have a well-rounded entrance, the radius of the curve of the section at entrance being half to three-fourths of the diameter of the smallest section or throat; from the throat the nozzles should expand gradually to the exit, avoiding any rapid change of velocity, as such a change is likely to roughen the surface where it occurs. The longitudinal section may well be a straight line joined to the entrance section by a curve of long radius. The taper of

* *Meitteilungen über Forschungrarbeiten Heft*, 18, p. 43.

the cone may be one in ten or twelve; this will give for the total angle at the apex of the cone 5° to 6°; if the entrance to the nozzle is not well rounded there will be a notable acceleration of the steam approaching the nozzle and this acceleration outside of the nozzle appears to diminish the amount of steam that the nozzle can deliver. The expansion should preferably be sufficient to reduce the steam to the pressure into which the nozzle delivers; otherwise the acceleration of the steam will continue beyond the nozzle, but the steam tends more and more to mingle with the adjacent fluid through which it moves, and a poorer effect is likely to be obtained.

If the expansion in the nozzle is not enough to reduce the pressure of the steam to (or nearly to) the external pressure into which the nozzle delivers, sound waves will be produced and there will be irregular action, loss of energy, and a distressing noise. On the other hand if the expansion in the nozzle reduces the pressure of the steam below the external pressure at the exit, sound waves will be set up in the nozzle with added resistance. This latter condition is likely to be worse than the former, and if the pressures between which the nozzle acts cannot be controlled it should be so designed as to expand the steam to a pressure a little higher than that against which it is expected to deliver, allowing a little acceleration to occur beyond the nozzle.

Friction Head. — In dealing with a resistance to the flow of water through a pipe, such as is caused by a bend or a valve, it is customary to assume that the resistance is proportional to the square of the velocity and to modify equation (258), page 425 to read

$$h = \frac{V^2}{2g} + C\frac{V^2}{2g},$$

where C is a factor to be obtained experimentally. The term containing this factor is sometimes called the head due to the resistance or required to overcome the resistance, and the equation may be changed to

$$h = \frac{V^2}{2g} + h';$$

it being understood that of the available head h, a certain portion h' is used up in overcoming resistances and the remainder is used in producing the velocity V. This aspect is well expressed by shifting h' to the other side of the equation and writing

$$\frac{V^2}{2g} = h - h' = h\left(1 - \frac{h'}{h}\right) = h\ (1 - y).$$

This method has been used by writers on steam turbines to allow for frictional and other resistance and losses. It must be admitted that it is a rough and unsatisfactory method, but perhaps it will serve. The value of y probably varies between 0.05 and 0.15 for flow through a single nozzle or set of guide blades or moving buckets in a steam turbine.

There is one difference between the behavior of water and an elastic fluid like air or steam that must be clearly understood, and kept in mind. Frictional resistance and other resistances to the flow of water, transform energy into heat and that heat is lost, or if it is kept by the water is not available afterwards for producing velocity; on the other hand the energy which is expended in overcoming frictional or other resistances of like nature by steam or air, is changed into heat and remains in the fluid, and may be available for succeeding operations.

Experiments on Flow of Steam. — There are five ways of experimenting on the flow of steam through orifices and nozzles that have been applied to test the theory of flow. Some of them, used separately or in combination, can be made to give values of the friction factor y.

(1) Steam flowing through an orifice or a nozzle may be condensed and weighed.

(2) The pressure at one or several points in a nozzle may be measured by side orifices or by a searching-tube; the latter may be used to investigate the pressure in the region of the approach to the entrance, or in the region beyond the exit, and may also be used with an orifice.

(3) The reaction of steam escaping from a nozzle oı an orifice may be measured.

(4) The jet of steam may be allowed to impinge on a plate or curved surface and the impulse may be measured.

(5) A Pitot tube may be introduced into the jet and the pressure in the tube can be measured.

Of course two or more of the methods may be used at the same time with the greater advantage. It will be noted that none of the methods alone or in combination can be made to determine the velocity of the steam, and that all determinations of velocity equally depend on inference from calculations based on the experiments.

Formerly the weight of steam discharged was considered of the greatest importance, as in the design of safety-valves, or in the determination of the amount of steam used by auxiliary machines during an engine-test. The first method of experimenting was obviously the most ready method of determining this matter, and was first applied by Napier in 1869, and on his results were based Rankine's equations.

Since the development of steam turbines much importance is given to determination of steam velocities, though it is probable that the determination of areas is still the more important method, as on it depends the distribution of work and pressure, while a considerable deviation from the best velocity will have an unimportant influence on turbine efficiency. The first experiments on reaction were by Mr. George Wilson in 1872, but as his tests did not include the determination of the weight discharged they are less valuable.

Büchner's Experiments. — A number of experimenters have determined the weight of steam discharged by nozzles and tubes and at the same time measured the pressure in side-orifices at one or more places. The most complete appear to be those of Dr. Karl Büchner * on the flow through tubes and nozzles. Omitting the tests on tubes and on nozzles with a very small

* *Mitteilungen über Forschungsarbeiten Heft* 18, p. 47.

taper, the nozzles for which results will be quoted have the following designations and dimensions:

NOZZLES TESTED BY DR. BÜCHNER, ALL DIMENSIONS IN INCHES.

Designation.	Total length.	Cylindrical part.	Conical part.	Diameter at throat.	Taper one in.	Distance first side orifice from entrance.	Distance last side orifice from exit.
2a	1.97	0.36	1.61	0.158	20	0.33	0.17
2b	1.97	0.36	1.61	0.158	13	0.33	0.17
3a	0.945	0.37	1.365	0.159	7.2	0.34	0.14
3b	0.945	0.37	1.365	0.159	4.9	0.34	0.14
5c	1.37	0.37	1.00	0.200	20.3	0.24	0.11
5d	1.37	0.37	1.00	0.200	14.2	0.24	0.11

All the nozzles had a cylindrical portion for which the length is given in the above table *including* the rounding at entrance. Excluding the rounding, this cylindrical portion was two or three times the diameter at the throat and appears to have had considerable influence on the distribution of the pressure. There were from one to three additional side orifices evenly distributed; from pressure in these orifices Büchner makes interesting computations concerning the behavior of the fluid in the tube, but the results are not different from those that are brought out by the investigations of Stodola and are not included in this discussion. The data and results from such of the tests as appear to bear on our present purpose of investigating the discharge and friction of nozzles are given on page 439.

Steam for these tests was taken from a boiler through a separator which probably delivered steam with a fraction of a per cent of priming. The pressures were all measured on one gauge by aid of an eight-way-cock. The steam from the nozzles was condensed and weighed; the experimenter estimates the error due to uncertainty of draining the condenser at two per cent, which appears to be the maximum error to be attributed to any of the

results. The discharge was also computed by Grashoff's equation on page 432, and the ratio to the actual discharge is that set down in the table; the variation from unity is not greater than the probable maximum error. The method of the computation of velocities at throat and exit by the experimenter is not very clear, but it was made to depend on the equation (268), using the proper pressure and the discharge computed by Grashoff's equation.

TESTS ON FLOW OF STEAM.

DR. KARL BÜCHNER.

Number and designation.	Pressure pounds absolute.				Ratio of throat to initial.	Discharge pounds per second.	Ratio of actual to computed discharge.	Velocity at throat	Velocity at exit.	Ratio of actual to computed velocity.
	Initial	Throat.	Exit.	External						
1-2a	182	104.4	25.3	13.6	0.573	0.0503	0.964 to 0.980	1800	3030	0.928
2-2a	160.5	94.4	21.7	13.6	0.577	0.0449		1790	3020	0.930
3-2a	147.3	83.0	20.7	13.8	0.564	0.0411		1820	2990	0.926
4-2a	131.3	75.1	18.5	...	0.572	0.0370		1790	2990	0.929
5-2a	117.1	67.6	16.8	13.8	0.577	0.0331		1780	2960	0.925
33-2b	180.2	92.1	16.5	14.1	0.511	0.0494	...	1940	3260	0.920
36-3a	149.9	76.8	21.2	13.6	0.529	0.0394	0.964 to 0.980	1860	3060	0.957
37-3a	131.5	70.4	19.5	13.8	0.535	0.0363		1850	3020	0.950
38-3a	115.7	62.0	17.4	13.8	0.536	0.0219		1850	3020	0.944
39-3b	183.6	99.6	18.5	18.5	0.541	0.0501	...	1830	3430	0.987
41-5b	103.0	68.6	38.1	15.4	0.660	0.0483	0.988 to 1.022	1550	2190	0.932
42-5b	89.3	58.7	32.8	14.9	0.658	0.0419		1550	2180	0.932
43-5b	75.2	49.3	27.9	14.7	0.656	0.0343		1560	2150	0.923
44-5b	61.0	37.6	22.3	14.5	0.643	0.0282		1560	2160	0.929
45-5b	45.4	28.0	16.9	14.5	0.618	0.0211		1630	2130	0.923
47-5c	102.5	65.4	25.9	15.0	0.637	0.0549	1.017 to 1.021	1630	2520	0.927
48-5c	88.8	55.7	22.2	14.8	0.635	0.0410		1630	2530	0.931
49-5c	74.2	46.9	18.5	14.6	0.633	0.0344		1620	2530	0.935
50-5c	59.2	37.1	14.9	14.4	0.625	0.0277		1630	2490	0.932

The nozzles $3a$ and $3b$ had tapers of 1:7.2 and 1:4.9 which were probably too great, so that they may not have been filled with

steam; this might account for the small ratio of the throat to the initial pressure; the nozzle *2b*, which had a taper of 1:13, also shows a small ratio of throat to initial pressure.

The most interesting feature of the tests is the ratio of the velocity at exit, computed by the method referred to above, from the pressure at the side orifice near the exit from the nozzle. This does not appear to depend on the throat pressure. Leaving out tests on the nozzles *3a* and *3b* the mean value of this ratio is about 0.93 which corresponds to a value $y = 0.14$.

Rateau's Experiments. — These tests * have already been referred to in connection with Grashoff's formula. They differ from most tests on the discharge from orifices and nozzles in that the steam was condensed by a stream of cold water which formed a jet condenser; the amount of steam was computed from the rise of temperature and the amount of cold water used, which latter was determined by flowing it through an orifice. He estimates his error at something less than one per cent. The number of tests is too large to quote here; it may be enough to say that his diagrams show a very great regularity in his results, so that whatever error there may be is to be attributed to the method, which does avoid, as he claims, the uncertainty of draining a condenser.

Kneass' Experiments. — In order to determine the pressure in steam-nozzles such as are used in injectors, Mr. Strickland L. Kneass † made investigations with a searching-tube, having a small side orifice, both when the nozzles were performing their usual function in an injector and when discharging freely into the atmosphere. He also used side orifices bored through the nozzles for the same purpose. The most interesting feature of his investigation is that it makes practically no difference whether the discharge is free or into the combining tube of an injector.

* *Experimental Researches on Flow of Steam*, trans. H. B. Brydon.

† *Practice and Theory of the Injector*. J. Wiley & Sons, 1894.

For a well-rounded nozzle such as is used for an injector having a taper of one to six, he found the following results:

Absolute Pressure.		Ratio.	Calculated Velocity at Throat.
Initial.	Throat.		
135	82.0	0.606	1407
105	61.5	0.585	1448
75	42	0.559	1491
45	24.5	0.546	1504

Stodola's Experiments. — In his work on *Steam Turbines*, Professor Stodola gives the results of tests made by himself on the flow of steam through a nozzle, having the following proportions: diameter at throat 0.494, diameter at exit 1.45, and length from throat to exit 6.07, all in inches. The nozzle had the form of a straight cone with a small rounding at the entrance; the taper was 1:6.37. Four side orifices and also a searching-tube were used to measure the pressure at intervals along the nozzle; the searching-tube was a brass tube 0.2 of an inch external diameter closed at the end and with a small side orifice. This orifice was properly bored at right angles; two other tubes with orifices inclined, one 45° against the stream and one 45° down stream, gave results that were too large and two small by about equal amounts.

Stodola made calculations with three assumptions (1) with no frictional action, (2) with ten per cent for the value of y, and (3) with twenty per cent; comparing curves obtained in this way for the distribution of pressures with those formed by experiments, he concludes that the value of y for this nozzle was fifteen per cent.

Rosenhain's Experiments. — The most recent and notable experiments on flow of steam with measurement of reactions were made at Cambridge by Mr. Walter Rosenhain.* Steam was brought from a boiler through a vertical piece of cycle-tubing to a chamber which carried the orifices and nozzles at its side; the reaction was counteracted by a wire that was attached to the chamber passed over an antifriction pulley to a scale pan, to which the proper weight could be added. Afterwards he determined the discharge by collecting and weighing steam

* *Proc. Inst. Civ. Eng.*, vol. cxl, p. 199.

under similar conditions. The steam pressure was controlled by a throttle-valve. It is probable that there was some moisture in the steam at high pressures and that at low pressures the steam was slightly superheated. The following table gives the dimensions of the nozzles:

ROSENHAIN'S EXPERIMENTS DIMENSIONS.

Designation.	Least Diameter.	Greatest Diameter.	Taper.
I	0.1873	...	...
II	0.1840	0.287	1 : 20
IIA	0.1866	...	...
IIB	0.1849	0.287	1 : 20
III	0.1882	0.368	1 : 12
IIIA	0.1882	0.255	1 : 12
IIIB	0.1882	0.241	1 : 12
IV	0.1830	0.255	1 : 30
IVA	0.1830	0.242	1 : 30
IVB	0.1830	0.230	1 : 30
IVC	0.1830	0.217	1 : 30
IVD	0.1830	0.205	1 : 30

I was an orifice with sharp edge; IIA had a sharp edge at entrances; the several orifices numbered III and IV had slightly rounded entrances.

DATA AND RESULTS.

Nozzle.	Ratio of diameter.	Proper initial pressure.	Velocities.			Coefficient of friction.
			Adiabatic	Expt.	Ratio.	
II	1.56	150	2900	2740	0.946	0.105
III	1.96	275	3280	...	...	...
IIIA	1.36	97½	2600	2530	0.972	0.045
IIIB	1.28	80	2460	2220	0.903	0.185
IV	1.39	105	2630	2400	0.913	0.166
IVA	1.32	90	2520	2340	0.929	0.137
IVB	1.26	77½	2440	2200	0.901	0.188
IVC	1.19	62½	2220	2030	0.914	0.165
IVD	1 12	50	2100	1920	0.914	0.165

A calculation has been made by the adiabatic method to determine the pressures for which the several nozzles tested would expand the steam down to the pressure of the atmosphere;

a direct calculation cannot be made, but a curve can readily be determined from which the pressure can be interpolated. The velocities corresponding to these pressures have been taken from Rosenhain's curves and the velocities were calculated also by the adiabatic method. Since the diagrams in the Proceedings are to a small scale the deduction of pressures from them cannot be very satisfactory, but the results are probably not far wrong. The table on page 442 gives the coefficient of friction obtained by this method.

Lewicki's Experiments. — These experiments were made by allowing the jet of steam to impinge on a plate at right angles to the stream, and measuring the force required to hold the plate in place; from this impulse the velocity may be determined. It was found necessary to determine by trial the distance at which the greatest effort was produced. One of his nozzles had for the least diameter 0.237 and for the greatest diameter 0.305 of an inch or a ratio of 1.28, which is proper for a pressure of 80 pounds per square inch absolute. His experiments gave the following results as presented by Büchner:

Steam pressure	77	99	108
Ratio of computed and expt. velocities	0.96	0.96	0.955
Coefficient of friction	0.08	0.08	0.09

These experiments like those for reaction are liable to be vitiated by expansion and acceleration of the steam beyond the orifice.

Pressure in the Throat. — Some of the tests by Büchner show rather a low pressure in the throat of the nozzle, but in general tests on the flow of steam show a pressure in the throat about equal to 0.58 of the initial pressure provided that the back pressure has less than ratio 3/5 to the initial pressure; this corresponds with Fliegner's results and should be expected from his comparison with molecular velocity on page 430. The following table gives results of tests made by Mr. W. H. Kunhardt * in the laboratories of the Massachusetts Institute of Technology:

The excess of the throat pressure above 0.58 of the initial

* *Transactions Am. Soc. Mech. Engs.*, vol. xi, p. 187.

pressure for the tests numbered 1 to 9 is to be attributed to the excessive length of the tube. Longer tubes tested by Büchner, showed the same effect in an exaggerated degree.

FLOW OF STEAM THROUGH SHORT TUBES WITH ROUNDED ENTRANCES.

Diameters 0.25 of an inch.

			Pressure above atmosphere, pounds per square inch.				Ratio of absolute pressures.				Flow in pounds per hour.			
	Length of tube, inches.	Duration, minutes.	Above the tube.	Below the tube.	At small orifice in side of tube.	Barometer, pounds per square inch.	Pressure above tube to pressure below.	Pressure at side orifice to pressure above tube.	Temperature of steam below tube. Fahrenheit.	Per cent of moisture in steam above tube.	By experiment weighed.	Calculated by Thermodynamic equation (268).	Calculated by Rankine's equation.	Coefficient of flow, equation (268).
1	1.5	30	74.1	14.8	41.2	14.7	0.332	0.630	126.2	1.2	221.0	217.0	224	1.018
2	"	30	71.0	13.2	39.6	14.8	0.326	0.634	138.7	1.5	213.0	207.8	215	1.025
3	"	20	72.6	19.7	40.6	14.7	0.394	0.634	141.4	0.5	216.0	211.4	220	1.022
4	"	20	75.9	20.4	42.6	14.7	0.387	0.632	139.8	0.7	228.0	219.3	227	1.040
5	"	20	71.9	24.5	40.6	14.7	0.454	0.638	140.6	0.7	213.0	209.7	218	1.016
6	0.5	30	72.8	14.8	39.0	14.8	0.338	0.614	138.7	0.3	225.0	213.6	221	1.053
7	"	20	72.1	20.4	38.8	14.8	0.405	0.617	142.2	0.5	223.5	211.7	219	1.056
8	"	30	72.6	24.7	39.0	14.8	0.452	0.616	144.0	0.5	223.0	213.1	220	1.046
9	"	30	73.1	29.9	39.2	14.8	0.509	0.615	145.2	0.5	225.5	213.0	222	1.054
10	0.25	30	72.6	24.8	36.1	14.9	0.454	0.583	143.8	0.4	225.0	213.5	220	1.054
11	"	30	72.6	19.9	36.1	14.9	0.398	0.583	141.6	0.4	225.0	213.5	220	1.054
12	"	30	72.7	14.9	36.2	14.8	0 339	0.583	140.5	0.4	227.0	213.0	220	1.066
13	"	30	126.3	27.8	69.0	14.7	0.295	0.594	155.0	0.5	358.8	338.9	355	1.058
14	"	30	125.0	40.8	67.9	14.7	0.398	0.598	157.0	0.2	355.0	334.8	352	1.060

Design of a Nozzle. — Required the dimensions of a nozzle to deliver 500 pounds of steam per hour with a steam pressure of 150 pounds by the gauge and a vacuum of 26 inches of mercury. The vacuum of 26 inches can be taken as substantially equivalent to 2 pounds absolute and the steam pressure may be taken as 165 pounds absolute. The throat pressure is then nearly 96 pounds absolute. Assuming the steam to be initially dry, the calculation can be arranged as follows:

$$x_2 r_2 = T_2\left(\frac{r_1}{T_1} + \theta_1 - \theta_2\right) = 784.4(1.0378 + 0.5237 - 0.4709) = 855.5$$

$$x_3 r_3 = T_3\left(\frac{r_1}{T_1} + \theta_1 - \theta_3\right) = 585.7\,(1.0378 + 0.5237 - 0.1753) = 811.8$$

$$r_1 + q_1 - x_2 r_2 - q_2 = 856.8 + 337.9 - 855.5 - 295.4 = 43.8$$

$$r_1 + q_1 - x_3 r_3 - q_3 = 856.8 + 337.9 - 811.8 - 94.2 = 288.6.$$

The quantities just obtained are the amounts of heat that would be available for producing velocity if the action were adiabatic. In order to find the probable velocity allowing for friction, they should be multiplied by $1 - y$, where y the coefficient for friction may be taken as 0.15 for the determination of the exit velocity V_3. As for the throat velocity, there are two considerations, the frictional effect is small because the throat is near the entrance, and all experiments indicate that orifices and nozzles which are not unduly long deliver the full amount of steam that the adiabatic theory indicates; therefore we may make the calculation for that part of the nozzle by the adiabatic method. The available heats for producing velocity may therefore be taken as

$$43.8 \text{ and } (1 - 0.15)\, 288.6 = 245,$$

and the velocities are therefore (see page 436)

$$V_2 = \sqrt{64.4 \times 778 \times 43.8} = 1480.$$

$$V_3 = \sqrt{64.4 \times 778 \times 245} = 3500.$$

The quality of steam in the throat is

$$x_2 = x_2 r_2 \div r_2 = 855.5 \div 889.9 = 0.961.$$

To find the quality of steam at the exit we may consider that if x_3' is the actual quality allowing for the effect of friction we have

$$r_1 + q_1 - x_3' r_3 - q_3 = 245$$

$$x_3' = (856.8 + 337.9 - 245 - 94.2) \div 1021.9 = 0.835.$$

Though not necessary for the solution of the problem it is interesting to notice that adiabatic expansion to the exit pressure would give for

$$x_3 = x_3 r_3 \div r_3 = 811.8 \div 1021.9 = 0.795.$$

Now 500 pounds of steam an hour gives

$$500 \div \overline{60}^2 = 0.139$$

of a pound per second; consequently the areas at the throat and the exit will be by equation (268), page 431, in square inches

$$a_2 = 144 \times 0.139 \frac{x_2 u_2 + \sigma}{V_2}$$

$$= 144 \times 0.139\,(0.961 \times 4.583 + 0.016) \div 1480 = 0.0597;$$

$$a_3 = 144 \times 0.139\,(0.835 \times 173.1 + 0.016) \div 3500 = 0.827.$$

The diameters are, therefore,

$$d_2 = 0.280 \qquad d_3 = 1.026.$$

If the taper is taken to be one in ten, the conical part will have a length of

$$10\,(1.026 - 0.280) = 7.46 \text{ inches;}$$

and allowing for the rounding at the entrance and for a fair curve joining the throat to the cone, the total length may be eight inches.

A nozzle to expand steam to the pressure of the atmosphere only, would have the computation for the exit made as follows:

$$x_3 r_3 = T_3\left(\frac{r_1}{T_1} + \theta_1 - \theta_3\right) = 671.5\,(1.0378 + 0.5237 - 0.3125) = 838.7;$$

$$r_1 + q_1 - x_3 r_3 - q_3 = 856.8 + 337.9 - 838.7 - 180.3 = 175.7.$$

Taking the coefficient for friction as 0.10 the available heat appears to be 157.5 and the velocity at exit will be

$$V_3 = \sqrt{64.4 \times 778 \times 157.5} = 2810.$$

The quality of the steam comes from the equation

$$r_1 + q_1 - x_3' r_3 - q_3 = 157.5.$$

$$\therefore\ x_3' = (856.8 + 337.9 - 157.5 - 180.3) \div 969.7 = 0.884.$$

The area at the exit will now become

$$144 a_3 = 144 \times .139\,(0.884 \times 26.80 + 0.016) \div 2810 = 0.169,$$

and the corresponding diameter is 0.464 of an inch. Taking the taper as one in ten, the length of the conical part of the nozzle becomes

$$10\,(0.464 - 0.280) = 1.84 \text{ inches,}$$

and its total length including throat and inlet may be 2.3 inches.

CHAPTER XVIII.

INJECTORS.

An injector is an instrument by means of which a jet of steam acting on a stream of water with which it mingles, and by which it is condensed, can impart to the resultant jet of water a sufficient velocity to overcome a pressure that may be equal to or greater than the initial pressure of the steam. Thus, steam from a boiler may force feed-water into the same boiler, or into a boiler having a higher pressure. The mechanical energy of the jet of water is derived from the heat energy yielded by the condensation of the steam-jet. There is no reason why an injector cannot be made to work with any volatile liquid and its vapor, if occasion may arise for doing so; but in practice it is used only for forcing water. An essential feature in the action of an injector is the condensation of the steam by the water forced; other instruments using jets without condensation, like the water-ejector in which a small stream at high velocity forces a large stream with a low velocity, differ essentially from the steam-injector.

Method of Working. — A very simple form of injector is shown by Fig. 91, consisting of three essential parts; *a*, the *steam-nozzle*, *b*, the *combining-tube*, and *c*, the *delivery-tube*. Steam is supplied to the injector through a pipe connected at *d*; water is supplied through a pipe at *f*, and the injector forces water out through the pipe at *e*. The steam-pipe must have on it a valve for starting and regulating the injector, and the delivery-pipe leading to the boiler must have on it a check-valve to prevent water from the boiler from flowing back through the injector when it is not working. The water-supply pipe commonly has a valve for regulating the flow of water into the injector.

This injector, known as a *non-lifting* injector, has the water-reservoir set high enough so that water will flow into the injector

through the influence of gravity. A *lifting* injector has a special device for making a vacuum to draw water from a reservoir below the injector, which will be described later.

To start the injector shown by Fig. 91, the steam-valve is first opened slightly to blow out any water that may have gathered above the valve, through the overflow, since it is essential to have dry steam for starting. The steam-valve is then closed, and the water-valve is opened wide. As soon as water appears at the overflow between the combining-tube and the delivery-tube the

Fig. 91.

steam-valve is opened wide, and the jet of steam from the steam-nozzle mingles with and is condensed by the water and imparts to it a high velocity, so that it passes across the overflow space between the combining-tube and the delivery-tube and passes into the boiler. When the injector is working a vacuum is liable to be formed at the space between the combining and delivery-tubes, and the valve at the overflow closes and excludes air which would mingle with the water and might interfere with the action of the injector.

Theory of the Injector. — The two fundamental equations of the theory of the injector are deduced from the principles of the conservation of energy and the conservation of momenta.

The heat energy in one pound of steam at the absolute pressure p_1 in the steam-pipe is

$$\frac{1}{A}(x_1r_1 + q_1),$$

where r_1 and q_1 are the heat of vaporization and heat of the liquid corresponding to the pressure p_1; $\frac{1}{A}$ is the mechanical equivalent of heat (778 foot-pounds), and x_1 is the quality of the steam; if there is two per cent of moisture in the steam, then x_1 is 0.98.

Suppose that the water entering the injector has the temperature t_3, and that its velocity where it mingles with the steam is V_w'; then its heat energy per pound is

$$\frac{1}{A}q_3,$$

and its kinetic energy is

$$\frac{V_w'^2}{2g},$$

where q_3 is the heat of the liquid at t_3, and g is the acceleration due to gravity (32.2 feet).

If the water forced by the injector has the temperature t_4, and if the velocity of the water in the smallest section of the delivery-tube is V_w, then the heat energy per pound is

$$\frac{1}{A}q_4,$$

and the kinetic energy is

$$\frac{V_w^2}{2g}.$$

Let each pound of steam draw into the injector y pounds of water; then, since the steam is condensed and forced through the delivery-tube with the water, there will be $1 + y$ pounds delivered for each pound of steam. Equating the sum of the heat and kinetic energies of the entering steam and water to the sum of the energies in the water forced from the injector, we have

$$\frac{1}{A}(x_1r_1 + q_1) + y\left(\frac{1}{A}q_3 + \frac{V_w'^2}{2g}\right) = (1 + y)\left(\frac{1}{A}q_4 + \frac{V_w^2}{2g}\right)$$

The terms depending on the velocities V_w' and V_w are never large and can commonly be neglected.

To get an idea of the influence of the former, we may consider that the pressure forcing water into a non-lifting injector is seldom, if ever, greater than the pressure of the atmosphere, and the corresponding pressure for a lifting injector is always less. Now, the pressure of the atmosphere is equivalent to a head of

$$h' = 144 \times 14.7 \div 62.4 = 34 \text{ feet.}$$

A liberal estimate of y (the pounds of water per pound of steam) is fifteen. Therefore,

$$y\frac{V_w'^2}{2g} = yh' = 15 \times 34 = 510.$$

In order that an injector shall deliver water against the steam-pressure in a boiler its velocity must be greater than would be impressed on cold water by a head equivalent to the boiler-pressure. Taking the boiler-pressure at 250 pounds by the gauge, or 265 pounds absolute, the equivalent head will be

$$h = 144 \times 265 \div 62.4 = 610 \text{ feet.}$$

Again taking fifteen for y, the value of the term depending on V_w will be

$$(1 + y)\frac{V_w^2}{2g} = (1 + 15)\,610 = 9150.$$

But the steam supplied to an injector is nearly dry and at 265 pounds absolute

$$r_1 + q_1 = 826.2 + 379.6 = 1205.8,$$

so that the term depending on that quantity will have the value

$$778 \times 1206 = 939000.$$

It is, therefore, evident that the term depending on V_w has an influence of less than one per cent and that the term depending on V_w' can be entirely neglected.

For practical purposes we may calculate the weight of water delivered per pound of steam by the equation

$$y = \frac{x_1 r_1 + q_1 - q_4}{q_4 - q_3} \quad \ldots \ldots \quad (270)$$

This equation may be applied to any injector including double injectors with two steam-nozzles.

The discussion just given shows that of the heat supplied to an injector only a very small part, usually less than one per cent, is changed into work. When used for feeding a boiler, or for similar purposes, this is of no consequence, because the heat not changed into work is returned to the boiler and there is no loss.

For example, if dry steam is supplied to the injector at 120 pounds by the gauge or 134.7 pounds absolute, if the supply-temperature of the water is 65° F., and if the delivery-temperature is 165° F., then the water pumped per pound of steam is

$$y = \frac{r_1 + q_1 - q_4}{q_4 - q_3} = \frac{869.9 + 321.5 - 133.0}{133.0 - 33.1} = 10.6 \text{ pounds.}$$

From the conservation of energy we have been able to devise an equation for the weight of water delivered per pound of steam; from the conservation of momenta we can find the relation of the velocities.

The momentum of one pound of steam issuing from the steam-nozzle with the velocity V_s is $V_s \div g$; the momentum of y pounds of water entering the combining-tube with the velocity V_w' is $yV_w' \div g$; and the momentum of $1 + y$ pounds of water at the smallest section of the delivery-tube is $(1 + y)\, V_w \div g$. Equating the sum of the momenta of water and steam before mingling to the momentum of the combined water and steam in the delivery-tube,

$$V_s + yV_w' = (1 + y)\, V_w \quad \ldots \ldots \quad (271)$$

This equation can be used to calculate any one of the velocities provided the other two can be determined independently. Unfor-

tunately there is some uncertainty about all of the velocities so that the proper sizes of the orifices and of the forms and proportions of the several members of an injector have been determined mainly by experiment. The best exposition of this matter is given by Mr. Strickland Kneass,* who has made many experiments for William Sellers & Co. The practical part of what follows is largely drawn from his work.

Velocity of the Steam-jet. — Equation (269), page 433, gives

$$V_s = \left\{ \frac{2g}{A} (x_1 r_1 - x_2 r_2 + q_1 - q_2) \right\}^{\frac{1}{2}}, \quad . \quad . \quad (272)$$

where r_1 and q_1 are the heat of vaporization and the heat of the liquid of the supply of steam at the pressure p_1, and r_2 and q_2 are corresponding quantities at the pressure p_2 for that section of the tube for which the velocity is calculated; x_1 is the quality of the steam at the pressure p_1 (usually 0.98 to unity) and x_2 is the quality at the pressure p_2 to be calculated by aid of the equation

$$\frac{x_1 r_1}{T_1} + \theta_1 = \frac{x_2 r_2}{T_2} + \theta_2.$$

Here T_1 and T_2 are the absolute temperatures corresponding to the pressures p_1 and p_2, and θ_1 and θ_2 are the entropies of the liquid at the same pressure. Also $\frac{1}{A}$ is the mechanical equivalent of heat and g is the acceleration due to gravity.

Some steam-nozzles for injectors are simple converging orifices and others have a throat and a diverging portion. It will be found in all cases including double injectors, that the pressure beyond the steam-nozzle is less than half the pressure causing the flow, and consequently the pressure at the narrowest part of the steam-nozzle and also the velocity at that place, depend only on the initial pressure. As was developed in the preceding chapter, the pressure and velocity at any part of an expanding nozzle depend on the ratio of the area at that part to the throat area, and are consequently under control. Also, as was empha-

* *Practice and Theory of the Injector*, J. Wiley & Sons.

sized by Rosenhain's experiments, the steam will expand and gain velocity beyond the nozzle, if it escapes at a pressure higher than the back-pressure. For an injector this last action is influenced by the fact that the jet from the steam-nozzle mingles with water and is rapidly condensed. Some injector makers use larger tapers than those recommended in the preceding chapter for expanding nozzles. The throat pressure may be assumed to be about 0.6 of the initial pressure; with the information in hand it is probably not worth while to try to make any allowance for friction.

The calculation of the area at the throat of a steam nozzle by the adiabatic method will be found fairly satisfactory; the calculation of the final velocity of the steam will probably not be satisfactory, as complete expansion in the nozzle seldom takes place, but it is easy to show that the velocity is sufficient to account for the action of the instrument.

For example, the velocity in the throat of a nozzle under the pressure of 120 pounds by the gauge or 134.7 pounds absolute is

$$V_s = \left\{ \frac{2g}{A} (x_1 r_1 - x_2 r_2 + q_1 - q_2 \right\}^{\frac{1}{2}}$$
$$= \{2 \times 32.2 \times 778 (869.9 - 0.965 \times 899.2 + 321.1 - 282.9)\}^{\frac{1}{2}}$$
$$= 1430 \text{ feet per second,}$$

having for x_2

$$x_2 = \frac{T_2}{r_2}\left(\frac{r_1}{T_1} + \theta_1 - \theta_2\right) = \frac{1}{1.1645}(1.0745 + 0.5035 - 0.4547)$$
$$= 0.965,$$

provided that $p_2 = 0.6 p_1 = 80.8$ pounds absolute.

If, however, the pressure at the exit of an expanded nozzle is 14.7 pounds absolute, then

$$x_s = \frac{1}{1.4441}(1.0745 + 0.5035 - 0.3125) = 0.877,$$

and

$$V_s = \{2 \times 32.2 \times 778 (869.9 - 0.877 \times 969.7 + 321.1 - 180.3)\}^{\frac{1}{2}}$$
$$= 2860 \text{ feet per second,}$$

which is nearly twice that just calculated for the velocity at the smallest section of the steam-nozzle. Since there is usually a vacuum beyond the steam-nozzle, the final steam velocity is likely to be considerably larger, but this computed velocity will suffice for explaining the dynamics of the case.

Velocity of Entering Water. — The velocity of the water in the combining-tube where it mingles with the steam depends on (*a*) the lift or head from the reservoir to the injector, (*b*) the pressure (or vacuum) in the combining-tube, and (*c*) on the resistance which the water experiences from friction and eddies in the pipe, valves, and passages of the injector. The first of these can be measured directly for any given case; for example, where a test is made on an injector. In determining the proportions of an injector it is safe to assume that there is neither lift nor head for a non-lifting injector, and that the lift for a lifting-injector is as large as can be obtained with certainty in practice. The lift for an injector is usually moderate, and seldom if ever exceeds 20 feet.

The vacuum in the combining-tube may amount to 22 or 24 inches of mercury, corresponding to 25 or 27 feet of water; that is, the absolute pressure may be 3 or 4 pounds per square inch. The vacuum after the steam and water are combined appears to be limited by the temperature of the water; thus, if the temperature is 165° F., the absolute pressure cannot be less than 5.3 pounds. But the final temperature is taken in the delivery-pipe after the water and condensed steam are well mixed and are moving with a moderate velocity.

The resistance of friction in the pipes, valves, and passages of injectors has never been determined; since the velocity is high the resistance must be considerable.

If we assume the greatest vacuum to correspond to 27 feet of water, the maximum velocity of the water entering the combining-tube will not exceed

$$\sqrt{2gh} = \sqrt{2 \times 32.2 \times 27} = 42 \text{ feet.}$$

If, on the contrary, the effective head producing velocity is as small as 5 feet, the corresponding velocity will be

$$\sqrt{2 \times 32.2 \times 5} = 18 \text{ feet.}$$

It cannot be far from the truth to assume that the velocity of the water entering the combining-tube is between 20 and 40 feet per second.

Velocity in the Delivery-tube. — The velocity of the water in the smallest section of the delivery-tube may be estimated in two ways; in the first place it must be greater than the velocity of cold water flowing out under the pressure in the boiler, and in the second place it may be calculated by aid of equation (271), provided that the velocities of the entering steam and water are determined or assumed.

For example, let it be assumed that the pressure of the steam in the boiler is 120 pounds by the gauge, and that, as calculated on page 451, each pound of steam delivers 10.5 pounds of water from the reservoir to the boiler. As there is a good vacuum in the injector we may assume that the pressure to be overcome is 132 pounds per square inch, corresponding to a head of

$$\frac{132 \times 144}{62.4} = 305 \text{ feet.}$$

Now the velocity of water flowing under the head of 305 feet is

$$\sqrt{2gh} = \sqrt{2 \times 32.2 \times 305} = 140 \text{ feet per second.}$$

The velocity of steam flowing from a pressure of 120 pounds by the gauge through a diverging-tube with the pressure equal to that of the atmosphere at the exit has been calculated to be 2830 feet per second. Assuming the velocity of the water entering the combining-tube to be 20 feet, then by equation (271) we have in this case

$$V_w = \frac{V_s + yV_w'}{1 + y} = \frac{2860 + 10.6 \times 20}{1 + 10.6} = 265 \text{ feet;}$$

this velocity is sufficient to overcome a pressure of about 470 pounds per square inch if no allowance is made for friction or losses.

Sizes of the Orifices. — From direct experiments on injectors as well as from the discussion in the previous chapter, it appears

that the quantity of steam delivered by the steam-nozzle can be calculated in all cases by the method for the flow of steam, through an orifice, assuming the pressure in the orifice to be $\frac{6}{10}$ of the absolute pressure above the orifice.

Now each pound of steam forces y pounds of water from the reservoir to the boiler; consequently if w pounds are drawn from the reservoir per second the injector will use $w \div y$ pounds of steam per second.

The specific volume of the mixture of water and steam in the smallest section of the steam-nozzle is

$$v_2 = x_2 u_2 + \sigma,$$

where x_2 is the quality, u_2 is the increase of volume due to vaporization, and σ is the specific volume of the water. The volume of steam discharged per second is

$$\frac{wv_2}{y},$$

and the area of the orifice is

$$a_s = \frac{wv_2}{yV_s} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (273)$$

where V_s is the velocity at the smallest section.

For example, for a flow from 134.7 pounds absolute to 80.8 pounds absolute x_2 is 0.965 and V_s is 1430 feet, as found on page 453. Again, for an increase of temperature from 65° F. to 165° F., the water per pound of steam is 10.6. Calculating the specific volume at 80.8 pounds, we have

$$v_2 = x_2 u_2 + \sigma = 0.965\,(5.38 - 0.016) + 0.016 = 5.23 \text{ cubic feet.}$$

If the injector is required to deliver 1200 gallons an hour, or

$$\frac{1200 \times 231 \times 62.4}{1728 \times 60 \times 60} = 2.78$$

pounds per second, the area of the steam-nozzle must be

$$a_s = \frac{wv_2}{yV_s} = \frac{2.78 \times 5.23}{10.6 \times 1430} = 0.000959 \text{ square feet.}$$

The corresponding diameter is 0.420 of an inch, or 10.6 millimetres.

In trying to determine the size of the orifice in the delivery-tube we meet with two serious difficulties: we do not know the velocity of the stream in the smallest section of the delivery-tube, and we do not know the condition of the fluid at that place. It has been assumed that the steam is entirely condensed by the water in the combining-tube before reaching the delivery-tube, but there may be small bubbles of uncondensed steam still mingled with the water, so that the probable density of the heterogeneous mixture may be less than that of water. Since the pressure at the entrance to the delivery-tube is small, the specific volume of the steam is very large, and a fraction of a per cent of steam is enough to reduce the density of the steam to one-half. Even if the steam is entirely condensed, the air carried by the water from the reservoir is enough to sensibly reduce the density at the low pressure (or vacuum) found at the entrance to the delivery-tube.

If V_w is the probable velocity of the jet at the smallest section of the delivery-tube, and if d is the density of the fluid, then the area of the orifice in square feet is

$$a_w = \frac{w\,(1 + y)}{V_w dy}, \quad \ldots \ldots \quad (274)$$

for each pound of steam mingles with and is condensed by y pounds of water and passes with that water through the delivery-tube; w, as before, is the number of pounds of water drawn from the reservoir per second.

For example, let it be assumed that the actual velocity in the delivery-tube to overcome a boiler-pressure of 120 pounds by the gauge is 150 feet per second, and that the density of the jet is about 0.9 that of water; then with the value of $w = 2.78$ and $y = 10.6$, we have

$$a_w = \frac{w\,(1 + y)}{V_w dy} = \frac{2.78 \times 11.6}{150 \times 0.9 \times 62.4 \times 10.6} = 0.000361 \text{ sq. ft.}$$

The corresponding diameter is 0.257 of an inch, or 6.5 millimetres. If this calculation were made with the velocity 266 (computed for expansion to atmospheric pressure) and with

clear water the diameter would be only 0.183 of an inch; this is to be considered rather as a theoretic minimum than as a practical dimension.

Steam-nozzle. — The entrance to the steam-nozzle should be well rounded to avoid eddies or reduction of pressure as the steam approaches; in some injectors, as the Sellers' injector, Fig. 92, the valve controlling the steam supply is placed near the entrance to the nozzle, but the bevelled valve-seat will not interfere with the flow when the valve is open.

It has already been pointed out that the steam-nozzle may advantageously be made to expand or flare from the smallest section to the exit. The length from that section to the end may be between two and three times the diameter at that section.

Consider the case of a steam-nozzle supplied with steam at 120 pounds boiler-pressure: it has been found that the velocity at the smallest section, on the assumption that the pressure is then 80.8 pounds, is 1430 feet per second, and that the specific volume is 5.23 cubic feet. If the pressure in the nozzle is reduced to 14.7 pounds, at the exit, the velocity becomes 2860 feet per second, the quality being $x_2 = 0.877$. The specific volume is consequently

$$v_2 = x_2 u_2 + \sigma = 0.877\ (26.78 - 0.016) + 0.016 = 23.5 \text{ cu. ft.}$$

The areas will be directly as the specific volumes and inversely as the velocities, so that for this case we shall have the ratio of the areas

$$\left.\begin{matrix} 5.23 : 23.5 \\ 2860 : 1430 \end{matrix}\right\} = 1 : 2.25;$$

and the ratio of the diameter will be

$$\sqrt{1} \cdot \sqrt{2.25} = 1 : 1.5.$$

Combining-tube. — There is great diversity with different injectors in the form and proportions of the combining-tube. It is always made in the form of a hollow converging cone, straight or curved. The overflow is commonly connected to a space between the combining-tube and the delivery-tube; it is,

however, sometimes placed beyond the delivery-tube, as in the Sellers' injector, Fig. 92. In the latter case the combining- and delivery-tubes may form one continuous piece, as is seen in the double injector shown by Fig. 93.

The Delivery-tube. — This tube should be gradually enlarged from its smallest diameter to the exit in order that the water in it may gradually lose velocity and be less affected by the sudden change of velocity where this tube connects to the pipe leading to the boiler.

It is the custom to rate injectors by the size of the delivery-tube; thus a No. 6 injector may have a diameter of 6 mm. at the smallest section of the delivery-tube.

Mr. Kneass found that a delivery-tube cut off short at the smallest section would deliver water against 35 pounds pressure only, without overflowing; the steam-pressure being 65 pounds. A cylindrical tube four times as long as the internal diameter, under the same conditions would deliver only against 24 pounds. A tube with a rapid flare delivered against 62 pounds, and a gradually enlarged tube delivered against 93 pounds.

If the delivery-tube is assumed to be filled with water without any admixture of steam or air, then the relative velocities at different sections may be assumed to be inversely proportional to the corresponding areas. This gives a method of tracing the change of velocity of the water in the tube from its smallest diameter to the exit.

A sudden change in the velocity is very undesirable, as at the point where the change occurs the tube is worn and roughened, especially if there are solid impurities in the water. It has been proposed to make the form of the tube such that the change of velocity shall be uniform until the pressure has fallen to that in the delivery-pipe; but this idea is found to be impracticable, as it leads to very long tubes with a very wide flare at the end.

Efficiency of the Injector. — The injector is used for feeding boilers, and for little else. Since the heat drawn from the boiler is returned to the boiler again, save the very small part which is changed into mechanical energy, it appears as though the

efficiency was perfect, and that one injector is as good as another provided that it works with certainty. We may almost consider the injector to act as a feed-water heater, treating the pumping in of feed-water as incidental. It has already been pointed out

Fig. 92.

on page 450 that the kinetic energy of the jet in the delivery-tube is less than one per cent of the energy due to the condensation of the steam. On this account the injector is used wherever cold water must be forced into a boiler, as on a locomotive, or when sea-water is supplied to a marine boiler. Considering only the advantage of supplying hot water to the boiler, it almost seems as though the more steam an injector uses the better it is. Such a view is erroneous, as it is often desirable to supply water without immediately reducing the steam-pressure and then it is necessary to use as little steam as may be. It is, however, true that simplicity of construction and certainty of action are of the first importance in injectors.

Lifting Injector. — The injector described at the beginning of

this chapter was placed so that water from the reservoir would run in under the influence of gravity. When the injector is placed higher than the reservoir a special device is provided for lifting the water to start the injector. Thus in the Sellers' injector, Fig. 92, there is a long tube which protrudes well into the combining-tube when the valves w and x are both closed. When the rod B is drawn back a little by aid of the lever H the valve w is opened, admitting steam through a side orifice to the tube mentioned. Steam from this tube drives out the air in the injector through the overflow, and water flows up into the vacuum thus formed, and is itself forced out at the overflow. The starting-lever H is then drawn as far back as it will go, opening the valve x and supplying steam to the steam-nozzle. This steam mingles with and is condensed by the water and imparts to the water sufficient velocity to overcome the boiler-pressure. Just as the lever H reaches its extreme position it closes the overflow valve K through the rod L and the crank at R.

Since lifting-injectors may be supplied with water under a head, and since a non-lifting injector when started will lift water from a reservoir below it, or may even start with a small lift, the distinction between them is not fundamental.

Double Injectors. — The double injector illustrated by Fig. 93, which represents the Körting injector, consists of two complete injectors, one of which draws water from the reservoir and delivers it to the second, which in turn delivers the water to the boiler. To start this injector the handle A is drawn back to the position B and opens the valve supplying steam to the lifting-injector. The proper sequence in opening the valves is secured by the simple device of using a loose lever for joining both to the valve-spindle; for under steam-pressure the smaller will open first, and when it is open the larger will move. The steam-nozzle of the lifter has a good deal of flare, which tends to form a good vacuum. The lifter first delivers water out at the overflow with the starting lever at B; then that lever is pulled as far as it will go, opening the valve for the second injector or forcer, and closing both overflow valves.

Self-adjusting Injectors. — In the discussions of injectors thus far given it has been assumed that they work at full capacity, but as an injector must be able to bring the water-level in a boiler up promptly to the proper height, it will have much more than the capacity needed for feeding the boiler steadily. Any injector may be made to work at a reduced capacity by simply reducing the opening of the steam-valve, but the limit

Fig. 93.

of its action is soon reached. The limit may be extended somewhat by partially closing the water-supply valve and so limiting the water-supply.

The original Giffard injector had a movable steam-nozzle to control the thickness of the sheet of water mingling with the steam, and also had a long conical valve thrust into the steam-nozzle by which the effective area of the steam-jet could be regulated. Thus both water and steam passages could be controlled without changing the pressures under which they were supplied, and the injector could be regulated to work through a wide range of pressures and capacities. The main objection was that the injector was regulated by hand and required much attention.

In the Sellers' injector, Fig. 92, the regulation of the steam-supply by a long cone thrust through the steam-nozzle is retained, but the supply of water is regulated by a movable combining-tube, which is guided at each end and is free to move forwards and backwards. At the rear the combining-tube is affected by the pressure of the entering water, and in front it is subjected to the pressure in the closed space *O*, which is in communication with the overflow space between the combining-tube and the delivery-tube, in this injector the space is only for producing the regulation of the water-supply by the motion of the combining-tube, as the actual overflow is beyond the delivery-tube at *K*. When the injector is running at any regular rate the pressures on the front and the rear of the combining-tube are nearly equal, and it remains at rest. When the starting-lever is drawn out or the steam-pressure increases, the inflowing steam is not entirely condensed in the combining-tube as it is during efficient action; lateral contraction of the jet therefore occurs when crossing the overflow chamber, causing a reduction of pressure in *O*, which causes the tube to move toward *D* and increase the supply of water. When the starting-lever is pushed inward, reducing the flow of steam, the impulsive effort is insufficient to force a full supply of water through the delivery-tube, and there is an overflow into the chamber *O* which pushes the combining-tube backwards and reduces the inflow of water. The injector is always started at full capacity by pulling the steam-valve wide open, as already described; after it is started the steam-supply is regulated at will by the engineer or boiler attendant, and the water is automatically adjusted by the movable combining-tube, and the injector will require attention only when a change of the rate of feeding the boiler is required on account of either a change in the draught of steam from the boiler, or a change of steam-pressure, for the capacity of the injector increases with a rise of pressure.

A double injector, such as that represented by Fig. 93, is to a certain extent self-adjusting, since an increase of steam-pressure causes at once an increase in the amount of water drawn in by

the lifter and an increase in the flow of steam through the steam-nozzle of the forcer. Such injectors have a wide range of action and can be controlled by regulating the valve on the steam-pipe.

Restarting Injectors. — If the action of any of the injector thus far described is interrupted for any reason, it is necessary to shut off steam and start the injector anew; sometimes the injector has become heated while its action is interrupted, and there may be difficulty in starting. To overcome this difficulty various forms of restarting injectors have been devised, such as the Sellers, Fig. 94. This injector has four fixed nozzles in line, the steam-nozzle 3, the draft-tube 11, the combining-tube 2, and the delivery-tube at the bottom. There is also a sliding bushing 5 and an overflow valve 15. The steam-nozzle has a wide flare and makes a vacuum which draws water from the supply-tank under all conditions; the water passes through the draught-tube and out at the overflow until the condensation of steam in the combining-tube makes a partial vacuum that draws up the bushing 5 against the draught-tube and shuts off the passage to the overflow; the injector then forces water to the boiler. If the injector stops for any cause the bushing falls and the injector takes the starting position and will start as soon as supplied with water and steam.

Fig. 94.

Self-acting Injector. — The most recent type of Sellers' injector invented by Mr. Kneass and represented by Fig. 95 is both self-starting and self-adjusting. It is a double injector with all the jets in one line; *a*, *b*, and *c* are the steam-nozzle, the combining-tube, and the delivery-tube of the forcer; the lifter is composed of the

Fig. 95.

annular steam-nozzle *d*, and the annular delivery-tube *e*, surrounding the nozzle *a*. The proportions are such that the lifter can always produce a suction in the feed-pipe even when there is a discharge from the main steam-nozzle, and it is this fact that establishes the restarting feature. When the feed-water rises to the tubes it meets the steam from the lifter-nozzle and is forced in a thin sheet and with high velocity into the combining-tube of the forcer, where it comes in contact with the main steam-jet, and mingling with and condensing it, receives a high velocity which enables it to pass the overflow orifices and proceed through the delivery-tube to the boiler.

Like any double injector, the lifter and forcer have a considerable range of action through which the water is adjusted to the steam-supply; but there is a further adjustment in this injector, for when a good vacuum is established in the space surrounding the combining-tube, water can enter through the check-valve *f*, and flowing through the orifices in the combining-tube mingles with the jet in it, and is forced with that jet into the boiler.

The steam-valve is seated on the end of the lifter-nozzle, and it has a protruding plug which enters the forcer-nozzle. When the valve is opened to start the injector, steam is supplied first to the starter, and soon after, by withdrawing the plug, to the forcer. If the steam is dry the starting-lever may be moved back promptly; if there is condensed water in the steam-pipe, the starting-handle should be moved a little way to first open the valve of the lifter, and then it is drawn as far back as it will go, as soon as water appears at the overflow. The water-supply may be regulated by the valve *g*, which can be rotated a part of a turn. The minimum delivery of the injector is obtained by closing this valve till puffs of steam appear at the overflow, and then opening it enough to stop the escape of steam.

When supplied with cold water this injector wastes very little in starting. If the injector is hot or is filled with hot water when started, it will waste hot water till the injector is

cooled by the water from the feed-supply, and will then work as usual. If air leaks into the suction-pipe or if there is any other interference with the normal action, the injector wastes water or steam till normal conditions are restored, when it starts automatically.

Exhaust Steam Injectors. — Injectors supplied with exhaust-steam from a non-condensing engine can be used to feed boilers up to a pressure of about 80 pounds. Above this pressure a supplemental jet of steam from the boiler must be used. Such an injector, as made by Schäffer and Budenberg, is represented by Fig. 96; when used with low boiler-pressure this injector has a solid cone or spindle instead of the live-steam nozzle. To provide a very free overflow the combining-tube is divided, and one side is hung on a hinge and can open to give free exit to the overflow when the injector is started. When the injector is working it closes down into place. The calculation for an exhaust-steam injector shows that enough velocity may be imparted to the water in the delivery-tube to overcome a moderate boiler-pressure.

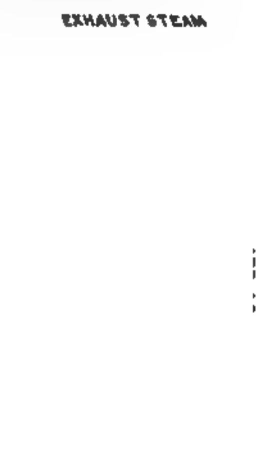

FIG. 96.

For example, an injector supplied with steam at atmospheric pressure, and raising the feed-water from 65° F. to 145° F., will draw from the reservoir

$$y = \frac{r_1 + q_1 - q_4}{q_4 - q_3} = \frac{969.7 + 180.3 - 113.0}{113.0 - 33.1} = 13.0$$

pounds of water per pound of steam. In this case as the steam-nozzle is converging we will use for computing the velocity the pressure

$$0.6 \times 14.7 = 8.8 \text{ pounds.}$$

This will give

$$x_2r_2 = T_2\left(\frac{r_1}{T_1} + \theta_1 - \theta_2\right) = 646.7\ (1.4441 + 0.3125 - .2745) = 958.6,$$

consequently

$$V = \sqrt{\frac{2g}{A}(r_1 + q_1 - x_2r_2 - q_2)}$$

$$= \sqrt{2 \times 32.2 \times 778\ (969.7 + 180.3 - 958.6 - 155.3} = 1340.$$

Assuming the velocity of the water entering the combining-tube will give for the velocity of the jet in the combining-tube

$$V_w = \frac{1340 + 13.0 \times 30}{1 + 13.0} = 124 \text{ feet.}$$

This velocity is equivalent to that produced by a static pressure of

$$\frac{124^2 \times 62.4}{64.4 \times 144} = 103$$

pounds absolute, or a gauge pressure of 88 pounds. No allowance is made for reduction of density by bubbles of steam in the combining-tube or for resistance of pipes and valves. If

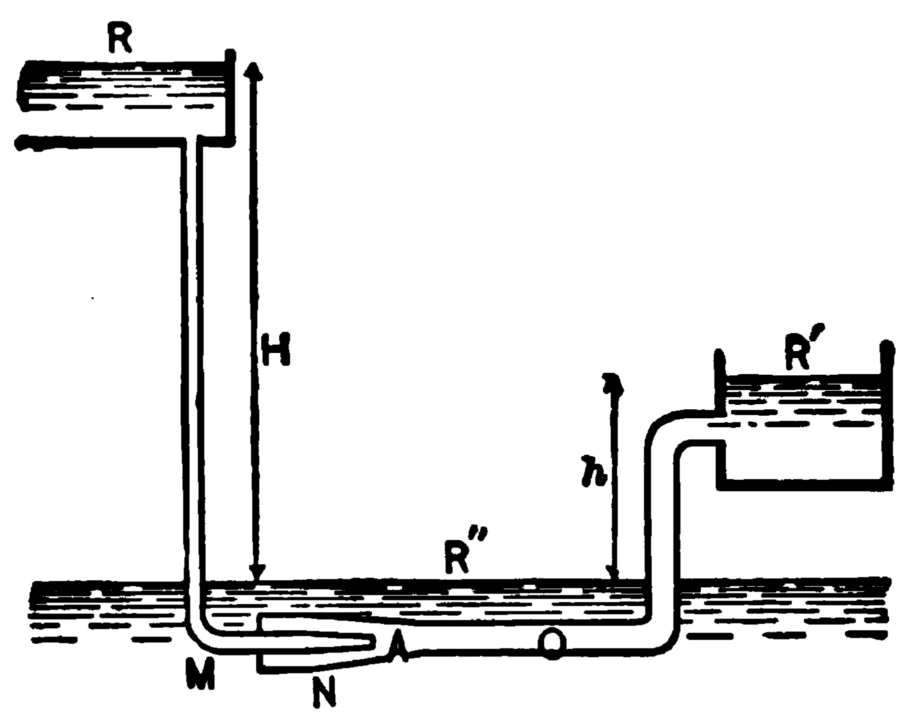

FIG. 97.

such an injector can take advantage of further expansion either in the steam-nozzle or beyond, the velocity may be greater than that computed and a better action might ensue.

Unless the exhaust-steam is free from oil its use for feeding

the boiler with an exhaust-steam injector will result in fouling the boiler.

Water-ejector. — Fig. 97 represents a device called a water-ejector, in which a small stream of water in the pipe M flowing from the reservoir R raises water from the reservoir R'' to the reservoir R'.

Let one pound of water from the reservoir R draw y pounds from R'', and deliver $1 + y$ pounds to R'. Let the velocity of the water issuing from A be v; that of the water entering from R'' be v_2 at N; and that of the water in the pipe O be v_1. The equality of momenta gives

$$v + yv_2 = (1 + y) v_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad (275)$$

Let x be the excess of pressure at M above that at N expressed in feet of water; then

$$v_2^2 = 2gx;$$
$$v^2 = 2g (H + x);$$
$$v_1^2 = 2g (h + x)$$

Substituting in equation (275),

$$\sqrt{H + x} + y\sqrt{x} = (1 + y) \sqrt{h + x};$$

$$\therefore \quad y = \frac{\sqrt{H + x} - \sqrt{h + x}}{\sqrt{h + x} - \sqrt{x}} \quad . \quad . \quad . \quad (276)$$

It is evident from inspection of the equation (276) that y may be increased by increasing x; for example, by placing the injector above the level of the reservoir so that there may be a vacuum in front of the orifice A.

If the weight G of water is to be lifted per second, then $\frac{G}{y}$ pounds per second must pass the orifice A, G pounds the space at N, and $\left(1 + \frac{1}{y}\right) G$ pounds through the section at O; which, with the several velocities v, v_2, and v_1, give the data for the calculation of the required areas.

Problem. — Required the calculation for a water-ejector

to raise 1200 gallons of water an hour, $H = 96$ ft., $h = 12$ ft., $x = 4$ ft.

$\sqrt{x} = \sqrt{4} = 2$; $\sqrt{H + x} = \sqrt{100} = 10$; $\sqrt{h + x} = \sqrt{16} = 4$;

$$y = \frac{10 - 4}{4 - 2} = 3.$$

The velocities are

$$v = \sqrt{2 \times 32.2 \times 100} = 80.25 \text{ feet per second;}$$
$$v_1 = \sqrt{2 \times 32.2 \times 16} = 32.10 \text{ feet per second;}$$
$$v_2 = \sqrt{2 \times 32.2 \times 4} = 16.05 \text{ feet per second.}$$

1200 gallons an hour = 0.04452 cubic feet per second.

The areas are

$$a = \frac{0.04452}{3 \times 80.25} = 0.000185 \text{ square feet;}$$
$$a_1 = \frac{4 \times 0.04452}{3 \times 32.10} = 0.06185 \text{ square feet;}$$
$$a_2 = \frac{0.04452}{16.05} = 0.00277 \text{ square feet.}$$

The diameters corresponding to the velocities v and v_1 are

$$d = 0.18 \text{ of an inch;}$$
$$d_1 = 0.58 \text{ of an inch.}$$

The area a_2 is of annular form, having the area 0.4 of a square inch.

Ejector. — When the ejector is used for raising water where there is no advantage in heating the water, it is a very wasteful instrument. The efficiency is much improved by arranging the instrument as in Fig. 98, so that the steam-nozzle A shall deliver a small stream of water at a high velocity, which, as in the water-ejector, delivers a larger stream at a less velocity. Each additional conical nozzle increases the quantity at the expense of the velocity, so that a large quantity of water may be lifted a small height.

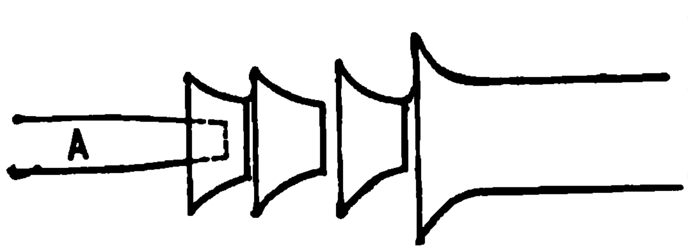

Fig. 98.

Ejectors are commonly fitted in steamships as auxiliary pumps in case of leakage, a service for which they are well fitted, since they are compact, cheap, and powerful, and are used only in emergency, when economy is of small consequence.

Ejector-condensers. — When there is a good supply of cold condensing water, an exhaust-steam ejector, using all the steam from the engine, may be arranged to take the place of the air-pump of a jet-condensing engine. The energy of the exhaust-steam flowing from the cylinder of the engine to the combining-tube, where the absolute pressure is less and where the steam is condensed, is sufficient to eject the water and the air mingled with it against the pressure of the atmosphere, and thus to maintain the vacuum.

For example, if the absolute pressure in the exhaust-pipe is 2 pounds, and if the temperatures of the injection and the delivery are 50° F. and 97° F., then the water supplied per pound of steam will be about 22 pounds. If the pressure at the exit of the steam-nozzle can be taken as one pound absolute, the velocity of the steam-jet will be 1490 feet per second. If the water is assumed to enter with a velocity of 20 feet, the velocity of the water-jet in the combining-tube will be 84 feet, which can overcome a pressure of 48 pounds per square inch.

CHAPTER XIX.

STEAM-TURBINES.

THE recent rapid development of steam-turbines may be attributed largely to the perfecting of mechanical construction, making it possible to construct large machinery with the accuracy required for the high speeds and close adjustments which these motors demand.

An adequate treatment of steam-turbines, including details of design, construction, and management, would require a separate treatise; but there is an advantage in discussing here the thermal problems that arise in the transformation of heat into kinetic energy, and the application of this energy to the moving parts of the turbine. For this purpose it is necessary to give attention to the action of jets of fluids on vanes and to the reaction of jets issuing from moving orifices, subjects that otherwise would appear foreign to this treatise.

The fundamental principles of the theory of turbines are the same whether they are driven by water or by steam; but the use of an elastic fluid like steam instead of a fluid like water, which has practically a constant density, leads to differences in the application of those principles. One feature is immediately evident from the discussion of the flow of fluids in Chapter XVII, namely, that exceedingly high velocities are liable to be developed. Thus, on page 444 it was found that steam flowing from a gauge pressure of 150 pounds per square inch into a vacuum of 26 inches of mercury (2 pounds absolute) through a proper nozzle, developed a velocity of 3500 feet per second, with an allowance of 0.15 for friction. This range of pressure corresponds to a hydraulic head of

$$163 \times 144 \div 62.4 = 376 \text{ feet};$$

and such a head will give a velocity of

$$V = \sqrt{2 \times 32.2 \times 376} = 156 \text{ feet per second.}$$

But so great a hydraulic head or fall of water is seldom, if ever, applied to a single turbine, and would be considered inconvenient. One hundred feet is a large hydraulic head, yielding a velocity of 80 feet per second, and twenty-five feet yielding a velocity of 40 feet per second is considered a very effective head.

If heads of 300 feet and upward were frequent, it is likely that compound turbines would be developed to use them; except for relatively small powers, steam-turbines are always compound, that is, the steam flows through a succession of turbines which may therefore run at more manageable speeds.

The great velocities that are developed in steam turbines, even when compounded, require careful reduction of clearances, and although they are restricted to small fractions of an inch the question of leakage is very important. Another feature in which steam turbines differ from hydraulic turbines is that steam is an elastic fluid which tends to fill any space to which it is admitted. The influence of this feature will appear in the distinction between impulse and reaction turbines.

Impulse. — If a well formed stream of water at moderate velocity flows from a conical nozzle, on a flat plate it spreads over it smoothly in all directions and exerts a steady force on it. If the velocity of the stream is V_1 feet per second, and if w pounds of water are discharged per second, the force will be very nearly equal to

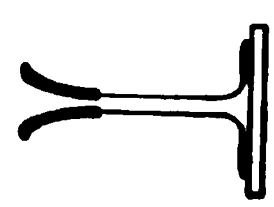

FIG. 99.

$$P = \frac{w}{g} V_1 \, .$$

Here we have the velocity in the direction of the jet changed from V_1 feet per second to zero; that is, there is a retardation, or negative acceleration, of V_1 feet per second; consequently the force is measured by the product of mass and the acceleration, g being the acceleration due to gravity. A force exerted by a jet or stream of fluid on a plate or vane is called an *impulse*. It

is important to keep clearly in mind that we are dealing with velocity, change of velocity or acceleration, and force, and that the force is measured in the usual way. The use of a special name for the force which is developed in this way is unfortunate but it is too well established to be neglected.

If the plate or vane, instead of remaining at rest, moves with the velocity of V feet per second, the change in velocity or negative acceleration will be $V_1 - V$ feet per second, and the force or impulse will be

$$P = \frac{w}{g}(V_1 - V).$$

This force in one second will move the distance V feet and will do the work

$$\frac{w}{g}(V_1 - V)\,V = \frac{w}{g}(V_1 V - V^2) \quad . \quad . \quad . \quad (276)$$

foot-pounds.

Since the vane would soon move beyond the range of the jet, it would be necessary, in order to obtain continuous action on a motor, to provide a succession of vanes, which might be mounted on the rim of a wheel. There would be, in consequence, waste of energy due to the motion of the vanes in a circle and to splattering and other imperfect action.

If the velocity of the jet of water is high it would fail to spread fairly over the plate in Fig. 99, when it is at rest, and a crude motor of the sort mentioned would show a very poor efficiency. Now steam has exceedingly high velocity when discharged from a nozzle, and the jet is more easily broken, so that adverse influences have even a worse effect than on water, and there is the greater reason for following methods which tend to avoid waste. Also, as pointed out on page 434, the nozzle must be so formed as to expand the steam down to the back pressure, or expansion will continue beyond the nozzle with further acceleration of the steam under unfavorable conditions.

It is easy to show that the best efficiency of the simple action of a jet on a vane, which we have discussed, will be obtained by making the velocity V of the vane half the velocity V_1 of the jet.

For if we differentiate the expression (276) with regard to V and equate the differential coefficient to zero we shall have

$$V_1 - 2V = 0; \quad V = \tfrac{1}{2}V_1;$$

and this value carried into expression (276) gives for the work on the vane

$$\frac{1}{4}\frac{w}{g}V_1^2;$$

but the kinetic energy of the jet is

$$\frac{1}{2}\frac{w}{g}V_1^2,$$

so that the efficiency is 0.5.

If the flat plate in Fig. 99 be replaced by a semi-cylindrical vane as in Fig. 99a, the direction of the stream will be reversed, and the impulse will be twice as great. If the vane as before has the velocity V the relative velocity of the jet with regard to the vane will be

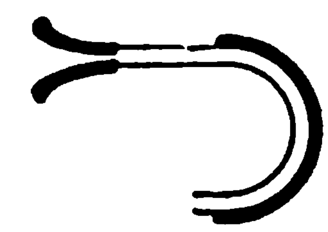
Fig. 99a.

$$V_1 - V$$

and neglecting friction this velocity may be attributed to the water where it leaves the vane. This relative velocity at exit will be toward the rear, so that the absolute velocity will be

$$V - (V_1 - V) = 2V - V_1.$$

The change of velocity or negative acceleration will be

$$V_1 - (2V - V_1) = 2(V_1 - V),$$

and the impulse is consequently

$$P = \frac{w}{g}.2(V_1 - V).$$

The work of the impulse becomes

$$\frac{w}{g}.2(V_1 - V)V = 2\frac{w}{g}(V_1V - V^2) \quad . \quad . \quad (277)$$

The maximum occurs when

$$\frac{d}{dV}(V_1V - V^2) = V_1 - 2V = 0 \quad \text{or} \quad V = \tfrac{1}{2}V_1.$$

But this value introduced in equation (277) now gives

$$\frac{1}{2}\frac{w}{g}V_1^2,$$

which is equal to the kinetic energy of the jet, and consequently the efficiency without allowing for losses appears to be unity.

Certain water-wheels which work on essentially this principle give an efficiency of 0.85 to 0.90. The method in its simplest form is not well adapted to steam turbines, but this discussion leads naturally to the treatment of all impulse turbines now made.

Reaction. — If a stream of water flows through a conical nozzle into the air with a velocity V_1 as in Fig. 100, a force

$$R = \frac{w}{g} V_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (278)$$

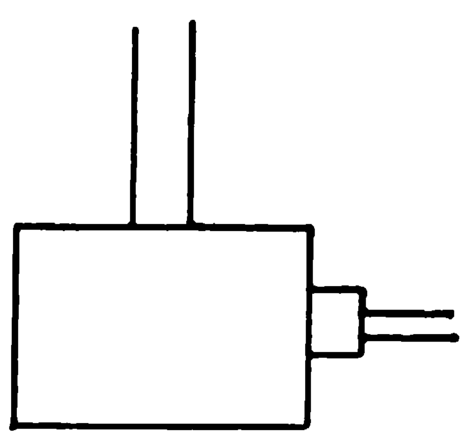

Fig. 100.

will be exerted tending to move the vessel from which the flow takes place, in the contrary direction. Here again w is the weight discharged per second, and g is the acceleration due to gravity. The force R is called the *reaction*, a name that is so commonly used that it must be accepted. Since the fluid in the chamber is at rest, the velocity V_1 is that imparted by the pressure in one second, and is therefore an acceleration, and the force is therefore measured by the product of the mass and the acceleration. However elementary this may appear, it should be carefully borne in mind, to avoid future confusion.

If steam is discharged from a proper expanding nozzle, which reduces the pressure to that of the atmosphere, its reaction will be very nearly represented by equation (278), but if the expansion is incomplete in the nozzle it will continue beyond, and the added acceleration will affect the reaction. On the other hand, if the expansion is excessive there will be sound waves in the nozzle and other disturbances.

The velocity of the jet depends on the pressure in the chamber, and if it can be maintained, the velocity will be the same relatively to the chamber when the latter is supposed to move. The work will in such case be equal to the product of the reaction, computed by equation (278), and the velocity of the chamber. There is no simple way of supplying fluid to a chamber which moves in a straight line, and a reaction wheel supplied with fluid at the centre and discharging through nozzles at the circumference is affected by centrifugal force. Consequently, as there is now no example of a pure reaction steam turbine, it is not profitable to go further in this matter. It is, however, important to remember that velocity, or increase of velocity, is due to pressure in the chamber or space under consideration, and is relative to that chamber or space.

General Case of Impulse. — In Fig. 101 let *ac* represent the velocity V_1 of a jet of fluid, and let V represent the velocity of a curved vane *ce*. Then the velocity of the jet, relative to the vane is V_2 equal to *bc*. This has been drawn in the figure coincident with the tangent at the end of the vane, and in general this arrangement is desirable because it avoids splattering.

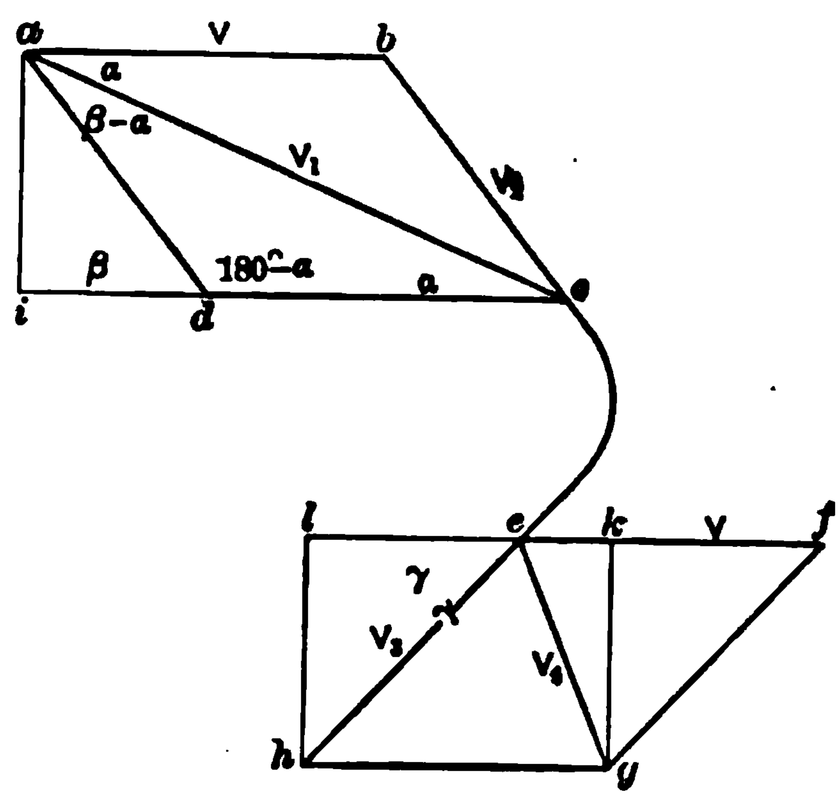

FIG. 101.

If it be supposed that the vane is bounded at the sides so that the steam cannot spread laterally and if friction can be neglected, the relative velocity V_3 may be assumed to equal V_2. Its direction is along the tangent at the end *e* of the vane. The absolute velocity V_4 can be found by drawing the parallelogram *efgh* with *ef* equal to V, the velocity of the vane.

The absolute entrance velocity V_1 can be resolved into the

two components *ai* and *ic*, at right angles to and along the direction of motion of the vane. The former may be called the velocity of flow, V_f, and the latter the velocity of whirl, V_w. In like manner the absolute exit velocity may be resolved into the components *ek* and *kg*, which may be called the exit velocity of whirl V_w', and the exit velocity of flow, V_f'.

The kinetic energy corresponding to the absolute exit velocity V_4 is the lost or rejected energy of the combination of jet and vane, and for good efficiency should be made small. The exit velocity of whirl in general serves no good purpose and should be made zero to obtain the best results.

The change in the velocity of whirl is the retardation or negative acceleration that determines the driving force or impulse; and the change in the velocity of flow in like manner produces an impulse at right angles to the motion of the vane, which in a turbine is felt as a thrust on the shaft.

Let the angle *acd* which the jet makes with the line of motion of the vane be represented by α, and let β and γ represent the angles *bcd* and *leh* which the tangents at the entrance and exit of the vane make with the same line.

The driving impulse is in general equal to

$$P = \frac{w}{g}(V_w - V_w'); \quad . \quad . \quad . \quad . \quad . \quad . \quad (279)$$

and the thrust is equal to

$$T = \frac{w}{g}(V_f - V_f'). \quad . \quad . \quad . \quad . \quad . \quad . \quad (280)$$

which may be written

$$T = \frac{w}{g}(V_1 \sin \alpha - V_3 \sin \gamma) \quad . \quad . \quad . \quad (281)$$

If there is no velocity of whirl at the exit the impulse becomes

$$P = \frac{w}{g} V_1 \cos \alpha \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (282)$$

The work delivered to the vane per second is

$$W = \frac{w}{g} V V_1 \cos \alpha, \quad . \quad . \quad . \quad . \quad . \quad . \quad (283)$$

and since the kinetic energy of the jet is $wV_1^2 \div 2g$ the efficiency is

$$e = 2\frac{V}{V_1}\cos\alpha \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (284)$$

To find the relations of the angles α, β, and γ, we have from inspection of Fig. 102 in which *el* is equal to *ef*,

$$V_1 \sin\alpha = V_2 \sin\beta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (285)$$

$$V = V_2 \cos\gamma \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (286)$$

$$V = V_1 \cos\alpha - V_2 \cos\beta;$$

from which

$$V_1 \cos\alpha - V_1\frac{\sin\alpha}{\sin\beta}\cos\beta = V_1\frac{\sin\alpha\cos\gamma}{\sin\beta}.$$

$$\therefore \quad \sin\beta\cos\alpha - \cos\beta\sin\alpha = \sin\alpha\cos\gamma$$

and

$$\sin(\beta - \alpha) = \sin\alpha\cos\gamma \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (287)$$

The equations given above may be applied to the computation of forces, work, and efficiency when w pounds of fluid are discharged from one or several nozzles and act on one or a number of vanes; that is, they are directly applicable to any simple impulse turbine.

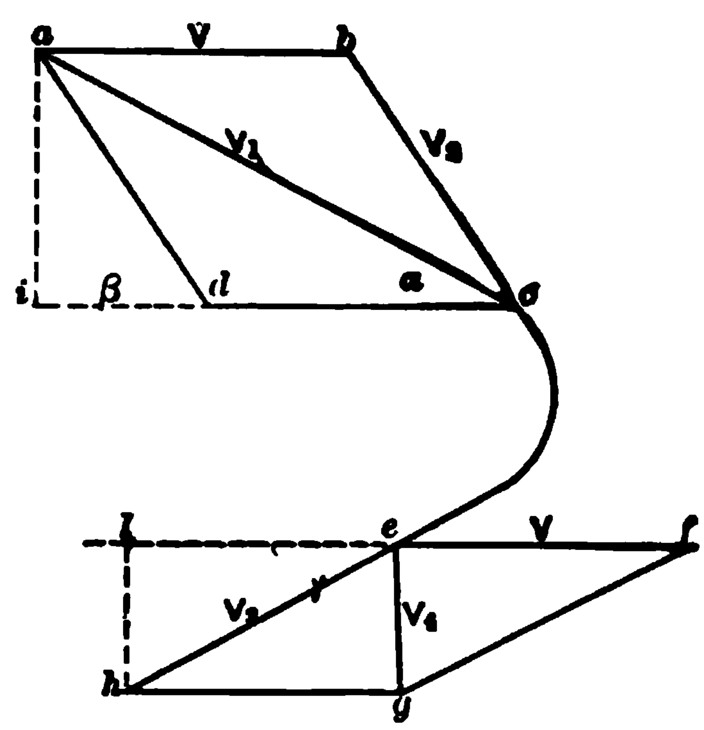

FIG. 102.

Example. Let V_1, the velocity of discharge, be 3500 feet per second as computed for a nozzle on page 444, and let $\alpha = \gamma = 30°$. By equation (287)

$$\sin(\beta - \alpha) = \sin\alpha\cos\gamma = 0.5 \times 0.866 = 0.433.$$

$$\therefore \quad \beta - \alpha = 25 \quad 40'; \; \beta = 55° \; 40'$$

$$V_2 = V_1\frac{\sin\alpha}{\sin\beta} = 3500\frac{0.5}{0.826} = 2120$$

$$V = V_2\cos\gamma = 2120 \times 0.826 = 1835$$

$$e = 2 \times 1835 \times 0.826 \div 3500 = 0.909.$$

No Axial Thrust. — The builders of impulse steam-turbines attribute much importance to avoiding axial thrust, which can be done by making the entrance and exit angles of the vanes equal, provided that friction and other resistances can be neglected. This is evident from equation (280), provided that γ is made equal to β and V_2 equal to V_3, and also that $V_1 \sin \alpha$ is replaced by $V_2 \sin \beta$. Or the same conclusion can be drawn from Fig. 103 because in this case

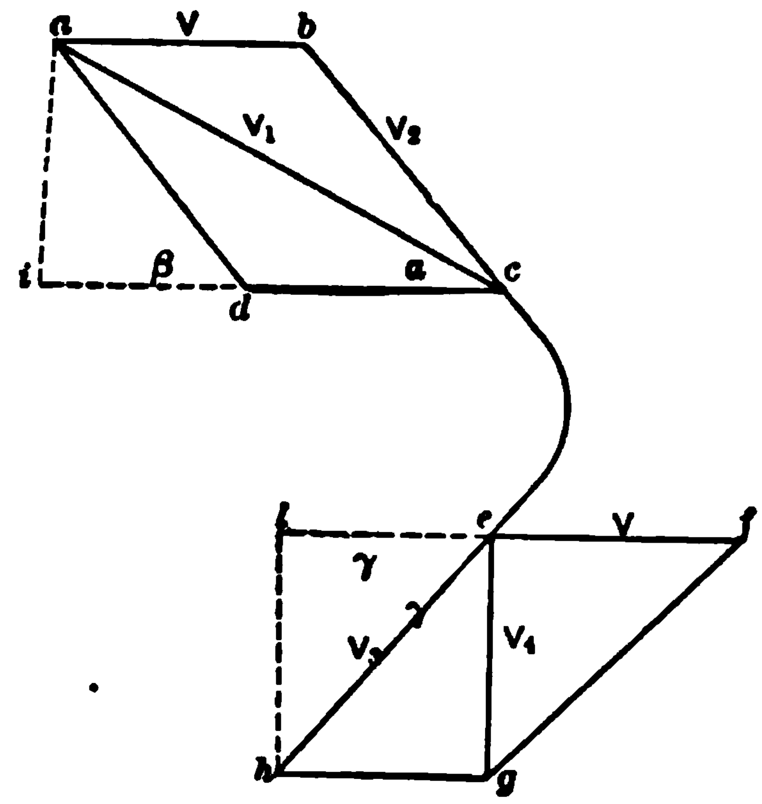

FIG. 103.

$$ai = V_1 \sin \alpha = V_2 \sin \beta = V_3 \sin \gamma = hl,$$

and consequently there is no axial retardation.

The de Laval turbine has only one set of nozzles which expand the steam at once to the back pressure, and consequently the velocity of the vanes is very high and even with small wheels it is difficult to balance them satisfactorily. This difficulty is met by the use of a flexible shaft, and consequently axial thrust is likely to be troublesome; as a matter of fact the turbine is so arranged that the axial force (if there is any) shall be a pull. The importance of avoiding axial thrust in other types of impulse turbines does not appear to be so great, and in some cases axial thrust may be an advantage, for example in marine propulsion.

If γ is made equal to β in equation (287) we have

$$\sin \beta \cos \alpha - \cos \beta \sin \alpha = \sin \alpha \cos \beta$$

$$\therefore \cot \beta = \tfrac{1}{2} \cot \alpha \quad . \quad . \quad . \quad . \quad . \quad (288)$$

and from inspection of Fig. 103 it is evident that V is half of the velocity of whirl or

$$V = \tfrac{1}{2} V_1 \cos \alpha \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (289)$$

If this value is carried into equations (283) and (284) the work and efficiency become

$$W = \frac{1}{2} \frac{w}{g} V_1^2 \cos^2\alpha \quad . \quad . \quad . \quad . \quad . \quad . \quad (290)$$

and

$$e = \cos^2 \alpha. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (291)$$

This freedom from axial thrust appears to be purchased dearly unless the accompanying reduction of velocity of the wheel is to be considered also of importance.

Example. If as in the preceding case the velocity of discharge is 3500 feet per second, and if α is 30°, we have now the following results,

$$\cot\beta = \frac{1}{2} \cot \alpha = \frac{1}{2} \times 1.732 = 0.866 \therefore \beta = 49° \ 10'$$

$$V = \frac{1}{2} V_1 \cos \alpha = \frac{1}{2} \times 3500 \times 0.866 = 1515$$

$$e = \cos^2 30° = 0.75.$$

Effect of Friction. — The direct effect of friction is to reduce the exit velocity from the vane; resistance due to striking the edges of the vanes, splattering, and other irregularities, will reduce the velocity both at entering and leaving. The effect of friction and other resistances is two-fold; the effect is to reduce the efficiency of the wheel by changing kinetic energy into heat, and to reduce the velocity at which the best efficiency will be obtained. There does not appear to be sufficient data to permit of a quantitative treatment of this subject. Small reductions from the speed of maximum efficiency will have but small effect.

The question as to what change shall be made in the exit angle (if any) on account of friction will depend on the relative importance attached to avoiding velocity of whirl and axial thrust. If the latter is considered to be the more important, then γ should be made somewhat larger so that the exit velocity of flow may be equal to the entrance velocity of flow. But if it is desired to make the exit velocity of whirl zero, then γ should be somewhat decreased.

Design of a Simple Impulse Turbine. — The following computation may be taken to illustrate the method of applying the

foregoing discussion to a simple impulse turbine of the de Laval type.

Assume the steam-pressure on the nozzles to be 150 pounds gauge and that there is a vacuum of 26 inches of mercury; required the principal dimension of a turbine to deliver 150 brake horse-power.

The computation on page 444 for a steam-nozzle under these conditions gave for the velocity of the jet, allowing 0.15 for friction, $V_1 = 3500$ feet per second. The throat pressure was taken to be 96 pounds absolute, giving a velocity at the throat of 1480 feet per second. The dryness factor was 0.967 at the throat; at the exit this factor was 0.835 for 0.15 friction and for adiabatic expansion was 0.795.

The thermal efficiency for adiabatic expansion with no allowance for friction or losses whatsoever, as for an ideal non-conducting engine, is given by equation (144), page 136, as

$$e = 1 - \frac{x_2 r_2}{r_1 + q_1 - q_2} = 1 - \frac{811.8}{856.8 + 337.9 - 94.2} = 0.262;$$

the corresponding heat consumption is

$$42.42 \div 0.262 = 162,$$

by the method on page 144.

Let the angle of the nozzle be taken as 30° as on page 481, then the angle β becomes 49° 10′, the efficiency is 0.75 and the velocity of the vanes must be 1515 feet per second.

Suppose that ten per cent be allowed for friction and resistance in the vanes, and that the friction of the bearings and gears is ten per cent; then, remembering that 0.15 was allowed for the friction in the nozzle, and that the efficiency deduced from the velocities is 0.75, the combined efficiency of the turbine should be

$$0.262 \times 0.75 \times 0.85 \times 0.9 \times 0.9 = 0.135;$$

which corresponds to

$$42.42 \div 0.135 = 314 \text{ B.T.U.}$$

per horse-power per minute.

Now it costs to make one pound of steam at 150 pounds by the gauge or 165 pounds absolute, from feed water at 126° F. (2 pounds absolute)

$$r_1 + q_1 - q_2 = 856.8 + 337.9 - 94.2 = 1101 \text{ B.T.U.},$$

consequently 314 B.T.U. per horse-power per minute correspond to

$$314 \times 60 \div 1101 = 17.1$$

pounds of steam per horse-power per hour.

The total steam per hour for 150 horse-power appears to be

$$150 \times 17.1 = 2570.$$

If the nozzle designed on page 444 be taken it appears that five would not be sufficient, as each would deliver only 500 pounds of steam per hour. But if allowance be made for a moderate overload, six could be supplied.

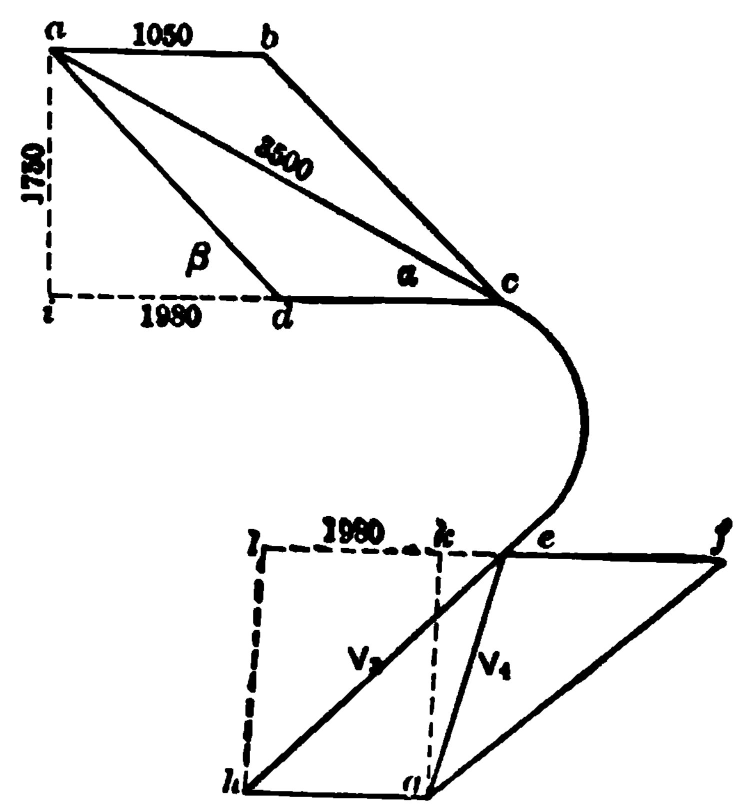

FIG. 104.

Not uncommonly turbines of this type are run under speed as a matter of convenience. Suppose, for example, the speed of the vanes is only 0.35 of the velocity of whirl, instead of 0.5; that is, in this case take $V = 1050$.

This case is represented by Fig. 104, from which it is evident that

$$V_f = V_f' = ai = V_1 \sin 30° = 3500 \times 0.5 = 1750$$

$$V_w = V_1 \cos 30° = 3500 \times 0.866 = 3030$$

$$\tan \beta = ai \div id = 1750 \div (3030 - 1050) = 0.884$$

$$\beta = 41° \, 30'.$$

The two triangles *aid* and *elh* are equal, and

$$le = id = 3030 - 1050 = 1980;$$

consequently the exit velocity of whirl is

$$W_f' = ek = 1050 - 1980 = -930.$$

Consequently the work delivered to the vane is

$$PV = \frac{w}{g}[3030 - (-930)]1050 = \frac{w}{g} 3960 \times 1050$$

$$= 416000 \frac{w}{g}.$$

But the kinetic energy is $wV_1^2 \div 2g$, so that the efficiency is

$$416000 \times 2 \div \overline{3500}^2 = 0.68.$$

The combined efficiency of the turbine therefore becomes

$$0.262 \times 0.68 \times 0.85 \times 0.9 \times 0.9 = 0.123$$

instead of 0.135; and the heat consumption becomes

$$42.42 \div 0.123 = 345 \text{ B.T.U.}$$

per horse-power per minute; and the steam consumption increases to

$$345 \times 60 \div 1099 = 18.8$$

pounds per horse-power per hour. The total steam per hour appears now to be about

$$18.8 \times 150 = 2820,$$

so that six nozzles like that computed on page 444 would give only a margin for governing.

If the turbine be given twelve thousand revolutions per minute the diameter at the middle of the length of the vanes will be

$$D = 1050 \times 12 \times 60 \div (3.14 \times 12000) = 20 \text{ inches.}$$

The computation on page 444 gave for the exit diameter of the nozzle 1.026 inches, and as the angle of inclination to the plane of the wheel is 30°, the width of the jet at that plane would be twice the exit diameter or somewhat more, due to the natural spreading of the jet. The radial length of the vanes may be made somewhat greater than an inch, perhaps $1\frac{1}{16}$ inches. The circumferential space occupied by the six jets will be about

12½ inches out of 62.8 inches (the perimeter), or somewhat less than one-fifth. The section of the nozzle is shown by

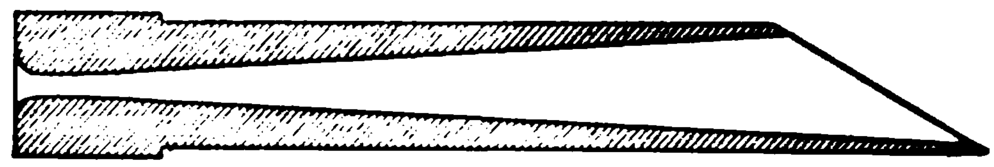

Fig. 105.

Fig. 105, and the form of the vanes may be like Fig. 106. In this case the thickness of a vane is made half the space from one vane to the next, or one-third the pitch from vane to vane. The normal width of the passage is made constant, the face of one vane and the back of the next vane being struck from the same centre. The form and spacing of vanes can be determined by experience only and appears to depend largely on the judgment of the designer. In deciding on the axial width of the vanes it must be borne in mind that increasing that width increases the length and therefore the friction of the passage; but that on the other hand, decreasing the width increases the curvature of the passage which may be equally unfavorable. Sharply curved passages also tend to produce centrifugal action, by which is meant now a tendency to crowd the fluid toward the concave side which tends to raise the pressure there, and decreases it at the convex side. Mr. Alexander Jude,* for a particular case with a steam velocity of 1000 feet per second, computes a change of pressure from 100 to 107.1 pounds on the concave side and a fall to 93.4 on the convex side. Even if this case should appear to be extreme there is no question that sharp curves are to be avoided in designing the steam passages.

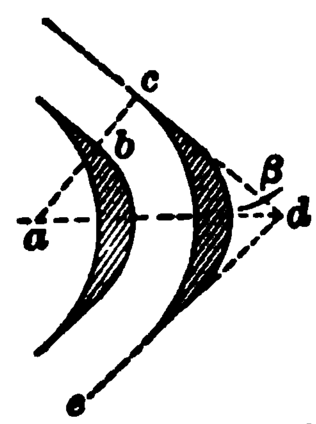

Fig. 106.

Tests on a de Laval Turbine. — The following are results of tests on a de Laval turbine made by Messrs. J. A. McKenna

* *Theory of the Steam Turbine*, p. 49.

and J. W. Regan * and by Messrs. W. W. Ammen and H. A. C. Small.†

	Regan and McKenna.			Ammen and Small.		
Number of nozzles	6	6	6	6	6	3
Boiler pressure gauge	153	154	154.7	148.8	148.8	150.7
Steam chest pressure	140	131.4	111.9	136.9	78.8	140.4
Vacuum, inches	24.3	25.2	25.1	26	26	26.4
Steam per brake, horse-power per hour	19.7	18.0	20.9	19.3	23.2	21.5
B.T.U. per brake horse-power per minute	355	326	379	350	426	374
Velocity of vanes	...	1016	...	1056	1037	...
Velocity of jet	...	3740	...	3470	3770	...
Ratio of velocities	...	0.271	...	0.305	0.275	...
Efficiency of electric generator	0.903	0.900	0.864	0.914	0.885	0.880

Compound Steam-Turbines. — There are three ways in which impulse-turbines have been compounded (1) the steam may be expanded at once to the back-pressure and then allowed to act on a succession of moving and stationary vanes, (2) the steam may flow through a succession of chambers each of which has in it one simple impulse-wheel or (3) a combination of these methods may be made, the steam flowing through a succession of chambers in each of which it acts on a succession of moving and stationary vanes. The first method which gives a very compact but an inefficient-wheel, is used for the backing-turbine of the Curtis marine-turbine. The second method is used in the Rateau turbine, which has usually a large number of chambers. The third method is found in the Curtis turbine which has from two to seven chambers in each of which are from two to four sets of revolving vanes.

The Parsons turbine, which is an impulse-reaction wheel, has a very large number of sets of moving vanes, i.e., from fifty to one hundred and fifty.

The various forms of compound turbines have been devised to reduce the speed of the vanes and the revolutions per minute to convenient conditions without sacrificing the efficiency.

* Thesis, M.I.T. 1903. † Thesis, M.I.T. 1905.

Velocity Compounding. — In Fig. 107, let V_1 represent the velocity of a jet of steam that is expanded in a proper nozzle down to the back-pressure. Suppose it acts on an equal-angled ($\beta = \gamma$) vane which has the velocity V. The relative velocity at entrance to that vane is V_2 and this velocity reversed and drawn at V_3 may represent the exit velocity, neglecting friction. V_4 is the absolute velocity at exit from the vane, which may be reversed by an equal-angled stationary guide, and then becomes the absolute velocity V_1' acting on the next vane. The diagram of velocities for the second moving vane is composed of the lines lettered V_1', V_2', V_3' and V_4'; the last of these is reversed by a stationary guide, and the velocities of the third vane are V_1'', V_2'', V_3'' and V_4''. The diagram is constructed by dividing the velocity of whirl

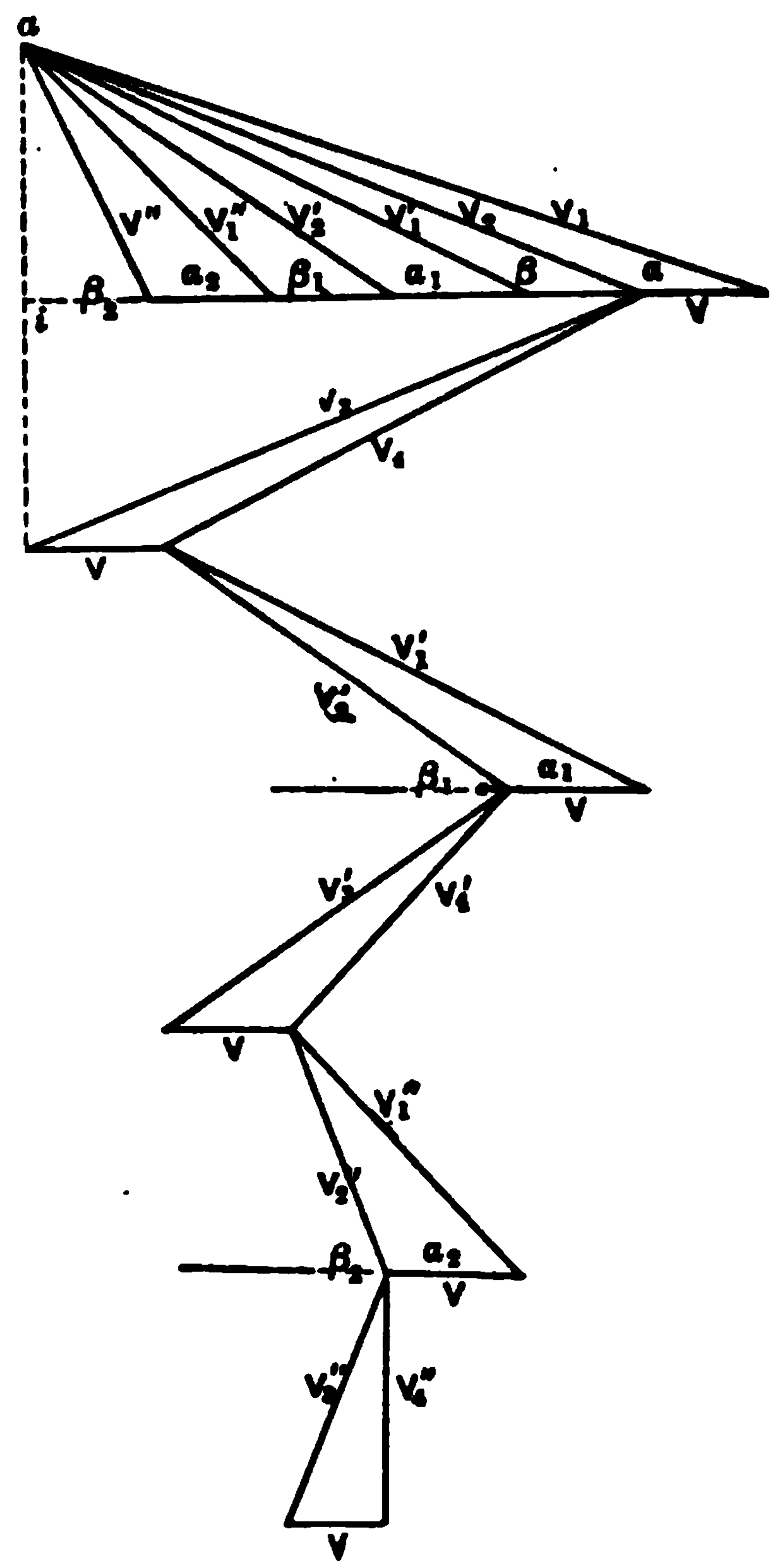

FIG. 107.

$$V_w = V_1 \cos \alpha$$

into six equal parts, and the final exit velocity V_4'' is vertical, indicating that there is no velocity of whirl at that place.

It is immediately evident, since the velocity of flow is unaltered in Fig. 107, and since there is no exit velocity of whirl that the efficiency neglecting friction is the same as for Fig. 103, namely

$$e = \cos^2 \alpha$$

as given by equation (291) page 481.

It is, however, interesting to determine the work done on each vane; the sum of the works of course leads to the same result. In Fig. 107 the velocity of whirl at entrance to the first vane is

$$V_1 \cos \alpha$$

and the velocity of whirl at exit is

$$- V_4 \cos \alpha_1 = - \frac{4}{6} V_1 \cos \alpha;$$

consequently the work done on the vane is

$$\frac{w}{g}\left[V_1 \cos \alpha - \left(-\frac{4}{6} V_1 \cos \alpha\right)\right]\frac{1}{6} V_1 \cos \alpha,$$

because V was made equal to one-sixth of the velocity of whirl. This expression reduces to

$$\frac{10}{36}\frac{w}{g} V_1^2 \cos^2 \alpha.$$

The second and third vanes receive the works

$$\frac{6}{36}\frac{w}{g} V_1^2 \cos^2 \alpha \text{ and } \frac{2}{36}\frac{w}{g} V_1^2 \cos^2 \alpha$$

so that the resultant work is

$$\tfrac{1}{2} \frac{w}{g} V_1^2 \cos^2 \alpha$$

and the efficiency is evidently given by the expression already quoted. The most instructive feature of this discussion is that the relation of the works done on the three vanes is

5, 3, 1.

A similar investigation will show that the distribution among four vanes is

7, 5, 3, 1.

The first figure in such a series is obtained by adding to the number of vanes one less than that number; and each succeeding term is two units smaller. Thus seven vanes give the distribution

13, 11, 9, 7, 5, 3, 1.

It is considered that this type of turbine cannot be made to give good efficiency in practice on account of large losses in passing through a succession of vanes and guides, especially as the steam in the earlier stages has high velocities. The turbine, however, has certain advantages when used as a backing device for a marine-turbine, in that it may be very compact, and can be placed in the low-pressure or exhaust chamber, so that it will experience but little resistance when running idle during the normal forward motion of the ship.

In dealing with this problem it is convenient to transfer the construction to the combined diagram at *abi*, Fig. 107; diagrams for guides like that made up of the velocities V_3, V_4 and V_1 being inverted for that purpose. It is clear that the absolute velocities at exit from the nozzle and the guides are represented by V_1, V_1' and V_1'', while the relative velocities are V_2, V_2' and V_2'' which with no axial thrust are equal to V_3, V_3' and V_3''. The absolute velocity at entrance to a given guide is taken as equal to the absolute velocity at exit from the preceding vane, thus V_1' is equal to V_4, etc. The last absolute velocity V_4'' is equal to *ai* the constant velocity of flow.

The angles α, β, α_1, β_1, α_2 and β_2 are properly indicated as may be seen by comparing the original with the combined diagram.

If the diagram is accurately drawn to a large scale, the velocities and angles can be measured from it, or they may readily be calculated trigonometrically. Thus

$$\tan \beta = \frac{\sin \alpha}{\frac{5}{6} \cos \alpha}; \ \tan \alpha_1 = \frac{\sin \alpha}{\frac{4}{6} \cos \alpha} \text{ etc.,}$$

$$V_2 = V_1 \sin \alpha \operatorname{cosec} \beta; \quad V_1' = V_1 \sin \alpha \operatorname{cosec} \alpha_1, \text{ etc.}$$

The radial length of the vanes and guides must be increased inversely proportional to the velocities, using relative velocities for the vanes and absolute velocities for the guides.

There appears to be no reason why the guides should be relieved from axial thrust provided they can be properly supported.

Except that the passages in the guides might become too long or too sharply curved, they might all be given the same delivery angle as the nozzle, and thus a notable improvement in economy could be realized. In Fig. 108 the velocities V_1, V_2 and V_4, are drawn in the usual manner, V_3 being equal to V_2; the velocity V_4 is laid off along the same line as V_1 and is lettered V_1' and serves as the initial velocity for a new construction as indicated. V_4' is in like manner laid off for V_1'', and thus the diagram is completed. The velocity of the vanes of course remains constant with the value V.

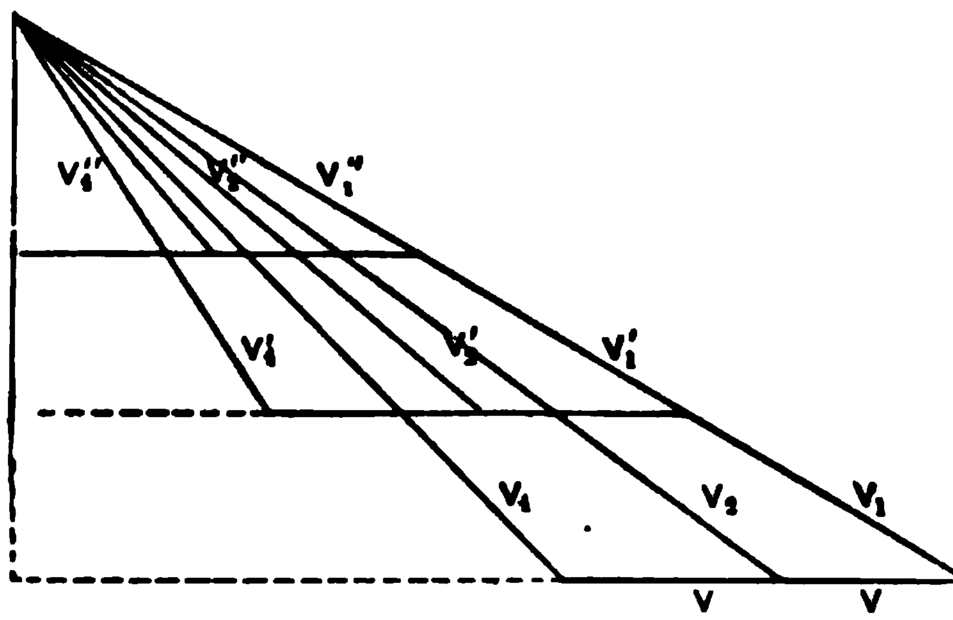

FIG. 108.

Following the problem on page 444 for a nozzle discharging from 150 pounds by the gauge into 26 inches of vacuum we have $V_1 = 3500$ feet per second with $y = 0.15$. The value of V may be taken as 620 feet per second, which gives a diagram with no final velocity of whirl.

The exit velocity of whirl from the first set of vanes is -1830 feet per second as measured on the diagram, and since the initial velocity of whirl is

$$V_1 \cos \alpha = 3500 \times 0.866 = 3030$$

the retardation is

$$3030 - (-1830) = 4860.$$

The retardation for the second set of vanes is

$$2160 - (-880) = 3040,$$

and for the third set is 1320, so that the work of the impulse is

$$(4860 + 3040 + 1320) \times 620 \frac{w}{g} = 5720000 \frac{w}{g},$$

and as the intrinsic energy of the jet is

$$\frac{w}{2g} V_1^2 = \tfrac{1}{2}\, \overline{3500}^2 \frac{w}{g} = 6125000 \frac{w}{g}$$

the efficiency of this arrangement without losses and friction appears to be

$$5720 \div 6125 = 0.92.$$

Effect of Friction. — The effect of friction is to change some of the kinetic energy into heat, thereby reducing the velocity and at the same time drying the steam and increasing the specific volume so that the length of the guides and vanes must be increased at a somewhat larger ratio than would otherwise be required.

A method of allowing for friction is to redraw the diagram of Fig. 107, shortening the lines that represent the velocities to allow for friction.

In order to bring out the method clearly an excessive value will be assigned to the coefficient for friction, namely, $y = 0.19$, so that the equation for velocity may have for its typical form

$$V_0 = \sqrt{2gh\,(1 - y)} = 0.9\sqrt{2gh}.$$

Again the coefficient will be assumed to be constant for sake of simplicity, more especially as but little is known with regard to its real value.

The diagram shown by Fig. 109 was drawn by trial with $V_1 = 3500$ and with $\alpha = 30°$. It appeared necessary to reduce V to 380 feet per second, instead of 505 feet, which would be proper without friction, this latter quantity being one-sixth of the initial velocity of whirl,

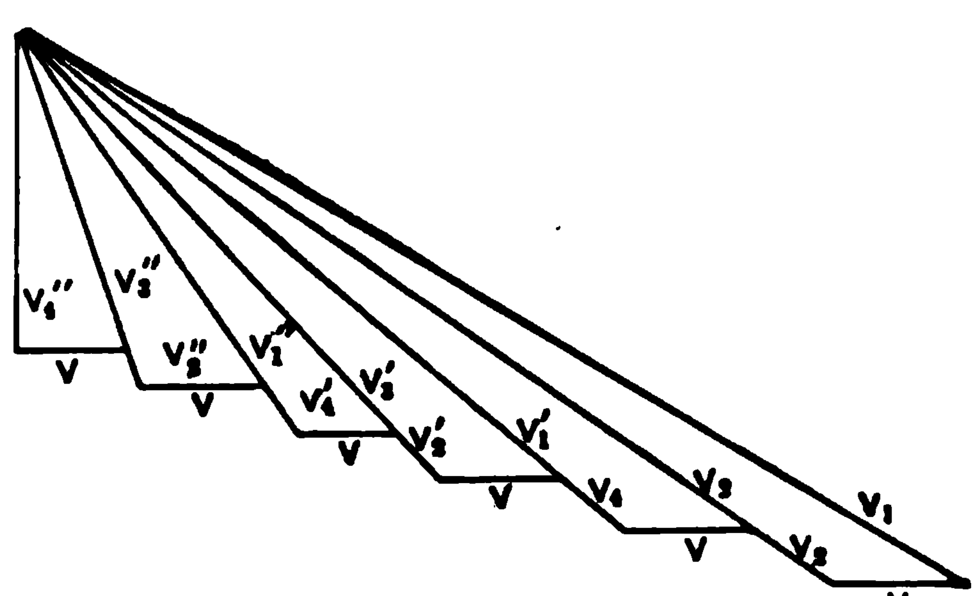

FIG. 109.

$$V_w = V_1 \cos\alpha = 3500 \times 0.866 = 3030.$$

Starting with V_1 the velocity of the jet, the triangle V_1, V, V_2 is drawn to determine the initial relative velocity for the first set of vanes. The exit velocity V_3 is made equal to 0.9 V_2, and the triangle V_3, V, V_4 is drawn to determine the absolute velocity at exit V_4 from the guides. This is taken to be the velocity at entrance to the guides, but the exit velocity from them is taken to be $V_1' = 0.9\ V_4$. Two repetitions of this process complete the diagram. The velocities of whirl at entrance to the three sets of vanes as measured on the diagram are

$$3030 \qquad 1780 \qquad 800,$$

and the velocities of whirl at exit from those vanes are

$$-\ 1890 \qquad -\ 880 \qquad -\ 0,$$

so that the negative accelerations are

$$4920 \qquad 2660 \qquad 800,$$

making a total of 8380. Since the velocity of the vanes is 380 feet per second the work delivered to the turbine is

$$8380 \times 380 \frac{w}{g} = 3180000 \frac{w}{g},$$

and consequently, using the kinetic energy already computed for the jet on the preceding page, the efficiency is

$$3180000 \div 6125000 = 0.52.$$

This method preserves the equality of the angles of the vanes and guides, but does not avoid axial thrust, for Fig. 109 shows a large reduction of the velocity of flow, and as there are no reversals of flow, the reduction is a measure of the impulse producing axial thrust. Nearly half of the thrust is borne by the fixed guides, and it is to be borne in mind that the assumption of an exaggerated coefficient for friction greatly exaggerates this feature, which in practice may not be very troublesome.

To entirely avoid axial thrust it appears to be necessary only to slightly increase the angle γ at the exit from the vane; the angles of the guides may be reduced if desired as an offset.

In Fig. 110 an attempt is made to avoid axial thrust on the vanes, and at the same time to retain a fair efficiency by making the delivery angle of the guides constant.

A calculation like that on page 492 indicates that an efficiency of 0.76 might be expected in this case. It is quite likely that in practice there might be difficulty in making the delivery angle of the guide as small as 30°, but it appears as though the common idea that it is practically impossible to make an economical turbine on this principle is not entirely justified.

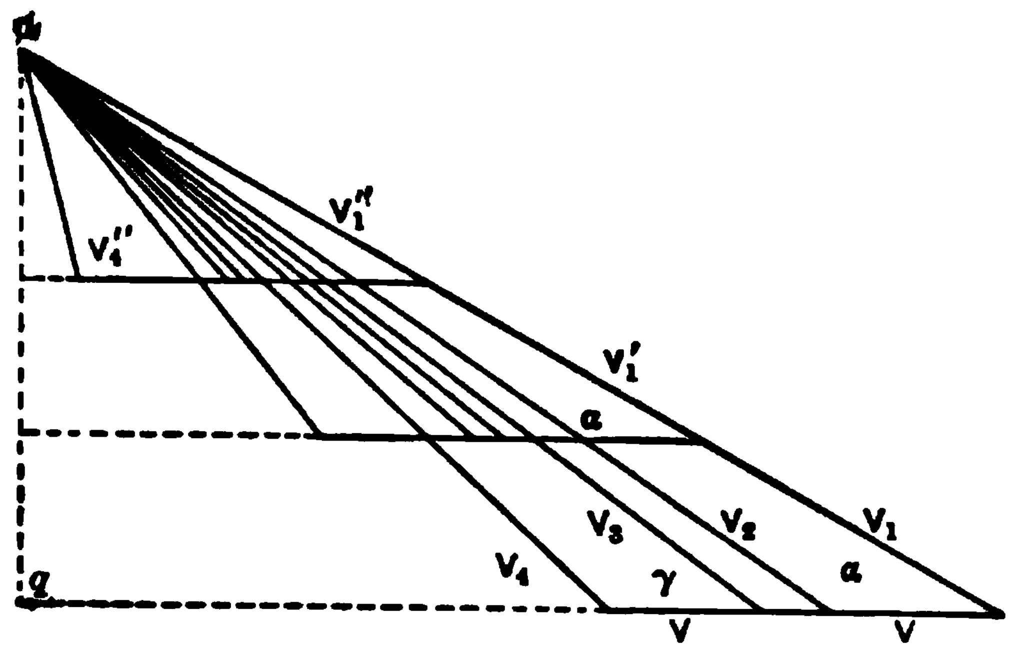

FIG. 110.

Pressure Compounding. — The second method of compounding impulse turbines with a number of chambers each containing a single impulse wheel like that of the de Laval turbine requires a large number of stages to give satisfactory results. For sake of comparison with preceding calculation we will take the same initial and final pressure and the same angle for the nozzles, namely, 150 pounds by the gauge and 26 inches vacuum, and $\alpha = 30°$.

Nine stages in this case will give approximately the same speed of the vanes as in the problem on page 490. The temperature-entropy table which was made for work of this nature is most conveniently used with temperature, and in this case the initial and final temperature can be taken as 366° F. and 126° F. At 366° F. the steam is found to be nearly dry for the entropy 1.56 and that column will be taken for the solution of this problem. The heat contents is 1193.3 instead of 1194.6 as found for 366° F. in Table I of the "Tables of Properties of Steam." On the other hand the table gives at

$126°$ for the heat contents 904.9, and the difference is

$$1193.3 - 904.9 = 288.$$

If we divide the available heat into nine portions we have for each

$$288 \div 9 = 32 \text{ B.T.U.}$$

If again we take $y = 0.1$ which may be excessive in this case since, as will be evident, simple converging nozzles will be required, the velocity of the steam jet will be

$$V_1 = \sqrt{2 \times 32.2 \times 778 \times 32 \times (1 - 0.1)} = 1200$$

feet per second. This is of course the velocity for all the stages.

The choice of $\alpha = 30°$ gives for the velocity of whirl

$$1200 \cos 30° = 1200 \times 0.866 = 1040,$$

and the velocity of the vanes to give the maximum economy is half of this or 520 feet per second or somewhat less if allowance be made for friction and other losses.

Since we have to deal with a single impulse wheel in each chamber and since the wheels are usually designed to avoid axial thrust, all the conclusions concerning that type of wheel may be assumed at once as has already tacitly been done.

One of the important conclusions is that the efficiency without friction as given by equation (291) page 481 is

$$e = \cos^2 \alpha;$$

with $\alpha = 30°$, this gives $e = 0.75$.

It is but fair to say that a smaller angle of α is used for this type of turbine and that the range of temperature is likely to be extended at both limits, and that in particular great importance is attached to securing a good vacuum; 28 inches of mercury, corresponding to one pound absolute, is commonly obtained in good practice with all compound turbines.

If the peripheral speed of the wheel must be kept down, this type of turbine is likely to have a very large number of chambers. For example, if the speed must be no more than 260 feet per second (half of 520), there must be 36 chambers instead of 9.

This will give for the available heat for each chamber 8 thermal units, and using as before $y = 0.1$ we shall have

$$V_1 = \sqrt{2 \times 32.2 \times 778 \times 8 \times 0.9} = 600$$

feet per second. With $\alpha = 30°$ the velocity of whirl is now 520 feet and the velocity of the vanes as stated is 260 feet per second.

The next question in the discussion of this turbine is the distribution of pressure. If the coefficients for friction and other losses are taken to be constant, then the pressure can be approximately determined by the adiabatic method.

In the problem already discussed 32 B.T.U. are assigned to each stage, and if this figure be subtracted nine times in succession from the heat contents 1194 at the initial temperature we shall have the values which may be used in determining the intermediate temperatures from the temperature-entropy table. Also from that table or from Table I in the "Tables of Properties of Steam," the corresponding pressures can be determined. The work is arranged in the following table:

DISTRIBUTION OF PRESSURE.

	Values of $xr+q$.	Temperatures.	Pressures absolute.	Ratios of pressures.
0	1193	366	165	0.68
1	1161	336	112	0.66
2	1129	306	73.2	0.65
3	1097	278	47.6	0.64
4	1065	251	30.4	0.61
5	1033	224	18.6	0.61
6	1001	199	11.3	0.58
7	969	174	6.56	0.57
8	937	150	3.72	0.53
9	905	126	1.99	0.

The last column gives the ratio of any given pressure to the preceding pressure, i.e. $112 : 165 = 0.68$. These ratios indicate that simple conical converging nozzles will be sufficient for all but the last stage. With the usual number of stages, twenty or more, the ratios are certain to be larger than 0.6 in all cases, indicating the use of converging nozzles throughout.

To determine the sizes of the nozzles or the passages in the guides it is necessary to estimate the quality of the steam in order to find the specific volume. To do this we may consider that, of the heat supplied to a certain stage of the turbine, a portion is changed into work on the turbine vanes, some part is radiated, and the remainder is in the steam that flows from the chamber of that stage; if there is appreciable leakage, special account must be taken of it, but both radiation and leakage can be left at one side for the present.

Now in the case under consideration, 32 thermal units were assigned to each stage in the adiabatic calculation for the distribution of pressure. But 0.10 part was assigned to y to allow for friction so that only 0.9 was applied to the calculation of velocity; of the kinetic energy of the jet 0.75 only was assumed to be applied to moving the vanes without friction, the remainder being in the kinetic energy of the flow from the vanes which was assumed to be changed into heat again; and further there was an allowance of 0.1 for losses in the vanes, leaving a factor, 0.9, to be applied for that action. Consequently instead of 32 thermal units changed into work per stage, our calculation gives only

$$32 \times 0.9 \times 0.75 \times 0.9 = 19.44 \text{ B.T.U.}$$

will be changed into work. A method of determining the qualities and specific volumes at the several nozzles is illustrated in the table on the following page.

The quantity of heat changed into work per stage is subtracted successively, giving the apparent remaining heat contents as set down in the tables. At a given temperature we may find the quality by subtracting the heat of the liquid from the heat contents and dividing the remainder by the value of r. The specific volumes are determined by the equation

$$v = xu + \sigma,$$

but as x is in all cases large, the effect of σ may be neglected altogether.

FIRST COMPUTATION OF QUALITIES AND VOLUMES.

	Temperature (t)	Heat contents ($xr+q$)	Heat of liquid (q)	Value of xr	Heat of vaporization (r)	Quality (x)	Specific volumes (sx)	
0	366	1193	338	855	857	0.998	2.75	2.74
1	336	1174	307	867	881	0.984	3.99	3.93
2	306	1154	276	878	904	0.971	5.94	5.77
3	278	1135	247	888	925	0.960	8.90	8.54
4	251	1115	220	895	944	0.948	13.6	12.9
5	224	1096	192	904	962	0.940	21.5	20.2
6	199	1076	167	909	978	0.930	34.3	31.9
7	174	1057	142	915	993	0.922	56.9	52.5
8	150	1037	118	919	1007	0.913	96.9	88.5
9	126	1018	94	924	1021	0.905	174	157.4

By the aid of the temperature-entropy table, the qualities and specific volumes may be determined directly with good approximation, it being necessary only to follow the line of the temperature to an entropy column, having nearly the proper heat contents.

There is a serious objection to the adiabatic method, because it does not take any account of the fact that as the steam passes from stage to stage losing less heat than it would with adiabatic action, the entropy increases, and that with increased entropy the difference of heat contents between two given temperatures increases. This will be very apparent from inspection of a temperature-entropy diagram or the temperature-entropy table. This matter will be discussed more at length in connection with the Curtis type of turbine.

It has been assumed that the same amount of heat should be assigned to each stage for the adiabatic calculation and that the values of y to allow for friction and losses remain constant. As to the values that should be assigned to y, we have very little published information; it may be noted in passing that our allowance for friction in the nozzles and guides is probably too large. It will be evident that there is no difficulty in maintaining the amount assigned to each stage in its proper proportion even

though y shall be varied from stage to stage. For example, our choice of 0.1 for both y and y_1 gives

$$32 \times 0.9 \times 0.9 = 25.92 \text{ B.T.U.},$$

which multiplied by 0.75, the efficiency due to the angles and velocities, gives 19.44 B.T.U. as above. Let it be assumed for the moment that the above product shall be kept constant, so as to obtain the same velocity of jet in each stage. Then the following table exhibits a way of accomplishing this purpose while varying y and y_1:

Stage	1	2	3	4	5	6	7	8	9
y	0.08	0.085	0.09	0.095	0.10	0.105	0.11	0.115	0.12
y_1	0.088	0.091	0.094	0.097	0.10	0.103	0.106	0.109	0.112
$(1-y)(1-y_1)$	0.839	0.832	0.824	0.817	0.81	0.803	0.796	0.787	0.781
B.T.U.	30.9	31.2	31.5	31.7	32	32.3	32.6	33.0	33.2

The last line shows the proper assignment of thermal units for this condition. For simplicity both y and y_1 are assumed to vary uniformly, but other variations can be worked out with a little more trouble. Evidently the sum of the figures in the last line should be equal to

$$9 \times 32 = 288;$$

it is a trifle larger in the table.

Now it is probable that the best values of the factor for friction and resistance are to be derived from investigations on turbines rather than from separate experiments on nozzles and vanes, and it is evident that the use of the methods of representing the friction by a factor y is rather a crude way of trying to attain in a new design favorable conditions found in a turbine already built.

Since the general conditions of this problem are the same as those on page 481, the efficiency due to adiabatic action will be the same as is also the efficiency due to the angles and velocities. Taking the factors for friction in the guides and blades as each

0.1, the corresponding factors are 0.9 and 0.9. The efficiency due to velocities is 0.75, and the mechanical efficiency may be estimated as 0.9. The combined efficiency of the turbine is

$$0.262 \times 0.75 \times 0.9 \times 0.9 \times 0.9 = 0.143.$$

A computation like that on page 483 with this efficiency gives for the probable steam consumption 16.2 pounds per brake horse-power per hour.

Assume that the turbine is to deliver 500 brake horse-power; then the steam consumption per second will be

$$16.2 \times 500 \div 3600 = 2.25 \text{ pounds.}$$

We can now determine the principal dimensions of the turbine to suit the conditions of its use. Suppose that it is desired to restrict the revolutions to 1200 per minute or 20 per second; then with nine stages and a peripheral velocity of 520 for the vanes the diameter will be

$$520 \div 20\pi = 8.28 \text{ feet.}$$

For a turbine of the power assigned this diameter will be found to be inconveniently large. If, however, the number of stages can be made 36, the velocity will be reduced to 260 feet per second as computed on page 495. This will give for the diameter

$$260 \div 20\pi = 4.14 \text{ feet.}$$

The remainder of our calculation will be carried out on these assumptions, namely, that the power is to be 500 brake-horse-power, and that there are to be 36 stages. If the method of the table on page 497 were applied to a turbine having the full 36 stages now contemplated, it would have 37 lines; namely, the ten already set down, and three intermediate entries between each pair of consecutive lines; but the temperatures found in that table would be found in the more extended table together with their specific volumes. We can, therefore, use that table to calculate areas and lengths of vanes for 9 out of the 36 stages,

which will suffice for illustration. Beginning with the lowest stage the area to be supplied will be

$$2.25 \times 157.4 + 600 = 0.590 \text{ square feet;}$$

where 600 is the velocity of the jet computed on page 495.

The circumference of a circle having the diameter of 4.14 feet is 13 feet; but of this a portion, one-fourth or one-third, must be assigned to the thickness of the guides. If we take one-fourth in this case the effective perimeter becomes 9.75 feet. But as is evident from Fig. 111 the peripheral space assigned to the distance between guides must be multiplied by sin α in order to find the effective opening. As α is taken to be 30°, the sine is one-half, so that the total width of spaces between guides is reduced to 4.88 feet. The radial length of the guides for the last stage will consequently be

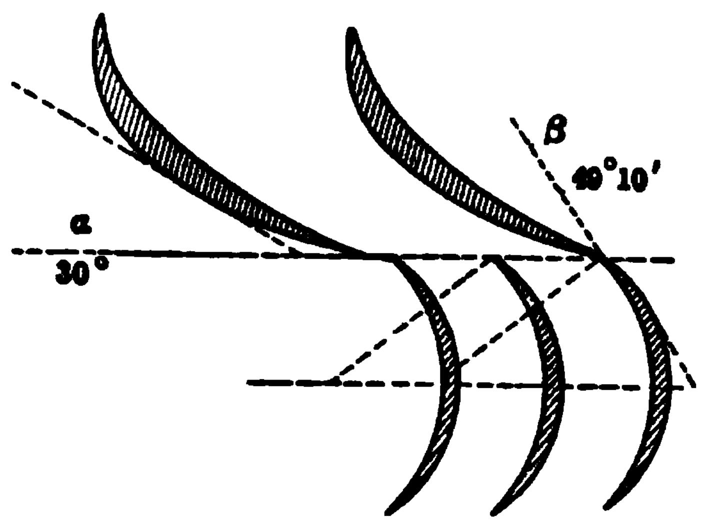

FIG. 111.

$$0.590 \div 4.88 = 0.121 \text{ of a foot} = 1.45 \text{ inches,}$$

provided that there is full peripheral admission to the guides.

Now the angles for this case are the same as those on page 481 and β is 49° 10′. Consequently the relative velocity is

$$V_2 = V_1 \sin \alpha \div \sin \beta = 600 \sin 30° \div \sin 49° \, 10' = 397 \text{ feet.}$$

If the passages between the vanes are made of constant width, as shown in Fig. 111, the effective perimeter will be the entire perimeter of the wheel less the allowance for thickness. An allowance like that for the guides will make the vanes shorter than the guides in this case. Let us try making the thickness equal to a space; then the effective perimeter will be 6.5 feet. If the density of the steam is assumed to be constant for a given stage. then the lengths of the guides and vanes will be inversely as the product of the velocities by the effective perimeters, so that the length of the vane will be

$$1.45 \times \frac{600 \times 4.88}{397 \times 6.5} = 1.65 \text{ inches.}$$

Conversely, if desired, the thickness of the vanes could be adjusted to give the same length. Such a construction as this leads to is likely to give too sharp a curvature to the backs of the vanes, and it may be better to give only the thickness demanded for strength and take the chance that the passage between the vanes shall not be filled. If allowance is made for friction and the consequent reduction in velocity the lengths of the vanes should be correspondingly increased.

The lengths of the guides for the other stages will be directly proportional to the specific volumes in the table on page 497, because the velocities have been made the same for all the stages. For example, at 199° the length for full admission will be

$$1.45 \times 31.9 \div 157 = 0.295 \text{ inch,}$$

which will be the proper length for the twenty-fourth stage. If it is considered undesirable to further reduce the length we may resort to admitting steam through guides for only a portion of the periphery. Making the arc of admission vary as the specific volumes, the fourth stage (line 1 of the table on page 497) will have admission for

$$360 \times 3.93 \div 31.9 = 44°.$$

Intermediate lengths of vanes and arcs of admission may be computed by filling out a table like that on page 497 for all the stages, or a diagram may be drawn from which the required information can be had by interpolation; the values on the line numbered 0 are for this purpose, there being of course no corresponding stage. In fact the method of computing at convenient intervals and interpolating from curves is likely to be more accurate as well as more convenient, as the error of adiabatic calculations for steam with small change of temperature is liable to be excessive.

Leakage and Radiation. — This type of turbine, as will be seen in the description of the Rateau turbine, has a number of wheels each in its own chamber, and the chambers are separated by stationary disks that extend to the shaft. Reduction of leakage must be attained by a small clearance between the disk and the

shaft for a proper bearing or stuffing box cannot be placed in so inaccessible a place. The leakage can be estimated by aid of Rankine's equation on page 432 or from Rateau's experiments on page 433; but both methods are likely to give results that are too large, and a factor less than unity should be applied; but the value of such a factor for a long, narrow, annular passage is not known, and any estimate must be crude. For a turbine of the Rateau type the leakage is likely to be less than five per cent at the high pressure end. Now the leakage is proportional nearly to the difference of pressure between successive chambers, and as the difference decreases so also does the leakage till it becomes of no account at the lower end. To allow for leakage the length of guides or the arc of admission may be increased at the high pressure end of the turbine. There does not appear to be any information concerning the radiation from steam-turbines. On the one hand the area of radiating surface is larger than for steam-engines and on the other the temperatures are less for the greater part if not all of that area. For compact steam-engines the radiation is likely to be from five to ten per cent. For turbines of the Rateau and Curtis types the effect of radiation is to require larger areas in guides and passages at the high pressure end.

Lead. — Turbines with pressure-compounding usually have some space between the vanes of one wheel and the next set of guides or nozzles, and consequently the absolute exit velocity is mainly if not entirely dissipated, so that the steam enters those nozzles with no appreciable velocity. If this action is complete it would appear to be of little consequence where the guides or nozzles are placed. Nevertheless considerable importance is attached to locating the guides so that steam from the wheel shall flow directly into them. Clearly, as it takes an appreciable time to flow through the passages between the vanes, the steam will be discharged at some distance from the place at which it was received and the general path of the steam is a spiral wound around the turbine case in the direction of rotation.

Let *abcde* represent a vane which has steam entering it tangentially with the velocity V_2, while it has itself the velocity V. Assuming that the relative velocity is constant we may divide the curve into a number of equal small parts that are approximately straight. From b lay off

$$bb' = ab\,\frac{V}{V_2},$$

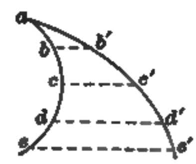

Fig. 112.

then b' will be a point in the trajectory of the particle of steam. In like manner

$$cc' = 2ab\,\frac{V}{V_2}\,,\ \text{etc.}$$

The path $ab'c'd'e'$ may be taken as the trajectory of the steam, and ee' is the lead as defined above. Properly a similar construction should be made also for the back of the vane, and the mean path should be taken to establish the lead. Extreme refinement is probably neither necessary nor justifiable in this work.

Rateau Turbine. — The construction of this turbine, which is of the pure pressure-compound type is represented by Fig. 113, which is a half section through the shaft, wheels and casing. The wheels are light dished plates which are secured to hubs that are pressed onto the shaft and which carry the moving vanes. The chambers are separated by diaphragms of plate steel, riveted to a rim and to a hub casting. The hubs are bushed with anti-friction metal that is expected to wear away if it by chance touches the shaft. This turbine is sometimes divided into two sections to provide a middle bearing for the shaft, which has considerable length and should preferably have a small

Casing

Fig. 113.

diameter to reduce leakage. The high pressure portion may have a smaller diameter to facilitate arrangement of guides and vanes. Sometimes there are three diameters for the same purpose. But little extra complication of computation is introduced by such change of diameter; all that is necessary is to make the portion of available heat per stage larger in proportion to the increase in peripheral speed.

TESTS ON RATEAU TURBINE.

Dr. A. Stodola.*

Duration minutes	40	50	35	180	30
Revolutions per minute	2184	2181	2190	2101	2200
Steam pressure at stop valve absolute pounds	176.1	175.1	170.5	168.4	181.1
Superheating degrees F.	5.2	9	14.8	20	14.6
Steam pressure at first guides	44.7	63.9	95.4	119.9	143.7
Superheating degrees F.	32.6	32.2	20.9	18.9	17.1
Absolute exhaust pressure	1.29	1.33	1.51	1.64	1.89
Effective power	172	257	417	531	634
Steam per horse-power per hour, exclusion of air-pump	19.0	17.6	15.7	15.6	15.2

The accompanying table gives results of tests on a Rateau turbine by Professor Stodola. To compare with results from steam-engines, these latter should be referred to brake horsepower, with a mechanical efficiency of 0.85 to 0.90.

This type of turbine has been applied successfully to use exhaust steam from reciprocating engines which for some purpose exhaust at atmospheric pressure or into a poor vacuum. Such application can, however, be but local or accidental.

Steam Friction of Rotating Disks. — The resistance which a turbine wheel experiences while rotating in steam can be divided into two parts: first, that due to the friction of the smooth disk, and second, that due to the action of the vanes, which have an effect comparable to that of a centrifugal pump.

From a consideration of tests made by Odell † on cardboard disks, and by Lewecki ‡ on a de Laval turbine wheel driven in

* *Steam Turbines*, trans. Dr. L. C. Löwenstein.

† *Engineering*, January, 1904. ‡ *Zeitschr. d. V. deutsch Ing.*, 1903.

its casing, and from tests of his own, Professor Stodola gives the following equations for the horse-power required to drive smooth wheels and to drive wheels with vanes forward:

Smooth wheels

$$\text{H.P.} = 0.02295\ \alpha_1\ D^{2.5}\left(\frac{V}{100}\right)^3\gamma.$$

Wheels with vanes

$$\text{H.P.} = [0.02295\ \alpha_1\ D^{2.5} + 1.4346\ \alpha_2\ L^{1.25}]\left(\frac{V}{100}\right)^3\gamma$$

where D is the diameter in feet, L is the blade length in inches, V is the peripheral speed in feet per second, and γ is the density of the medium. The values of the other factors are

$$\alpha_1 = 3.14 \qquad \alpha_2 = 0.42.$$

These formulæ explain why the backing turbine for marine propulsion is always run in a vacuum when idle.

Turbines which have only a partial admission must be affected by some such action for that part of the revolution during which steam is not admitted; but this matter is obscure and such a resistance must be combined with friction and other resistances. It is therefore very difficult to assign the proper value to the friction factor y for steam in the vanes or in the guides and vanes of a velocity-compound turbine. In particular any change of the angle γ (Fig. 103, page 480) to avoid end thrust must be made with caution and should be checked by experiment.

Side Thrust. — If admission is restricted to only a part of the periphery of a turbine, then in order to preserve a balance and avoid unnecessary pressure on the bearings of the shaft, the arc of admission should be divided into two equal portions, that are diametrically opposite. Some builders, however, prefer to ignore this effect, and concentrate the admission at one side, because there is tendency for the steam to spread which will have double the effect if the arc is divided as suggested. The amount of side thrust can be estimated from the powers developed at the several wheels, having partial admission, together with the dimensions and speed of revolutions, making allowance of course for the distribution of the torque over an arc of a circle.

Pressure and Velocity Compounding. — A favorable combination may be made of the two methods of compounding already discussed; that is, the pressure and temperature range may be divided between two or more chambers in each of which shall be two or three sets of moving vanes. This has been done on a large scale with the Curtis turbine which appears to have a wider range of economical application than any other type.

Since the principles of each method have been discussed already, we will illustrate the application to a comparatively simple problem avoiding too great minutiæ of detail.

Let us take for the principle conditions the delivery of 500 kilowatts of electrical energy, which, with an efficiency of the dynamo of about 0.87, will correspond to nearly 770 brake horse-power.

Let the initial pressure be 150 pounds by the gauge, and the vacuum be 28 inches of mercury. Let the angle of the nozzles be $\alpha = 20°$. The absolute pressures will be about 165 pounds and one pound absolute, and the compounding temperatures are 366° and 102° F. Dry saturated steam at the given pressure will have nearly 1.56 units of entropy, and for this the temperature-entropy table gives for adiabatic expansion with the above limits of temperature the heat contents as 1193 and 871. The value of q_2 is 70 at the lower temperature, and consequently $x_2 r_2$ is equal to 801 B.T.U.

The thermal efficiency of adiabatic expansion without allowing for any losses is

$$e = 1 - \frac{x_2 r_2}{r_1 + q_1 - q_2} = 1 - \frac{801}{1125} = 0.288;$$

the corresponding heat consumption is

$$42.42 \div 0.288 = 147 \text{ B.T.U.}$$

per horse-power per minute.

The efficiency for the turbine without friction by equation (291), page 481 is

$$e = \cos^2 \alpha = 0.883.$$

The efficiency of the nozzles has already been determined to be 0.85 by the selection of 0.15 for y. Let us further assume that

the combined effect of losses in the vanes may be taken to be equivalent to making y_0 equal to 25 so that $1 - y_0$ is 0.75; this is in effect the efficiency factor for the vanes as affected by friction. If, further, we take the mechanical efficiency of the machine as 0.9, then the combined efficiency for the turbine will be

$$0.288 \times (0.883 \times 0.85 \times 0.75) \times 0.9 = 0.144.$$

This corresponds to

$$42.42 \div 0.144 = 295 \text{ B.T.U.}$$

per horse-power per minute. Now it costs to make steam from water at 102°, and at an absolute pressure of 165 pounds, 1125 $(r_1 + q_1 - q_2)$ thermal units, as already calculated in the deduction of the efficiency of adiabatic action. Consequently the steam per horse-power per hour will be

$$295 \times 60 \div 1125 = 15.7$$

pounds per brake horse-power per hour. To this should properly be added a fraction, to allow for leakage and radiation, amounting to five or ten per cent; this added amount of steam will affect the size of the high pressure nozzles only in this case, and as extra nozzles are sure to be provided we will take no further account of it than to say that the steam consumption may amount to 16.5 to 17.3 pounds per brake horse-power per hour.

The heat contents which have already been found give for the adiabatic available heat

$$1193 - 871 = 322,$$

and if this be divided equally we have 161 thermal units per stage. Using 0.15 for y in the nozzles, the velocity of the jet becomes

$$V = \sqrt{2 \times 32.2 \times 778 \times 161 \times 0.85} = 2610$$

feet per second.

Assuming that we may use three sets of moving vanes the velocity for them will be

$$2610 \div (2 \times 3) = 435$$

feet per second.

If we choose a diameter of $4\frac{1}{2}$ feet for the pitch surface of the vanes it will lead to the use of 1850 revolutions per minute.

To find the intermediate pressure we may take for the heat contents at that pressure

$$1193 - 161 = 1032,$$

which in the temperature-entropy table corresponds to 223° F., or 18.2 pounds. Since the back-pressure for the nozzles is relatively small in each case, the nozzles will have throats for which the velocities must be determined in order to find the areas. The throat pressures may be taken to be

$$165 \times 0.58 = 95.7; \quad 18.2 \times 0.58 = 10.6,$$

and the corresponding temperatures are 325° and 196° F.

Since the rounding of the nozzle is likely to give but small area for friction compared with the cone for expanding to the back-pressure, we may assume adiabatic expansion to the throat and allow the entire value of $y = 0.15$ for the computation for the exit. This appears to agree with tests showing that such nozzles give nearly full theoretical discharge. The heat contents by the temperature-entropy table at entropy 1.56 and 325° F. amounts to 1150 B.T.U., the value of x is 0.960 and the specific volume is 4.41 cubic feet. The apparent available heat is

$$1193 - 1150 = 43 \text{ B.T.U.}$$

giving a throat velocity of

$$V = \sqrt{2 \times 32.2 \times 778 \times 43} = 1470.$$

The apparent available heat for producing velocity at the exit with y taken at 0.15 is

$$0.85 \times 161 = 137 \text{ B.T.U.},$$

leaving for the heat contents

$$1193 - 137 = 1056 \text{ B.T.U.}$$

The heat of the liquid is 191 so that with 963 for r we have

$$x' = x'r' \div r' = (1056 - 191) \div 963 = 0.898.$$

The specific volume is

$$v = (xu + \sigma) = 0.898\ (21.9 - 0.016) + 0.016 = 19.7.$$

With 15.7 pounds of steam per brake horse-power per hour and 770 horse-power the steam per second is

$$w = 15.7 \times 770 \div 3600 = 3.36 \text{ pounds.}$$

The combined area of discharge of all the first stage nozzles is therefore, with the velocity at exit equal to 2610 feet,

$$3.36 \times 19.7 \times 144 \div 2610 = 3.65 \text{ square inches.}$$

The nozzles of turbines of this type are sometimes made square at the exit so as to give a continuous sheet of steam to act on the vanes. If the side of such a nozzle were made half an inch there would appear to be fourteen and a half such nozzles; the turbines would probably be given 16 or 18 of them, which could be arranged in two groups. Since the angle of the nozzle is 20° the width of the jet measured along the perimeter of the wheel will be

$$0.5 \div \sin 20° = 0.5 \div 0.3420 = 1.46 \text{ inch.}$$

Allowing one-fourth of the width of the orifice for the thickness of the walls, the width occupied by eight nozzles would be

$$1.46 \times 1.25 \times 8 = 14\tfrac{1}{2} \text{ inches.}$$

The combined throat area of all the nozzles will be

$$3.36 \times 4.41 \times 144 \div 1470 = 1.45 \text{ square inch.}$$

Dividing by 14½, the number of necessary nozzles, gives for the throat area of one nozzle

$$1.45 \div 14.5 = 0.1000 \text{ square inch,}$$

so that the diameter will be about 0.36 of an inch.

A method of calculation for the second set of nozzles consistent with the method of determining the intermediate pressure is as follows: The pressure in the throat has already been found to be 10.6 pounds, corresponding to 196° F., for which the temperature-entropy table at 1.56 units of entropy gives for heat

contents 998. The heat contents at 18.2 pounds (223°) has already been found to be 1032, so that the available heat for adiabatic flow appears to be 34 B.T.U., which gives for the velocity in the throat

$$V = \sqrt{2 \times 32.2 \times 778 \times 34} = 1300 \text{ feet.}$$

The next step is the determination of the qualities at the throat and exit, and from them the specific volumes. Now of the 161 B.T.U. available for adiabatic flow in the first nozzles only a part has actually been changed into work, because there was allowed 0.15 for friction in the nozzle, and 0.25 for losses in the guides and vanes, while the efficiency due to angles and velocities was 0.883. The heat changed into work was therefore

$$161 \times 0.85 \times 0.75 \times 0.883 = 90.6 \text{ B.T.U.}$$

Consequently the heat left in the steam as it approaches the second nozzle is

$$1193 - 91 = 1102 \text{ B.T.U.}$$

per pound. Now r has the value 963 at 223° F., and q is 191, so that the quality is

$$x = (1102 - 191) \div 963 = 0.946.$$

In the flow from the entrance to the throat 34 B.T.U. are assumed to be changed into kinetic energy leaving for

$$xr + q = 1102 - 34 = 1068,$$

and as r is equal to 980 and q is 164 at 196° F., we have

$$x = (1068 - 164) \div 980 = 0.923$$

at the throat of the second nozzle.

Allowing as before 0.15 for the friction of the nozzle there will be

$$0.85 \times 161 = 137 \text{ B.T.U.}$$

changed into kinetic energy for the entire nozzle leaving

$$xr + q = 1102 - 137 = 965 \text{ B.T.U.};$$

and at 1 pound or 102° F., the values of r and q are 1035 and 70

$$x = (965 - 70) \div 1035 = 0.865.$$

at exit from the second set of nozzles. The volume of saturated steam at 102° is 332 cubic feet, and with x equal to 0.865 the specific volume is 287 cubic feet. Consequently, with a weight of 3.36 pounds per second, and a velocity of 2610 feet, the united areas of all the nozzles at exit will be

$$3.36 \times 287 \times 144 \div 2610 = 53.2 \text{ square inches.}$$

Now the perimeter of a circle having a diameter of $4\frac{1}{2}$ feet is about 170 inches. Allowing for the sine of the angle 20° and one-fourth for thickness of guides there will be about 43.5 inches for the united width of passages between guides so that the radial length will be

$$53.2 \div 43.5 = 1.22 \text{ inch.}$$

The specific volume of saturated steam at 197° is 35.6 cubic feet, so that with x equal to 0.923 the specific volume is 32.9. Now the areas are proportional to the specific volumes and inversely as the velocities, consequently the length of guides at the throat is

$$1.22 \times \frac{2610}{1300} \times \frac{32.9}{287} = 0.28 \text{ inch.}$$

The length of the vanes and guides can be found by the method on page 500, using relative velocities for the vanes and absolute velocities for the guides. The velocities decrease as indicated by Fig. 107, page 487, and the lengths must be correspondingly increased. In this case, however, there are two considerations which influence the lengths that should be finally assigned to the guides and vanes. (1) The thickness may be diminished, which tends to decrease the length. (2) Friction reduces the velocity which tends to increase the length. Friction of course diminishes all velocities including the peripheral velocity of the wheel, but a proper discussion of that matter would be both long and uncertain.

Attention has already been called to the defect of this method of making all the calculations at a single value of entropy and trying to allow for friction and other losses by simple factors. The difficulty is aggravated in this case by the fact that the

second set of nozzles or guides have proper throats. The proper method after having selected a set of intermediate pressures appears to be to calculate the turbine step by step. The steam supplied to the second set of nozzles (or guides) has been found to have the quality 0.946, and this is probably a good approximation to the actual condition, even if allowance is made for radiation and leakage. The temperature-entropy table gives for steam having that quality and the temperature 223, the entropy as nearly 1.66. At that entropy the heat contents at the initial, throat and exit pressures, are given in the following table with also the quality and specific volume at the throat; the table also gives the quality and specific volumes at exit with y equal to 0.15.

Pressure.	Temperature.	Heat contents.	Quality.	Specific volume.
18.2	223	1100	0.94	...
10.6	196	1063	0.92	33.3
1.0	102	927	0.86	286

The apparent available heat for adiabatic flow to the throat is now

$$1100 - 1063 = 37,$$

which would give a velocity of

$$V = \sqrt{2 \times 32.2 \times 778 \times 37} = 1360,$$

instead of 1280 as previously found. The apparent available heat to the exit with 0.15 for the friction factor is now

$$(1100 - 927)\, 0.85 = 147,$$

which gives for the exit velocity

$$V = \sqrt{2 \times 32.2 \times 778 \times 147} = 2710,$$

instead of 2610 previously computed.

This comparison shows that the intermediate pressure determined by the customary method will be too high, and that to obtain the desired distribution of temperature the factors for

the lower stages must be modified arbitrarily as may be determined by comparison with practice.

Curtis Turbine. — Fig. 114 shows a partial elevation and section of the essential features of a Curtis turbine, which has four chambers and two sets of moving vanes in each chamber. The axis of the turbine is vertical which demands an end bearing, the difficulties of which construction appear to have been met by

Fig. 114.

pumping oil under pressure into the bearing, so that there is complete lubrication without contact of metal on metal. The condenser is placed directly under the turbine, and the electric-generator is above on a continuation of the shaft. The arrangement appears to be convenient, and in particular to demand small floor space only.

When used for marine propulsion the Curtis turbine has a horizontal shaft from necessity, and has a large number of stages.

A turbine developing 8000 horse-power has seven pressure stages, each of which but the first has three velocity stages, that one has four velocity stages. The diameter is ten feet and the peripheral velocity is 180 feet per second.

Tests on Curtis Turbines. — The following tables give tests on two Curtis turbines, having two and four pressure stages, respectively; both were made by students at the Massachusetts Institute of Technology.

TESTS ON A TWO-STAGE CURTIS TURBINE.

DARLING AND COOPER.*

Duration minutes	120	120	120	120	60
Throttle pressure gauge	146.3	145.3	143.2	143.9	149.3
Throttle temperature F.	512	520	464	502	512
Barometer inches	29.8	29.9	29.9	29.9	30.0
Exhaust pressure absolute pounds	0.82	0.79	0.92	0.84	0.85
Load kilowatts	161.4	255.7	374.0	512.9	731.9
Steam per kilowatt hour, pounds	21.98	19.63	19.98	18.43	17.75
Thermal units kilowatt minute	440	396	392	369	357

If the efficiency of the dynamo is taken at 0.9 and one kilowatt is rated as 1.34 horse-power, the steam and heat consumptions per brake horse-power are, for the best result,

11.8 pounds 239 B.T.U.

TESTS ON A FOUR-STAGE CURTIS TURBINE

COE AND TRASK.†

Duration minutes	60	60	60	180	120
Boiler pressure, pounds	152	149.6	152.1	150	150.4
Vacuum inches	28.5	28.2	28.8	28.4	28.3
Load kilowatts	282	380	523	562	788
Steam per kilowatt hour pounds	21.4	20.3	18.8	19.5	19.3
Thermal units per kilowatt (minute)	394	370	352	360	357

* *Thesis*, M. I. T., 1905.

† *Thesis*, M. I. T., 1906.

Taking the efficiency of the dynamo as 0.9 and a kilowatt as 1.34 horse-power, the best result is equivalent to a steam consumption of 12.6 pounds and a heat consumption of 237 thermal units.

Reaction Turbines. — The essential feature of a reaction turbine is a fall of pressure and a consequent increase of velocity in the passages among the vanes of the turbine. Since such wheels commonly are affected by impulse also they are sometimes called impulse-reaction wheels, but if properly understood the shorter name need not lead to confusion. In consequence of the feature named the relative exit velocity V_3 is greater than V_2. Another consequence is that steam leaks past the ends of the vanes which are usually open, and there is also leakage past the inner ends of the guides which are also open; this feature is shown by Fig. 115.

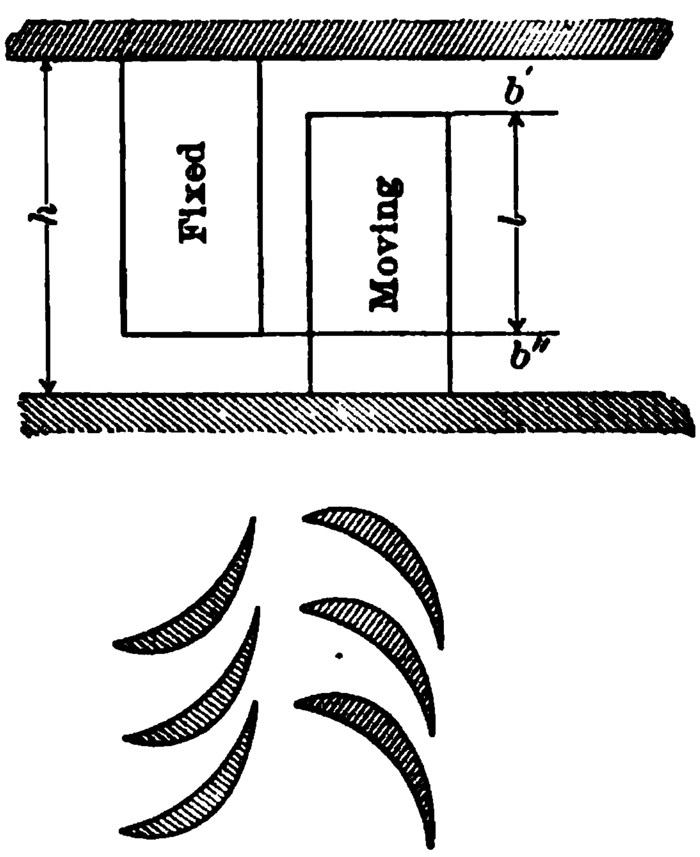

FIG. 115.

The reaction turbine is always made compound with a large number of stages, one set of guides and the following set of vanes being counted as a stage. In consequence the exit pressure either from the guides or the vanes is only a little less than the entrance pressure, and the passages are all converging.

There is no axial thrust because there is no change in velocity of flow; γ is commonly equal to the exit angle α from the guides. A common value for these angles is 20°.

The guides and vanes follow alternately in close succession leaving only the necessary clearance; the kinetic energy due to the absolute exit velocity from a given set of vanes is not lost but is available in the next set of guides. The turbines are usually

made in two or three sections as shown by Fig. 117, page 526, and it is only at the end of a section that the kinetic energy due to the absolute exit velocity is rejected; at the end of a section this kinetic energy is changed into heat and is in a manner available for the next section; at the end of the turbine it is of course wasted. Since there are usually sixty stages or more the influence of the kinetic energy rejected is likely to be less than five per cent and it may properly be combined with the general factor to allow for friction and leakage past the ends of the guides and vanes. Both influences reduce the change of heat into work applied to the turbine and increase the value of the quality x and also of the specific volume of the mixture of steam and moisture.

Since the exit absolute velocity from the vanes is applied to driving the steam into the next set of guides, there is no direct advantage in avoiding velocity of whirl at this place; it is only necessary to give the guides the proper angle at entrance to receive the steam. Indirectly it is disadvantageous to have a high velocity at the entrance to the guides, or, for that matter, in any part of the turbine, as the friction is probably proportional to the square of the velocity as has been assumed in the use of the friction factor y.

The steam enters a set of guides with a certain velocity, i.e., the exit absolute velocity from the preceding set of vanes. On account of the loss of pressure in the guides a certain amount of heat is changed into kinetic energy and the equivalent increase of velocity may be added to the entrance velocity to find the exit velocity which is of course an absolute velocity. This absolute velocity combined with the velocity of the blades gives the relative entrance velocity to the vanes. To this entrance velocity is to be added the gain in velocity due to change of heat into kinetic energy in the vanes, in order to find the relative exit velocity. The ratio of the heat used in the vanes to that used in the entire stage is called the degree of reaction. Commonly the degree of reaction is one-half; that is, the amount of heat used in the vanes is equal to that used in the guides; and

the gain of velocity in the vanes is equal to the gain in the guides.

In Fig. 116 let V_1 be the velocity of the steam leaving the guides and V the velocity of the vanes; then V_2 is the relative velocity of the steam entering the vanes. V_3 is the relative exit velocity which is greater than V_2 on account of the change of heat into work. V_4 is the absolute exit velocity from the vanes with which the steam enters the next set of guides. If the conditions for successive stages are the same, V_4 is also equal to the entrance velocity to the set of guides of the stage under discussion, and if *ce* is laid off at *ac'* then *c'b* is the gain of velocity in the

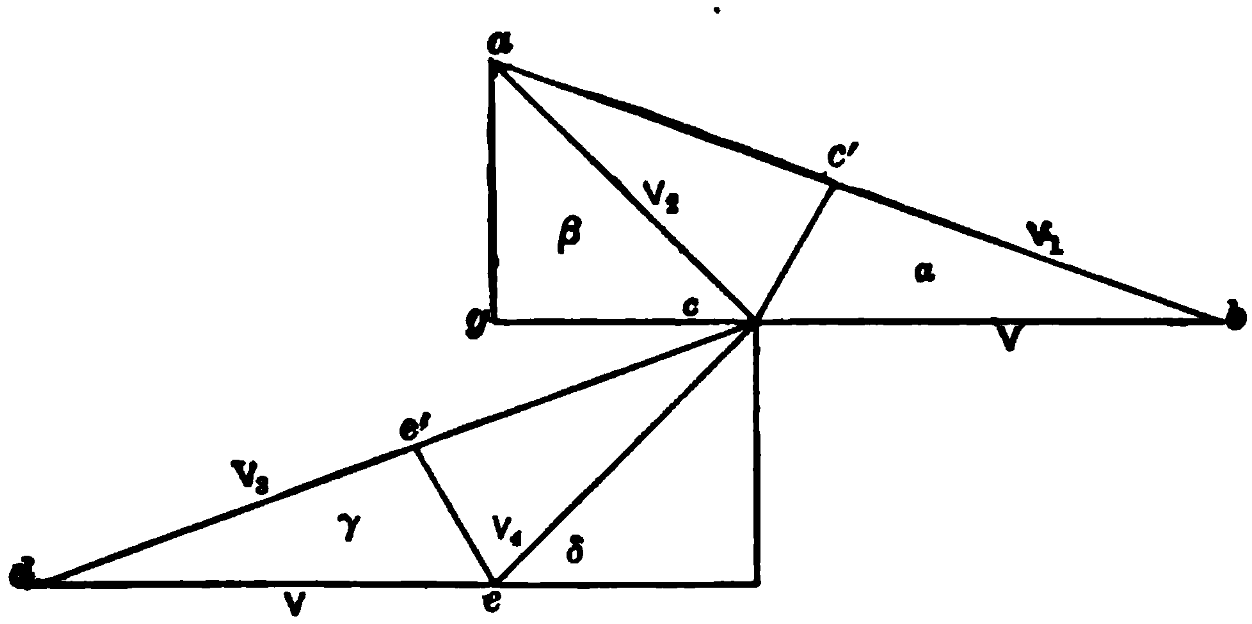

FIG. 116.

guides. Consequently to construct V_3 we may lay off *ce'* equal to *ac* and *e'd* equal to *c'b*. Now α and γ are commonly made equal, and therefore the triangles *abc* and *cde* are equal. Consequently the angle δ for the entrance to the guides is equal to β at the entrance to the vanes. In fact the guides and vanes have the same form.

Choice of Conditions. — The foregoing discussion shows that the designer is given a wider latitude in his choice of conditions for the compound reaction turbines than appeared possible for impulse turbines, though if the restriction of no axial thrust were removed from the latter the comparison would be quite different.

The most authoritative statement of the preferable conditions in practice for reaction turbines of the Parson's type is formed in a paper by Mr. E. M. Speakman,* but much of the information in the hands of the builders "being based on long and costly experiments, much reticence is observed regarding their publication." The statement of practical conditions is therefore based on such information as can be gleaned from his paper, with obvious applications by ordinary methods. Factors for friction and leakage are largely conjectural, as must in fact be the case at present for all turbines, and for our purpose may perhaps be limited to giving the student an idea of the nature of the problems.

The ratio of the velocity of the vanes to the velocity of the steam has varied in turbines built by the Parsons Company from 0.25 to 0.85. In general the ratio may be taken as 0.6.

These turbines are usually built with two or three diameters of the revolving cylinder or rotor as shown in Fig. 117. The following tables give the practice of that company with regard to peripheral speed and number of stages.

PARSONS TURBINES — ELECTRICAL WORK.

Normal output kilowatts.	Peripheral speed, feet per second.		Number of stages.	Revolutions per minute.
	First expansion.	Last expansion.		
5000	135	330	70	750
3500	138	280	75	1200
2500	125	300	84	1360
1500	125	360	72	1500
1000	125	250	80	1800
750	125	260	77	2000
500	120	285	60	3000
250	100	210	72	3000
75	100	200	48	4000

* *Trans. Inst. Eng. and Shipbld.*, Scot., vol. xlix, 1905–06.

PARSONS TURBINE — MARINE WORK.

Type of vessel.	Peripheral speed, feet per second.		Ratio of velocities, vanes to steam.	Number of shafts.
	H.P.	L.P.		
High speed mail steamers	70–80	110–130	0.45–0.5	4
Intermediate mail steamers	80–90	110–135	0.47–0.5	3–4
Channel steamers	90–105	120–150	0.37–0.47	3
Battleships and large cruisers . . .	85–100	115–135	0.48–0.52	4
Small cruisers	105–120	130–160	0.47–0.5	3–4
Torpedo crafts	110–130	160–210	0.47–0.51	3–4

The Westinghouse Company have used much higher velocities of vanes for electrical work than given in the above tables; as much as 170 feet per second for the smallest cylinder and 375 for the largest cylinder.

The blade height should be at least three per cent of the diameter of the cylinder in order to avoid excessive leakage over the tips. Mr. Speakman says that leakage over the tips of the blades is perhaps not so detrimental on account of actual loss by leakage as because it upsets calculations regarding passages by increasing the steam volume.

The following equation represents Mr. Speakman's diagram for clearances over tips of vanes,

$$\left.\begin{array}{l}\text{clearance in}\\ \text{inches}\end{array}\right\} = 0.01 + 0.008 \text{ diam. in feet.}$$

The proportions of blades may be taken from the following table:

PROPORTIONS OF BLADES — INCHES.

Height	1	2	3	4	6	8	10	12	15	18	21	24	30
Width	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{3}{4}$	1	1	$1\frac{1}{8}$	$1\frac{1}{4}$
Pitch	$1\frac{1}{8}$	$1\frac{1}{8}$	$1\frac{1}{4}$	$1\frac{3}{8}$	$1\frac{3}{4}$	$2\frac{1}{8}$	$2\frac{1}{4}$	$2\frac{1}{2}$	$2\frac{5}{8}$	$3\frac{1}{8}$	$3\frac{1}{4}$	$3\frac{5}{8}$	4
Axial clearance	$\frac{3}{16}$	$\frac{3}{16}$	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{11}{16}$	$\frac{3}{4}$

Mr. Parsons * gives for the efficiency of the steam in the turbine blades themselves 0.70 to 0.80.

* *Inst. Naval Arch.*, 1903.

In addition to the leakage past the tips of the blades which cannot in practice be separated in its effects from friction, there is likely to be a considerable leakage past the balance pistons which will be described in connection with Fig. 117. This leakage is in the end direct to the condenser, and no account need be taken of it in the design of the blading of the turbine; but allowance should be made in comparing theoretical calculations with results of tests.

Design for a Reaction Turbine. — Let us take for the principal conditions the delivery of 500 kilowatts of electrical energy, which with an efficiency of the dynamo of about 0.87 will correspond to 770 brake horse-power, as for the calculation on page 506. Let the initial pressure be 150 pounds by the gauge, and the vacuum be 28 inches. The absolute pressures corresponding are 165 pounds and one pound, and the temperatures are 365°.9 and 102° F. The calculation referred to gives for the thermal efficiency of adiabatic action 0.285, which corresponds to 149 B.T.U. per horse-power per minute. If we allow 0.60 for the turbine efficiency, and ten per cent for leakage to the condenser and radiation, and take 0.9 for the mechanical efficiency we shall have for the combined efficiency of the turbine

$$0.288 \times 0.60 \times 0.9 \times 0.9 = 0.140.$$

This will give for the heat and steam consumption per horse-power, 16.3 pounds per hour and 305 B.T.U. per minute. These are to be compared with results of tests to determine whether the constants assumed are proper.

For the estimate of the weight of steam to be used in determining the dimensions of the turbine we should omit the factor for leakage to condenser and radiation, which will give for the steam per horse-power per hour 14.7 pounds. The weight of steam per second to be used in computing passage therefore becomes

$$w = 14.7 \times 770 \div 3600 = 3.15 \text{ pounds.}$$

Let the peripheral speed of the smallest cylinder be taken as 148 feet per second, and let the intermediate and low-pressure

cylinders be $1\frac{1}{2}$ and $2\frac{1}{2}$ times the diameter of the small cylinders. Let the peripheral speed be 0.75 of the steam velocity, then the latter will be 300 feet per second. If the exit angles for guides and vanes be taken as 20° and if the degree of reaction is 0.5, the velocities and angles will be represented by Fig. 116, page 517. In that figure

$$gb = V_1 \cos 20^\circ = 0.940\ V_1;$$

and as $\quad V$ is $0.75\ V_1$,

we have $\quad gc = (0.940 - 0.75)\ V_1 = 0.190\ V_1.$

But $\quad ag = V_1 \sin 20^\circ = 0.342\ V_1;$

and $\quad \tan \beta = 0.342 \div 0.190 = 1.800 \quad \therefore \quad \beta = 61^\circ.$

The angle β is given to the *backs* of the blades, and the angle at the faces is somewhat larger, as will appear by Fig. 115, page 515; in consequence there is some impulse at the entrance to the vanes.

To get the relative velocity V_2 we have from Fig. 116

$$V_2^2 = \overline{ag}^2 + \overline{gc}^2 = (\overline{0.342}^2 + \overline{0.190}^2)V_1^2.$$

$$\therefore \quad V_2^2 = 0.1531\ V_1^2.$$

Now the absolute exit velocity V_4 from the wheel is the entrance velocity to the next guides, and further the exit velocity V_4 is equal to the relative velocity V_2. The kinetic energy of the steam entering the guides or the blades is therefore equal to

$$V_2^2 \div 2g = 0.1531\ V_1^2 \div 2g,$$

so that the increase in kinetic energy is

$$\frac{V_1^2}{2g} - \frac{0.1531\ V_1^2}{2g} = \frac{0.8469\ V_1^2}{2g};$$

and this is to be equated to the heat expended, so that

$$0.8469\ V_1^2 = 2 \times 32.2 \times 778 \times h.$$

$$\therefore \quad h = 0.8469 \times \overline{198}^2 \div (64.4 \times 778) = 0.66.$$

This is the amount with allowance for friction and leakage past the ends of the blades which has been assigned the factor 0.6, so that for the preliminary adiabatic computation we may take for one set of blades

$$0.66 \div 0.6 = 1.1 \text{ B.T.U.}$$

and for a stage, consisting of a set of guides and vanes, we may take for the basis of the determination of the proper number of stages 2.2 B.T.U. per pound of steam used.

It appears on page 507 that adiabatic expansion from 165 pounds absolute to one pound absolute gives 322 thermal units for the available heat. If this is to be distributed to the stages of a turbine with 2.2 units per stage, then the total number of stages will be

$$322 \div 2.2 = 146$$

stages. This is under the assumption that the turbine has a uniform diameter of rotor with 225 feet for the velocity of the vanes; we have, however, taken the intermediate diameter $1\frac{1}{2}$ times the high-pressure and the low-pressure $2\frac{1}{2}$ times. The peripheral velocities will have the same ratios, and the amounts of available heat per stage will be proportional to the squares of those ratios, namely, 2.25 and 6.25. Consequently the amounts of heat assigned per stage will be as follows:

High-pressure	Intermediate	Low-pressure.
2.2	4.95	13.75

If we decide to use ten low-pressures and twenty intermediate stages they will require

$$10 \times 13.75 + 20 \times 4.95 = 236.5 \text{ B.T.U.},$$

leaving 85.5 thermal units which will require somewhat less than 39 stages. Reversing the operation it appears that one distribution calls for

$$10 \times 13.75 + 20 \times 4.95 + 39 \times 2.2 = 322 \text{ B.T.U.}$$

For convenience of manufacture it is customary to make several stages identical, that is, with the same length of blades, clearances, etc.; this of course will derange the velocities to some extent and interfere with the realization of the best economy. That part of the cylinder which has the same length of blades is known technically as a *barrel*. Let there be three barrels for each cylinder, making nine in all, which may be conveniently numbered, beginning at the high-pressure end and may have

the number of stages assigned above. In that table is given also the number of the stage counting from the high-pressure end, which is at or near the middle of the length of the barrel, for which calculations will be made. The values of the heat contents $xr + q$ are readily found for each stage given in the table by subtracting the amounts of heat changed into kinetic energy, down to that stage, allowing 2.2 for each stage of the high-

COMPOUND REACTION TURBINE.

Cylinders.	Barrels.	Number of stages.	Middle stage.	Temperature. t	Pressure. p	Heat contents adiabatic. $xr+q$	Heat contents probable. $x'r+q$	Heat of liquid. q	Heat of vaporization. r	Quality. x	Specific volume. s	Specific volume. v	Length of blades.
I—	1—	14	7	351	136.3	1177.9	1184	322	869	.992	3.30	3.27	0.51
	2—	13	20	324.5	95.4	1149.3	1167	295	890	.980	4.62	4.53	0.71
	3—	12	33	298.5	65.5	1120.7	1150	268	910	.969	6.60	6.40	1.00
II—	4—	8	43	270	41.8	1087.7	1130	239	931	.957	10.05	9.62	0.67
	5—	6	50	240.5	25.2	1053.0	1109	209	952	.945	16.2	15.3	1.06
	6—	6	56	216	15.9	1023.3	1091	184	967	.938	24.9	23.4	1.62
III—	7—	4	61	183	8.02	981.0	1066	151	988	.926	47.1	43.6	1.09
	8—	3	65	141	2.96	926.0	1033	109	1013	.912	120	109.5	2.73
	9—	3	68	111.5	1.33	884.7	1008	80	1029	.903	255	230	5.74

pressure cylinder, 4.95 for each intermediate stage and 13.75 for each low-pressure stage. For example, the fiftieth stage has its heat contents found by subtracting from the initial heat contents 1193.3, the amount

$$39 \times 2.2 + 11 \times 4.95 = 140.3,$$

leaving for the heat contents after that stage 1053 thermal units. The probable heat contents allowing for friction and leakage is found by subtracting the product of the above quantity by the factor 0.6. Giving

$$1193.3 - 140.3 \times 0.6 = 1109 \text{ B.T.U.}$$

Having the values of $x'r + q$ obtained in this way, the values of x' can be found by subtracting the heat of the liquid q, and

dividing the remainder by r. Finally the specific volumes are computed by the equation

$$v = x'u + \tau;$$

but in practice σ may be neglected giving

$$u = x's$$

because we have either x nearly equal to unity or else s will be larger compared with σ.

The steam velocity for the first cylinder is 198 feet per second, the weight of steam per second is 3.15 pounds and the specific volume at the seventh stage, i.e., the middle of the first barrel, is 3.27 cubic feet. The effective area must therefore be

$$a = 144\frac{wv}{V} = 144\frac{3.15 \times 3.27}{198} = 7.48 \text{ square inches.}$$

To this must be added a fraction of one-third or one-fourth to allow for the thickness of the blades, and the result must be divided by sine α in order to find the area of the peripheral ring through which the steam will flow. Taking one-fourth for the fraction in this case, and 20° for α, we have

$$\frac{7.48 \times 5}{0.342 \times 4} = 27.4 \text{ square inches.}$$

It is recommended that the height of the blades shall be 0.03 of the diameter, which gives for the expression for the peripheral ring

$$0.03\ \pi d^2 = 27.4.$$

$$\therefore\ d = \sqrt{27.4 \div 0.03\ \pi} = 17.02 \text{ inches.}$$

The diameters of the intermediate and low-pressure cylinders will be

$$d_1 = 17.02 \times 1.5 = 25.5 \text{ in.};\ d_2 = 17.02 \times 2\tfrac{1}{2} = 42.6 \text{ in.}$$

The length of blade at the seventh stage will be

$$0.03 \times 17.02 = 0.51 \text{ inch,}$$

and this length will be assigned to all the blades of the first barrel. The blades of the second and third barrels will have their lengths increased in proportion to the specific volumes at the middle of those barrels, as set down in the table. The effect of increasing the diameters of the intermediate and low-pressure cylinders is to increase the steam velocity, and the peripheral length of the steam passage, both in proportion to the diameter. Consequently the lengths of the blades for these cylinders are directly proportional to the proper specific volumes and inversely proportional to the squares of the diameters. Thus the length of the blades at the forty-second stage, i.e., the middle of the fourth barrel is

$$\frac{0.51 \times 9.62}{3.27 \times \overline{1.5}^2} = 0.67 \text{ inch.}$$

The lengths are computed for the other barrels in the same way, using 2.5 for the ratio of the low-pressure diameter.

Since the diameter of the small cylinder is 13.85 inches and the speed of the vanes on it is 225 feet per second, the revolutions per minute are

$$\frac{148 \times 60 \times 12}{17.02\,\pi} = 2000 \text{ nearly.}$$

Parsons Turbine. — The essential features of the Parsons turbine are shown by Fig. 117. Steam is admitted at A and passes in succession through the stages on the high-pressure cylinder, and thence through the passage at E to the stages of the intermediate cylinder; after passing through the intermediate stages it passes through G to the low-pressure stages and finally by B to the condenser.

The axial thrust is counterbalanced by the dummy cylinders, C, C, C, the first receiving steam from the supply directly, the second from the passage between the high and intermediate cylinders through the pipe F, and the third through the pipe near G from the passage between the intermediate and low-pressure cylinders. Leakage past the dummy cylinders is checked by labyrinth packing, which is variously arranged to give a succession

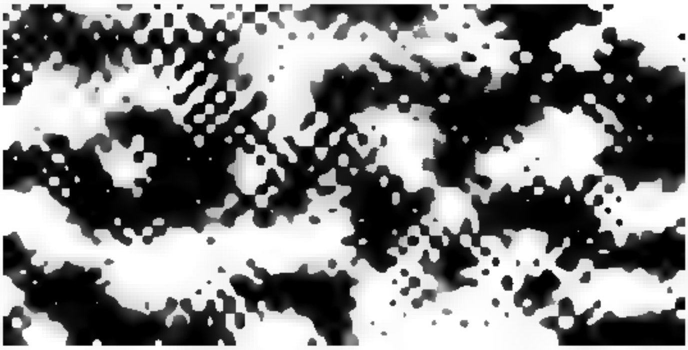

FIG. 117.

of spaces through which the steam must pass with narrow passages, which throttle the steam as it passes from chamber to chamber. One method is to let narrow strips of brass into the surface of the cylinder and into the surface of the case; these strips are adjusted to leave a very small axial clearance, so that the steam is strongly throttled as it passes through. It is reported that the labyrinth clearance is entirely successful in reducing the leakage past the dummy cylinder to a small amount. It is pointed out by Mr. Jude that the most effective throttling is at the last section of the labyrinth, and that the other sections are comparatively inefficient. This feature will be evident if an attempt is made to calculate the loss by continual application of Rankine's equations, page 432. Of course such a method can be but crude, and yet its indications should be of value for estimating leakage which should be small.

When applied to marine propulsion the dummy pistons are omitted and the axial thrust is usefully applied to the propeller-shaft. Since an absolute balance cannot be obtained, a thrust-bearing is provided but it may have small bearing area and will have but little friction. Stationary turbines also have a bearing for residual unbalanced thrust.

Test on a Parsons Turbine. — A test on a Westinghouse-

Parsons turbine in Savannah was made under the direction of Mr. B. R. T. Collins and reported by Messrs. H. O. C. Isenberg and J. Lage,* which is interesting because the steam consumption of the auxiliary machines was determined separately. The data and results of tests on the turbine are given in the following table.

The tests made at full load with varying degrees of vacuum show clearly the advantage obtained in this machine from a good vacuum, which amounted to a saving of

$$\frac{289 - 279}{289} = 0.035.$$

TESTS ON WESTINGHOUSE-PARSONS TURBINE.

COLLINS, ISENBERG AND LAGE.

	½ load.	¾ load.	Full load.			1¼ load.	1½ load.
Duration minutes	60	60	60	60	60	45	45
Steam pressures, gauge	131	129	128	127	128	127	125
Vacuum inches	28.1	28.1	25.7	26.7	28.0	26.7	26.6
Revolutions per minute	3616	3601	3602	3612	3562	3540	3537
Load kilowatts	260	379	493	501	499	629	733
Steam consumption, pounds							
per kilowatt-hour	24.3	21.2	20.7	19.8	19.7	19.8	20.2
per electric h.p. per hour	18.1	15.8	15.6	14.8	14.7	14.7	15.1
Heat consumption B.T.U.							
per kilowatt-minute	462	403	494	375	374	373	381
per horse-power per minute	345	301	289	284	279	278	283

A great importance is attributed by turbine builders to obtaining a low vacuum, in many cases special air-pumps and other devices being used for that purpose. Unless discretion is shown both in the design and operation of this auxiliary machinery, its size and steam consumption is likely to be excessive, and what appears to be gained from the vacuum may be entirely illusory.

* *Thesis*, M.I.T. 1906.

The steam consumption in pounds per hour for the several auxiliary machines was as follows:

Centrifugal pump for circulating water	881
Dry vacuum pump	212
Hot-well pump	42.8
	1135.8

This total was equivalent to 0.115 of the steam consumption of the turbine at full load and with 28 inches vacuum. Some tests of turbine installations show twice or three times this proportion.

Effect of Friction on Entropy. — Attention has been called to the fact that the effect of friction is to increase the entropy of steam, and that in consequence any method of designing compound turbines which depends on determination of intermediate temperatures and pressures at the initial entropy is liable to appreciable error.

In the discussion of the problem on page 506 for a two-stage turbine, the total available adiabatic heat is found to be 322 B.T.U., which being divided into equal parts assigns 161 units per stage. The intermediate temperature is found to be 223°, corresponding to 18.2 pounds absolute. But the effect of internal steam friction and other conditions is to reduce the heat actually changed into work to

$$0.883 \times 0.85 \times 0.75 \times 161 = 0.563 \times 161 = 90.6 \text{ B.T.U.}$$

The entropy is thus increased to 1.66 at which the adiabatic heat for the second stage is found to be

$$1100 - 927 = 173 \text{ B.T.U.}$$

In consequence the velocity of steam from the second set of nozzles is computed to be 2710 feet per second instead of 2610, as previously calculated.

It is evident that the intermediate temperature and pressure must be reduced if the two stages are to have the same peripheral velocity. A crude way of doing this would be to take the mean

of 161 and 173, namely, 167 units for determining the intermediate temperature. The heat contents then become

$$1193 - 167 = 1026,$$

which at 1.56 entropy corresponds to 219°.5 or 17 pounds. A step-by-step process will now show less discrepancy, and a third computation would be satisfactory; but such a cut-and-try method becomes very tedious when there are a large number of stages. By the aid of the entropy table it is possible to make a direct determination of the proper intermediate temperatures, which shall give concordant results from the step-by-step calculation.

Internal Heat Factor. — In the preliminary computation, on page 506, three factors were assigned, one (0.85) to allow for the friction in the nozzle, another (0.883) to take account of the angles of guides and blades, and a third (0.75) to take account of steam friction. A more precise analysis of the action of the steam shows that the latter two cannot be separated as in the preliminary calculation, but the general conception is sufficient for its use in that place. The product of these three factors,

$$0.85 \times 0.883 \times 0.75 = 0.563,$$

may be called the internal heat factor. It is also the ratio of the steam per horse-power per hour computed by Rankine's cycle, to the actual steam per turbine horse-power per hour; the latter corresponding roughly to the steam per indicated horse-power per hour for a reciprocating engine, which is the proper basis for heat comparisons. In the discussion on page 507, the factor 0.9 is used to allow for mechanical efficiency, leading to the determination of 15.7 as the steam per brake horse-power per hour, to which some addition is suggested for leakage and radiation.

Starting with the actual steam-consumption per brake horse-power as determined by a test on a turbine already installed, we may allow for radiation and gland leakage and for mechanical efficiency and so estimate the steam per turbine horse-power per hour. It will appear that considerable uncertainty in the factors assigned for this purpose will have but little effect on the deter-

mination of pressures. If it be assumed that such a test shows that 16.5 pounds of steam are required per turbine horse-power per hour, and if we assign 0.95 to allow for radiation and gland leakage and 0.9 for the mechanical efficiency we shall have

$$16.5 \times 0.95 \times 0.9 = 14.1$$

for the turbine steam consumption. The thermal units per horse-power per minute for Rankine's cycle were found to be 149; which corresponds to

$$149 \times 60 \div 1123 = 7.96$$

and $$7.96 \div 14.1 = .564$$

which corresponds sufficiently with the value previously assigned.

Having the value assigned to the internal heat factor we may determine that of the available adiabatic heat only

$$322 \times 0.563 = 181.4$$

will finally be changed into work per pound of steam. And if this be equally divided between two stages the heat yielded in each stage per pound of steam will be

$$181.4 \div 2 = 90.7$$

thermal units. The amount of heat changed into work per pound of steam being based on the actual steam consumption of the turbine, when once assigned by aid of the proper factors, must be accepted for all consequent computations.

Method for Determining Intermediate Pressures. — This method for determining the intermediate temperatures and corresponding pressures for a turbine with several pressure stages is to be justified by its convenience and concordance with a step-by-step calculation. It can be stated best by aid of an example. The same pressure will be chosen as on page 506, but the computation will be carried to the degree of precision possible with the entropy table, as is necessary for the best results. At 1.56 entropy, and at

temperatures 366° and 102° the heat contents and their difference are

$$1193.3 - 871.1 = 322.2.$$

Let this total available heat be divided into four equal parts which will give three intermediate temperatures, or five temperatures in all, as shown in the following table:

Heat contents, entropy 1.56.	Temperature.	Heat contents.	Differences.
1193.3 80.6	366	...	...
1112.7	292 $\begin{cases} 302 \\ 282 \end{cases}$	1124.4 1101.8	22.6
80.6 1032.1	223 $\begin{cases} 233 \\ 213 \end{cases}$	1043.8 1019.3	24.5
80.6 951.5	160 $\begin{cases} 170 \\ 150 \end{cases}$	964.2 937.7	26.5
80.6 870.9	102 $\begin{cases} 112 \\ 92 \end{cases}$	885.4 856.9	28.5

At each temperature, except the first, the variation of heat contents for twenty degrees is determined; for example, at 302° the heat contents is 1124.4 and at 282° it is 1101.8, giving a difference of 22.6 B.T.U.

The next step is to find the entropy of the steam at the several temperatures with an internal heat factor which can be taken as 0.563 for a convenient mean value.

For this purpose the amount

$$80.6 \times 0.563 = 45.4$$

is subtracted four times successively, and the nearest entropy column is sought in the entropy table. The variation is determined again in the entropy column.

Temperature.	Heat contents.	Nearest entropy.	Heat contents.	Differences.
366	1193.3	1.56	...	...
	45.4			
292	1147.9	1.61 {302	1162.5	
		282	1138.8	
	45.4			23.7
223	1102.5	1.66 {233	1113.1	
		213	1086.5	
	45.4			26.6
160	1057.1	1.73 {170	1071.2	
		150	1041.3	
	45.4			29.9
102	1011.7	1.81 {112	1028.2	
		92	994.7	
				33.5

The differences in this last table are divided by those in the preceding table, and the ratios are set down in the following table. The computation for the first step is omitted, as it would evidently give unity for the ratio.

Ratio of differences.	Mean value.	Factor.
$1.000 \times \frac{1}{2}$	$= 0.500$	1.087
1.049	1.049	
1.086	1.086	1.000
1.128	1.128	
$1.175 \times \frac{1}{2}$	$= 0.587$	0.913
	$4/\underline{4.350}$	
	1.087	

The mean value of the ratio is now to be obtained by taking the sum of half of the extreme values and all the other values and dividing by four. The factors in the last column are the mean value thus obtained, unity and

$$1 - 0.087.$$

The direct solution for intermediate temperature and pressure can now be made by aid of Fig. 118, in which h_1h_2 represents the

total available adiabatic heat. At h_1 is laid off the abscissa 1.087, and at h_2, 0.913 from h_1h_2 as the base unity, and the diagonal line is drawn as shown. Now divide the line h_1h_2 into portions representing the heat that would be assigned to each of the stages of a turbine having adiabatic action. In this case, since the turbine has the same pitch diameter for the two stages and since, consequently, the steam velocity at discharge from the nozzles must be the same, the line h_1h_2 may be bisected at o. When the pitch diameters are unequal the heat assignments would be unequal, and the line h_1h_2 would be divided unequally. At the midpoints of the sections abscissæ are drawn and measured; they give in the figure the factors 1.043 and 0.957. The total available adiabatic heat, 322.2 units, is now divided into equal parts, giving 161.1 for each. These parts are now to be multiplied by the factors just obtained giving

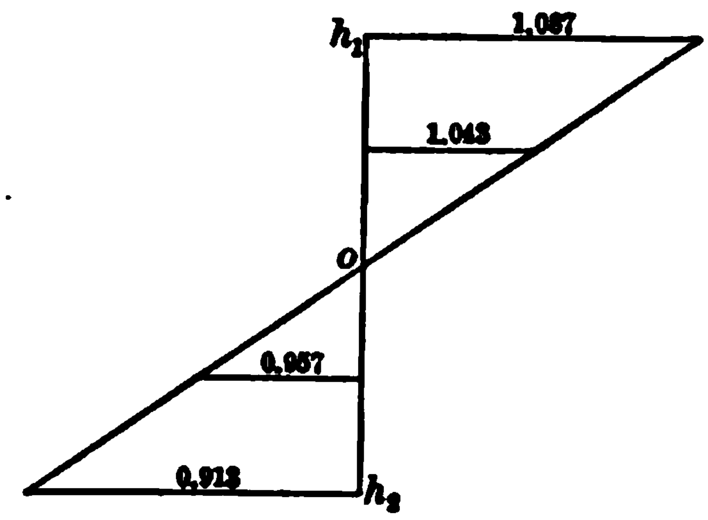

FIG. 118.

$$\begin{array}{lll} \text{First stage} & 161.1 \times 1.043 & = 168.0 \\ \text{Second stage} & 161.1 \times 0.957 & = \underline{154.2} \\ & & \ 322.2 \end{array}$$

The intermediate temperatures are found as follows:

Heat contents, entropy 1.56.	Temperature.	Pressure.
1193.3 168.0	366	164.8
1025.3 154.2	218	16.5
871.1	102	1.0

Making a calculation like that on page 512 we have for the heat applied to driving the turbine in each stage 90.7 thermal units.

The heat contents of the steam approaching the second set of nozzles is

$$1193.3 - 90.7 = 1102.6.$$

This is found most nearly at entropy 1.67 for the temperature 218°. The available heat for adiabatic action is

Temperature	218°	Heat contents	1099.9
	102°		932.9
			167.0

The difference between this figure and that assigned at entropy 1.56 to the first stage is 0.6 per cent. The probable error of this method, depending as it does on the precision of the entropy table, is about the same under all conditions. The percentage of error is, therefore, larger when there are many stages. But for turbine having many stages the computations are conveniently made at intervals so that the error need never be important.

Variation of Ratios. — To show the effect of varying the heat factor for internal operations the following table has been computed.

(Ratios to allow for influence of increase of entropy, due to steam friction. One hundred and fifty gauge pressure to 28 inches vacuum.)

Heat factor	0.55	0.60	0.65	0.70	0.75
Ratios, initial	1.090	1.078	1.065	1.058	1.050
Midpoint	1.000	1.000	1.000	1.000	1.000
Final	0.910	0.922	0.935	0.942	0.950

This table shows that probable variations in the heat factor have little effect on the determination of intermediate temperatures and pressures. The same fact is brought out in the following statement of the intermediate temperature for the two-stage turbine, discussed in the preceding section.

Heat factor	0.55	0.65	0.75
Intermediate temperature	218	219	220
Pressure	16.5	16.9	17.2

Application to Six-stage Turbine. — To further illustrate the method of determining temperatures the following application is made to a turbine with six equal stages. The factors are determined by aid of Fig. 119 for a heat factor 0.65. The total available heat is divided into six parts, using, as before, 150 pounds gauge pressure and a vacuum of 28 inches. The mean assignment per stage is therefore

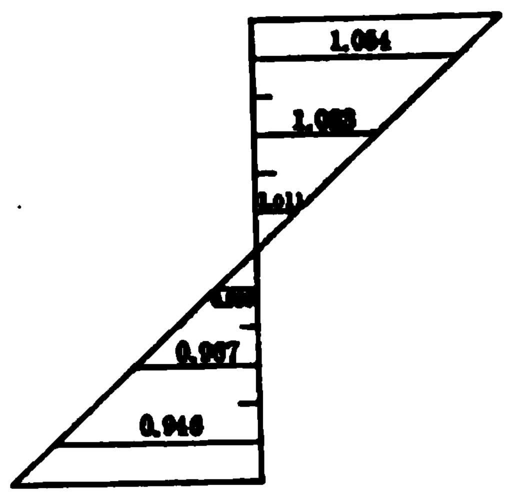

FIG. 119.

$$(1193.3 - 871.1) \div 6 = 53.7.$$

The heat assignments at entropy 1.56 are

$$53.7 \times 1.054 = 56.6$$
$$53.7 \times 1.032 = 55.4$$
$$53.7 \times 1.011 = 54.3$$
$$53.7 \times 0.989 = 53.1$$
$$53.7 \times 0.968 = 52.0$$
$$53.7 \times 0.946 = 50.8$$

The computation of the intermediate temperatures and the check computation are given in the following table:

Heat contents, entropy 1.56	Temperatures.	Pressures.	Heat contents, factor 0.65.	Entropy.	Heat drop.
1193.3	366.0	164.8	1193.3	1.560	1193.3
56.6			34.9		1136.7
					56.6
1136.7	313.0	80.1	1158.4	1.588	1158.4
53.4			34.9		1101.5
					56.9
1081.3	264.3	38.1	1123.5	1.618	1123.5
54.3			34.9		1066.4
					57.1
1027.0	219.3	16.9	1088.6	1.651	1088.6
53.1			34.9		1031.9
					56.7
973.9	177.4	7.09	1088.7	1.685	1053.7
52.0			34.9		996.7
					57.0
921.9	138.3	2.76	1053.8	1.722	1018.8
50.8			34.9		962.1
					56.7
871.1	102.0	1.00	983.9	1.761	

In the check calculation the heat changed into work per stage is taken to be

$$53.7 \times 0.65 = 34.9.$$

This quantity subtracted, successively gives the heat contents of the steam approaching the several sets of nozzles. The corresponding entropies are found by interpolation in the entropy table. At these entropies the heat contents are found for the initial and final temperatures for the several stages; the differences are the available heats for adiabatic action in these stages. The variation of the heat drops from the assigned value 56.6 is a measure of the precision of the method.

In this sample computation the work is carried to the limit of precision of the entropy table. Usually it will be sufficient to take the nearest temperature from that table and to make the check computation in the nearest entropy column, thus avoiding the labor of interpolating, which is considerable when cross-interpolation is undertaken. Occasionally it may be worth while to work to half degrees of temperature.

The discussion of the properties of steam in Chapter VI shows that our knowledge is not more precise than is represented by a single thermal unit; it is convenient to have tenths of units in our steam tables, and such a degree of precision is required in order to use the method explained for determining intermediate temperatures for compound turbines. The method can be depended on to give concordant results, and the check calculation is valuable mainly, as it gives a good arithmetical check on the calculation.

Unequal Stages. — In the examples given of the method of determining intermediate temperatures, the several stages have been assigned equal amounts of heat. If the amounts of heat assigned are unequal, the line h_1h_2, of Fig. 118, is to be divided correspondingly, and the abscissæ are to be drawn at the mid-points to determine the ratios. The check calculation is to be carried out with the true amount of heat changed into work for each stage, obtained by multiplying the adiabatic assignment by the heat factor.

Heat Factors; Overall, and per Stage. — The heat factors used in the method for determining the pressures and temperatures for a turbine are made to depend on the comparison of the steam per horse-power per hour for Rankine's cycle and the steam per *turbine* horse-power. This may be called the overall heat factor. Though its precise determination may be difficult it has been shown that a fair approximation is all that is required for determining pressures.

When we come to analyzing the heat losses in a stage of a turbine another heat factor is required. This is evident from the consideration that on page 533 the heat assigned to each stage of a two-stage turbine is 168 thermal units; while the heat changed into work is only 90.7 units. This gives a ratio

$$90.7 \div 168 = 0.540$$

instead of 0.563, which was taken for the overall factor. The factor can be obtained also as follows:

$$0.563 \div 1.043 = 0.540.$$

This factor can be taken to be the product of factors (1) for the nozzle, (2) for the action of the blades and girders, and (3) for the steam friction on the disk, and other similar effects. If a factor 0.85 be assigned to the nozzle, a factor 0.75 to the wheels, etc., and a factor 0.75 to the friction of the disk and similar actions, the product becomes

$$0.85 \times 0.75 \times 0.85 = 0.542.$$

An intelligent discussion of these matters can be had only with special data from experiments on turbines and on single stages of turbines.

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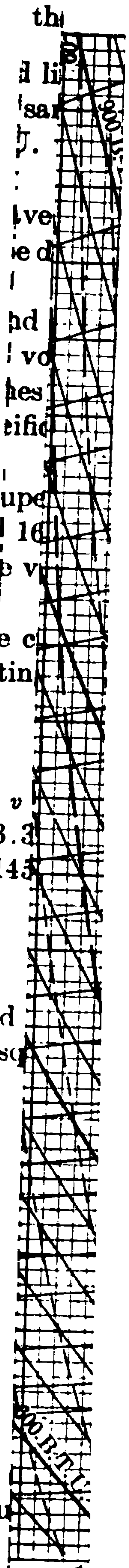

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